Braneworld Black Rings

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Plan of Talk

- Introduction
- Black Rings
- Black Ring Geodesics
- Braneworld Black Rings
- Braneworld Black Ring Geodesics
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Introduction

- Brane world scenario requires generic gravitational configurations on the brane to arise from a four dimensional bulk.The investigation of black hole configurations in this context has been an exciting aspect of study

- Chamblin *et al* [*] described a Schwarzschild black hole as a 3-brane intercept of a bulk black string in a Randall-sundrum brane world.This was soon generalized to the case of a rotating black hole on the 3-brane[†]

- The bulk black string solution was generalized to n dimensional RS branewrld with a single transverse AdS direction[‡] where it intercepted the (n-1)brane in a Myers Perry black hole

- This work attempts to describe five dimensional black ring solutions in an RS braneworld scenario

[*] A. Chamblin, S. Hawking, H. Reall, Phys. Rev. D 61, 065007 (2000)

[†] M.S. Modgil, S. Panda and G.Sengupta, Mod. Phys. Lett A, 17 (2002) 1479

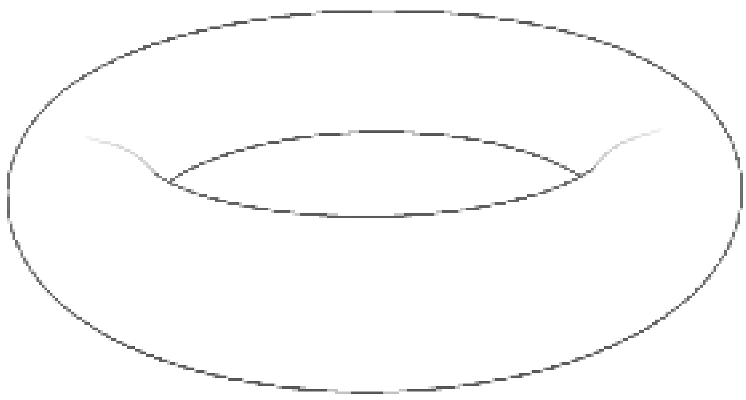
[‡] G.Sengupta, Int.J.Mod.Phys.D15 (2006) 171

Black Rings

- neutral rotating black rings* are stationary asymptotically flat solutions of five dimensional vacuum Einstein equations having a regular horizon of topology $S^1 \times S^2$.

- for a certain range of mass and angular momentum two black ring solutions can coexist with a rotating black hole solution. Thus, uniqueness doesn't hold in five dimensions.

*R. Emparan and H.S.Reall, Phys. Rev. Lett. 88 (2002)



Black ring cartoon. The azimuthal angle of S^2 is suppressed.

Co-ordinate system review

- $SO(4)$ has Cartan subgroup $U(1)^2$ thus implying two independent rotations.

- the four spatial co-ordinates can be paired as

$$\begin{aligned} x^1 &= r_1 \cos\phi \qquad x^3 = r_2 \cos\psi \\ x^2 &= r_1 \sin\phi \qquad x^4 = r_2 \sin\psi \,. \end{aligned} \tag{1}$$

  with the corresponding flat space metric

$$d\mathbf{x}_4^2 = dr_1^2 + r_1^2 d\phi^2 + dr_2^2 + r_2^2 d\psi^2 \,. \tag{2}$$

- *adapted co-ordinates*(x, y, ψ, ϕ) foliate flat space by the equipotential surfaces of the field created by a source resembling the black ring extended along (x^3, x^4) plane and rotating along ψ direction.

$$r_1 = R\frac{\sqrt{1-x^2}}{x-y} \,, \qquad r_2 = R\frac{\sqrt{y^2-1}}{x-y} \,. \tag{3}$$

