Warped Brane World and Flux Compactification

Subir Mukhopadhyay

Workshop on physics of warped extra dimensions,
February 23, 2008.

Introduction-1

- Two fundamental scales of nature are very disparate,

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- There are several known mechanisms to explain this hierarchy: Dimenstional Transmutation, Non-perturbative effects, (instantons whose contribution is exponentially small in inverse coupling), large extra dimensions etc.
- Randall and Sundrum [hep-ph/9905221,hep-th/9906064] proposed a scenario where space-time metric is a warped product rather than a direct product

$$ds^2 = e^{A(y)} ds_4^2 + g_{yy}(y) dy^2, \tag{1}$$

where ds_4^2 is the four dimensional metric.

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- How this warped Braneworld scenario can be incorporated in String theory?

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$$ds_5^2 = e^{-2kr_c\phi} ds_4^2 + r_c^2 d\phi^2. \quad (2)$$
- They place two branes at two fixed points $\phi = 0$ and $\phi = \pi$ and call them hidden brane and visible brane respectively.

RS model-2

- The metrics on these two branes will differ by a redshift factor:

$$g^{\text{vis}}_{\mu\nu} = G_{\mu\nu}|_{\phi=\pi}, \quad g^{\text{hid}}_{\mu\nu} = G_{\mu\nu}|_{\phi=0}. \tag{3}$$

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- Four dimensional Planck scale depends on r_c only weakly in large kr_c limit.

$$M_4^2 = \frac{M_5^3}{k}(1 - e^{-2\pi k r_c}) \tag{4}$$

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- The matter action on visible brane,

$$S = \int d^4x \sqrt{-\tilde{g}} \left(\tilde{g}^{\mu\nu} e^{2\pi k r_c} (D_\mu H)^\dagger (D_\nu H) - \lambda(|H|^2 - v_0^2)^2 \right). \tag{5}$$

RS model-3

- Redefine Higgs field H so that kinetic term has the canonical form. The action becomes,

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- New physical mass scale is

$$v = e^{-\pi k r_c} v_0. \tag{7}$$

So to generate required hierarchy one need $kr_c = 10$ which is a small number.

Verlinde-1

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- A two-torus can be thought of as a rectangle in a complex plane with opposite sides identified: $z = z + 1$, $z = z + \tau$ where τ is a complex number. The $\mathbf{Z}_2$ acts on each two-torus as $z \to -z$. On each two torus there will be $2^2 =$ four fixed points $z = 0, 1, \tau, 1 + \tau$.

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- Similarly on six torus there are $2^6 =$ sixty-four fixed points. At each fixed point there will be an orientifold plane.
- In addition there will be thirty-two D3 branes pairwise identified by $\mathbf{Z}_2$

Verlinde-2

- Generically the D3 barnes are separated and the gauge theory living on the worldvolume is $\mathcal{N} = 4$ Super Yang-Mills theory with gauge group $U(1)^{16}$. Making N of them coincident one gets $U(N)$ gauge theory.

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- In the vicinity of D3 branes, they backreact on the metric and changes it into $AdS_5 \times S^5$ which seamlessly glued to the $\mathcal{M}_4 \times T^6$ and thus provide a compact AdS.

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- In the vicinity of D3 branes, they backreact on the metric and changes it into $AdS_5 \times S^5$ which seamlessly glued to the $\mathcal{M}_4 \times T^6$ and thus provide a compact AdS.
- Thus the six-torus grows a throat of finite length which serves as the compact AdS required for warping. The six-torus and the tip of the throat can be thought of as hidden and visible brane respectively.