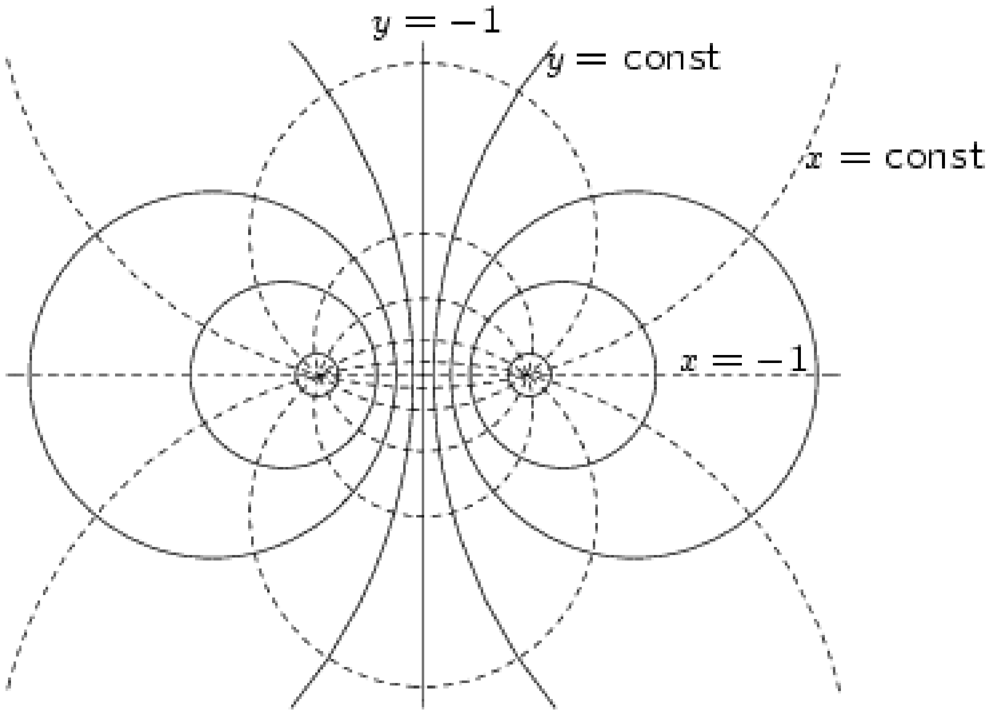
$$-\infty \leq y \leq -1 \,, \qquad -1 \leq x \leq 1$$

- asymptotic infinity is the limit $x, y \to -1$

- $y = -\infty$ or $r_1 = 0, r_2 = R$ is the location of the ring sorce

- $y = -1$, the fixed 'point' of ψ rotation, is the whole of (r_1, ϕ) plane.

- $x = 1$ and $x = -1$, the fixed 'points' of ϕ rotation are, respectively, $r_2 \leq R$ and $r_2 > R$ regions of (r_2, ψ) plane.



Ring coordinates for flat four-dimensional space, on a section at constant ϕ and ψ (and $\phi+\pi$, $\psi+\pi$). Dashed circles correspond to spheres at constant $|x|\in[0,1]$, solid circles to spheres at constant $y\in[-\infty,-1]$. Spheres at constant y collapse to zero size at the location of the ring of radius R, $y=-\infty$*.

flat space metric in ring co-ordinates

$$d\mathbf{x}_4^2 = \frac{R^2}{(x-y)^2}\left[(y^2-1)d\psi^2 + \frac{dy^2}{y^2-1} + \frac{dx^2}{1-x^2} + (1-x^2)d\phi^2\right]. \tag{4}$$

- constant y sections have ring like topology $S^1\times S^2$
- the S^2's are metrically round spheres for flat space.

*R.Emparan, H.S Reall Class.Quant.Grav. 23 (2006) R169

Black ring Metric

The metric of a neutral rotating black ring is

$$ds^2 = -\frac{F(y)}{F(x)}\left(dt - C\,R\,\frac{1+y}{F(y)}\,d\psi\right)^2 + \frac{R^2}{(x-y)^2}\,F(x)\left[-\frac{G(y)}{F(y)}d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\phi^2\right] \quad (5$$

$$F(\xi) = 1+\lambda\xi, \qquad G(\xi) = (1-\xi^2)(1+\nu\xi) \qquad C = \sqrt{\lambda(\lambda-\nu)\left(\frac{1+\lambda}{1-\lambda}\right)}\,.$$