Verlinde-3

- Explicit background metric with back reaction of D3 branes and orientifold planes can be written as

$$ds^2 = \frac{1}{H(x_\perp)^{\frac{1}{2}}} dx_4^2 + H(x_\perp)^{\frac{1}{2}} dx_\perp^2, \tag{8}$$

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- The harmonic function $H(x_\perp)$ is given by

$$H(x_\perp) = 1 + 4\pi g_s(\alpha')^2[f_D(x_\perp) - f_O(x_\perp)], \tag{9}$$

$$f_D(x_\perp) = 2\sum_{\vec{n}\in \mathbf{Z}^6} \frac{16}{|\vec{x}_\perp + \vec{n}R_c|^4}, \quad f_O(x_\perp) = 2\sum_{\vec{n}\in \mathbf{Z}^6} \frac{1/4}{|\vec{x}_\perp + (1/2)\vec{n}R_c|^4}. \tag{10}$$

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- Close to the D3 brane the geometry indeed reduces to AdS_5

Verlinde-4

- The four dimensional and ten dimensional Planck scales are related through

$$\frac{1}{M_{10}^8} = \frac{1}{M_4^2} \int_{T^6} d^6 x_\perp H(x_\perp), \tag{11}$$

where the RHS is finite even though there is a fourth order pole.

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- Strength of coupling to matter is obtained by inserting variation of four dimensional metric $h_{\mu\nu}$

$$\int d^{10}x \sqrt{-g_{10}} \delta g_{10}^{\mu\nu} T_{\mu\nu} = \int d^4 x_\parallel h^{\mu\nu} \tilde{T}_{\mu\nu} \qquad (12)$$

$$\text{where} \quad \tilde{T}_{\mu\nu}(x_\parallel) = \int d^6 x_\perp H(x_\perp) T_{\mu\nu} \qquad (13)$$

Verlinde-5

- Energy Momentum tensor for a $d = 10$ point particle with mass m is

$$T_{\mu\nu} = \frac{m}{\sqrt{-g_{10}}} \int d\tau \frac{\dot{x}_\mu \dot{x}_\nu}{\sqrt{\dot{x}^2}} \delta^{10}(x - x(\tau)). \qquad (14)$$

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- When located at $x_\perp$ in the transverse space, the effective Energy Momentum tensor becomes

$$\tilde{T}_{\mu\nu} = \frac{m}{H(x_\perp)^{1/4}} \int d\tau \frac{\dot{x}_\mu \dot{x}_\nu}{\sqrt{\dot{x}^2}} \delta^4(x_\perp - x_\perp(\tau)). \tag{15}$$

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- Physical $d = 4$ mass will be redshifted depending its position in the transvers space,

$$m_4(x_\perp) = \frac{m}{H(x_\perp)^{1/4}}. \tag{16}$$

Moduli-1

- In type-IIB string theory D3 branes and O3 planes are charged with respect to four form RR field $C^{(4)}_{\mu\nu\rho\sigma}$ Since T^6 is a compact manifold total charge on it should vanish.

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- In order to get a finite hierarchy one need to separate some of the D3 branes and keep them at a distance. The associated moduli need to be stabilized.
- Verlinde's model has $\mathcal{N} = 4$ superconformal symmetry, which gives rise to a large number of moduli. A realistic model should have $\mathcal{N} = 1$ or no supersymmetry and all the moduli should be fixed.

Moduli-2

- There are various ways to stabilize the moduli. One mechanism is to use background fluxes [Giddings-Kachru-Polchinski, hep-th/0105097; Kachru-Kallosh-Linde-Trivedi; hep-th/0301240, Dewolfe-Giryavets-Kachru-Taylor, hep-th/0505160; Douglas-Kachru, hep-th/0610102].

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- Another mechanism uses magnetic branes [Antoniadis-Maillard, hep-th/0610246, Antoniadis-Kumar-Maillard, hep-th/0505260, 0610246; Kumar-SM-Ray, hep-th/0605083, 0706.1825]. We will briefly discuss the moduli stabilization with beckground fluxes here.

Moduli-3

- It turned out turning on background fluxes also create a warped metric. It was shown in a slightly different context[Klebanov-Strassler, hep-th/0007191] that even though one begins with branes to motivate the construction of warped metric one ends up with NS-NS and R-R fluxes through the compactification manifold.

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- Can one obtain an explicit solution with warped metric and the moduli stabilized at least at the level of low energy effective action in a compact situation.

Action-1

- Consider low energy effective theory of type-IIB string theory given by type-IIB supergravity. The effective action (in string frame) is

$$\begin{aligned} S &= \frac{1}{2k_{10}^2}\int d^{10}x\sqrt{-G}e^{-2S}(R+4(\partial S)^2-1/2F_1^2-\frac{1}{12}G_3.\bar{G}_3 \\ &- \frac{1}{4.5!}\tilde{F}_5^2)+\frac{1}{8ik_{10}^2}\int e^S C_4\wedge G_3\wedge\bar{G}_3+S_{loc}. \end{aligned} \tag{17}$$

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- H_3 is NSNS field strength with potential B_2, $F_{1,3,5}$ are RR field strengths with potential $C_{0,2,4}$ and two scalar fields are combined as $\tau = C_0 + ie^{-S}$ called axiodilaton.

Action-2

- The three-form field G_3 and five-form field strength $\tilde{F}_5$ are

$$G_3 = F_3 - \tau H_3, \qquad \tilde{F}_5 = F_5 - 1/2 C_2 \wedge H_5 + 1/2 B_2 \wedge F_3. \quad (18)$$

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- In addition, $\tilde{F}$ is self-dual,

$$\tilde{F} = * \tilde{F} \quad (19)$$

Ansatz

- Metric in Einstein frame:

$$ds^2_{10} = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}(y)dy^m dy^n, \qquad (20)$$

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- Self-dual five-form field strength to be

$$\tilde{F}_5 = (1 + *)[d\alpha(y) \wedge dx^0 \wedge \cdots \wedge dx^3], \qquad (22)$$

- Three-form fluxes have only compact components

$$F_3, H_3 \in H^3(M, Z) \qquad (23)$$

No-Go Theorem

- Minkowski components of Einstein equation implies

$$\tilde{\nabla}^2 e^{4A} = e^{2A}\frac{G_{mnp}\bar{G}^{mnp}}{12\Im(\tau)} + e^{-6A}[(\partial\alpha)^2 + (\partial e^{4A})^2] + k_{10}^2 e^{2A}(T_m^m - T_\mu^\mu)_{\rm loc} \tag{24}$$

where $T_{\rm loc}$ is the Energy Momentum tensor from local terms.

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$$(T^m_m - T^\mu_\mu)_{\rm loc} < 0 \tag{25}$$

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- Integrating one gets the Tadpole cancellation condition

Constraints-1

- Combining Einstein equation and Bianchi Identity,

$$\begin{aligned}\tilde{\nabla}^2(e^{4A}-\alpha) &= \frac{e^{2A}}{12Im(\tau)}|iG_3 - *_6 G_3|^2 + e^{-6A}|\partial(e^{4A}-\alpha)|^2 \\ &+ 2k_{10}^2 e^{2A}[\frac{1}{4}(T_m^m - T_\mu^\mu)_{\mathrm{loc}} - T_3\rho_3^{\mathrm{loc}}] \qquad (28)\end{aligned}$$

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- One makes an assumption

$$\frac{1}{4}(T^m_m - T^\mu_\mu)_{\text{loc}} - T_3\rho_3^{\text{loc}} \geq 0, \qquad (29)$$

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- Then G_3 is Imaginary self-dual

$$*_6 G_3 = iG_3 \quad \text{and} \quad e^{4A} = \alpha \quad (30)$$

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- The extradimensional and axion-dilaton equations implies that this class of solutions describe F-theory models in supergravity approximations.
- In next few slides we consider some aspects of the effective four dimensional action obtained by compactification on a Calabi-Yau threefold K.

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- Complex structure moduli are associated with $2(h^{2,1}+1)$ three-cycles $\{A^a, B_b\}$ and their dual three-forms $\{\alpha_a, \beta^b\}$ where we choose a symplectic basis.

$$\int_{A^a} \alpha_b = \delta^a_b, \quad \int_{B_b} \beta^a = -\delta^a_b, \quad \int_K \alpha_a \wedge \beta^b = \delta^b_a. \qquad (31)$$

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- One defines a period vector $\vec{\Pi}$ with nowhere vanishing holomorphic $(3,0)$-form Ω

$$z^a = \int_{A^a} \Omega, \quad \mathcal{G}_b = \int_{B_b} \Omega, \quad \vec{\Pi}(z) = (\mathcal{G}_b, z^a). \tag{32}$$

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- Dynamics of CS moduli and the axion-dilaton τ is described by the Kähler potential

$$\mathcal{K} = -\log(i \int_K \Omega \wedge \bar{\Omega}) - \log[-i(\tau - \bar{\tau})] \tag{34}$$

CS Moduli-3

- In $\mathcal{N} = 2$ compactification special geometry governs CS moduli space (vector multiplets) which remains valid in this $\mathcal{N} = 1$ orientifold compactification too.

NS-NS & RR flux

- Turn on RR and NS-NS three-form field strengths through the three-cycles

$$F_3 = -(4\pi^2\alpha')[f_a\alpha^a + f^{a+h_{2,1}+1}\beta_a], \quad H_3 = -(4\pi^2\alpha')[h_a\alpha^a + h^{a+h_{2,1}+1}\beta_a]. \tag{35}$$

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- Supersymmetric vacua requires $D_\tau W = D_a W = 0$ implying

$$(\vec{f} - \bar{\tau}\vec{h}).\vec{\Pi} = 0. \tag{37}$$

There are $(h_{2,1}+1)$ variables and $(h_{2,1}+1)$ equations this will stabilize all CS moduli.

Tadpole

- The fluxes contribute to the tadpole for D3 brane charge due to Chern-Simmons term in the action,

$$N_{\text{flux}} = \frac{1}{(4\pi^2\alpha')^2} \int_K F_3 \wedge H_3 \tag{38}$$

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- The tadpole condition becomes

$$N_{\text{flux}} + N_{\text{D3}} = L \tag{39}$$

where L is total negative D3 charge need to be canceled.

Kähler moduli

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- The volume V of K is given by

$$V = \frac{1}{6} C_{\alpha\beta\gamma} t^{\alpha} t^{\beta} t^{\gamma}, \tag{41}$$

where $C_{\alpha\beta\gamma}$ is the triple intersection form.

Tree-level Potential

- The full tree-level flux potential appears as

$$V = e^{\mathcal{K}+\mathcal{K}_k}[g^{a\bar{b}}(D_a W)(\bar{D}_b W) \tag{42}$$

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- To stabilize the Kähler moduli in this set-up one need to use non-perturbative contributions to the super-potential.

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- The singualrity can be removed by deforming the conifold

$$P = x^2 + y^2 + u^2 + v^2 = z^2. \tag{44}$$

This amounts blowing up a three-sphere at the origin. If z^2 is real the real section of the equation represents a three sphere. We identify this as an A-cycle.

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- The periods are

$$\int_A \Omega = z, \qquad \int_B \Omega = \frac{1}{2i\pi} z \log(z) + \text{regular} = \mathcal{G}(z) \tag{46}$$

The regular part depends on the particular CY.

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- Superpotential takes the form

$$W(z) = -K\tau z + M\mathcal{G}(z). \tag{48}$$

Conifold-4

- In the limit K/g_s very large the condition for supersymmetry is

$$D_z W = \frac{M}{2i\pi}\log(z) - \frac{iK}{g_s} = 0. \qquad (49)$$

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- The axio-dilaton and other CS moduli can be stabilized in a similar way. To stabilize the Kähler moduli one need to turn on some non-perturbative effect.

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- There are several methods [Uranga: NPB (PS) 171,(2007),119] to obtain chiral gauge theory on the worldvolume of D brane, such as,

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- Using these methods many supersymmetric and non-supersymmetric standard-like models have been constructed. We briefly consider D brane at singularity.
- In this set up to obtain a chiral gauge theory living on the worldvolume of D brane at warped metric we need a singularity at the bottom of the throat.

Orbifold of SSP-1

- One way is to construct a geometry which admits a complex structure deformation (through blowing-up of a three-cycle) with left over geometry singular. Turning on fluxes through the three-cycle one can obtain a warped geometry and the chiral gauge theory comes from D branes at singularity.

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- While there are different possibilities we describe a particular example where the geometry is an orbifold of suspended pinch point (SSP)[Cascales et. al. hep-th/0503079].

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- While there are different possibilities we describe a particular example where the geometry is an orbifold of suspended pinch point (SSP)[Cascales et. al. hep-th/0503079].
- SSP is given by following hypersurface in C^4:

$$xy - uv^2 = 0 \tag{52}$$

Orbifold of SSP-1

- Consider $\mathbf{Z}_3$ quotient of it where $\mathbf{Z}_3$ action is given by

$$\theta: \quad x \rightarrow \omega . x, \quad y \rightarrow \omega . y, \quad u \rightarrow \omega . u, \quad v \rightarrow \omega^2 . x, \quad \omega = e^{\frac{2\pi}{3}}. \tag{53}$$

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- The $\mathbf{Z}_3$ quotient leaves the holomorphic three-form $\Omega = \frac{dx \wedge dy \wedge du}{uv}$ and so the quotient is also a CY.
- The SPP admits a complex structure deformation

$$xy - uv^2 = z.v \quad . \tag{54}$$

This is invariant under $\mathbf{Z}_3$ quotienting and so the deformation is inherited to the quotient space. The quotient geometry contains a finite size three-cycle.

Warped metric

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- On general ground it is expected that turning on M units of RR flux through the finite size three cycle and $-K$ units of NS-NS flux through the dual non-compact cycle leads to a warped throat.
- Unlike conifold, explicit metric and periods of Ω are not known for SSP. But again, on general ground, one expects the warp factor to be $e^{K/(Mg_s)}$. At the bottom of the throat it is cut off by finite size three-cycle and contains C^3/Z_3 singularity.

Semi-realistic model-1

- Now one can introduce a small number of D branes at the singularity to obtain chiral gauge theory. Number of D branes should be small compare to the flux so that it does not modify the geometry substantially.

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- Introducing three sets of N number of D3 branes one obtains the following gauge theory

$$\text{Vector Multiplet}: U(N) \times U(N) \times U(N) \tag{55}$$

$$\text{Chiral Multiplet}: 3 \times [(N, \bar{N}, 1) \oplus (1, N, \bar{N}) \oplus (\bar{N}, 1, N)]. \tag{56}$$

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- Two of the $U(1)$'s in the above gauge group gets massive due to Green-Schwarz mechanism and acquire mass through $B \wedge F$ coupling.

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- To get standard model gauge group from $\mathbf{Z}_3$ singularity one choose D3 branes with Chan-Paton embedding

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- In order to have supersymmetry D7 branes need to be wrapped on holomorphic four-cycles.

Matter	Q_3	Q_2	Q_1	$Q_{U(3)}$	$Q_{U(6)}$	Y
3(3,2)	1	-1	0	0	0	1/6
$3(\bar{3}, 1)$	-1	0	1	0	0	-2/3
3(1,2)	0	1	-1	0	0	1/2
$(3,1;\bar{3}, 1)$	1	0	0	-1	0	-1/3
$(\bar{3},1;1,6)$	-1	0	0	0	1	1/3
$(1,2;1,\bar{6})$	0	1	0	0	-1	-1/2
(1,1;3,1)	0	0	-1	1	0	1

Table: Matter fields coming from (33) and (37) sectors

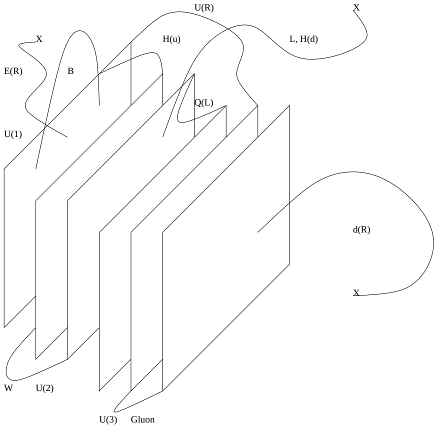

Figure: D3/D7 brane configuration leading to standard model on the Worldvolume.