$$0 < \nu \leq \lambda < 1 \quad (6)$$

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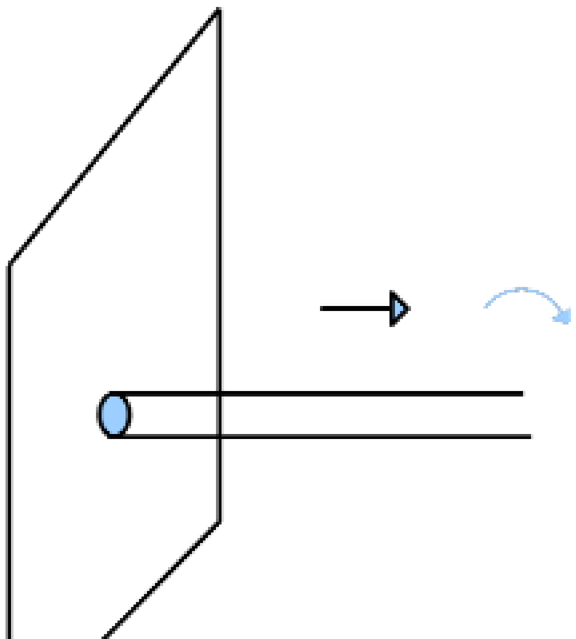
- three Killing isometries ∂_t,∂_ϕ,∂_ψ
- ν signifies shape of the rings
- λ/ν measures speed of rotation along ψ,*i.e* S^1
- $y = -1/\lambda$ section is the ergosurface
- $y = -1/\nu$ section is the black ring horizon
- $y = -\infty$ is a spacelike curvature singularity inside the horizon

- black ring tension and gravitational self attraction are internally balanced by the centrifugal force.This implies there are only two independent parameters of black ring, R and ν. Also,it means angular momentum is bounded from below.

- black ring of large radius R is like a Schwarzschild black string boosted along the translation invariant direction and periodically identified with a period $2\pi R$.The pressure of the boosted string is zero



Black ring geodesics

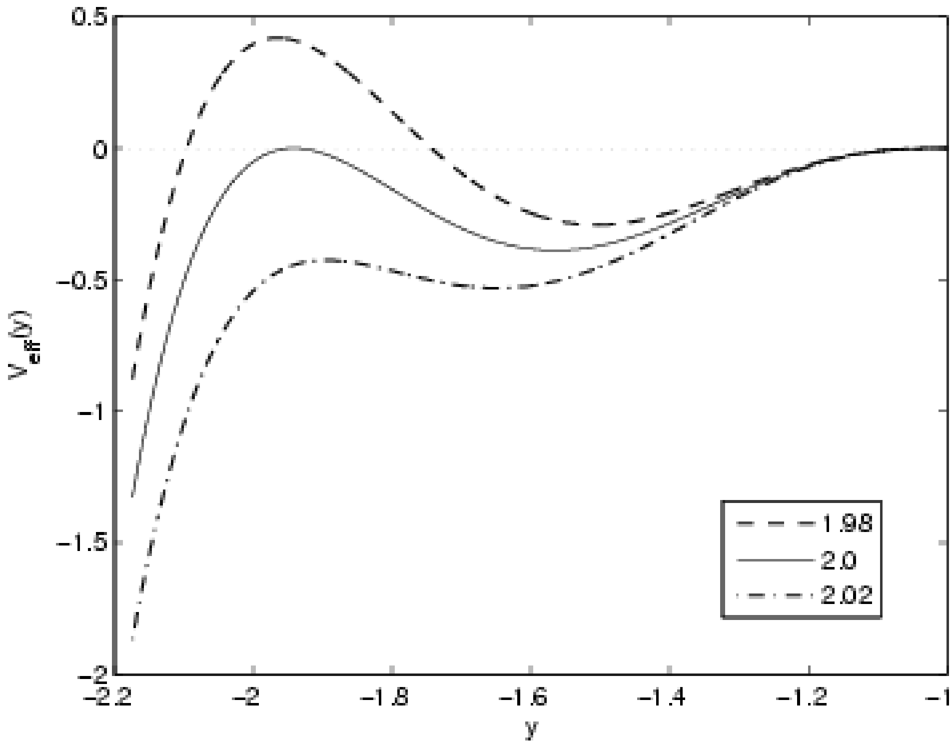
- equatorial plane: we consider geodesics restricted to the plane of rotation of the black ring outside the horizon, *i.e* $r_2 > R$ or, equivalently, the $x = -1$ section in the ring co-ordinates.

$$\dot{y}^2 + V_{eff}(y; E, \Psi) = 0 \tag{7}$$

- asymptotic behaviour $(y \to -1)$

$$V_{eff}(y; E, \Psi) \to -\frac{2(1-\nu)}{R^2(1+\lambda)}|1+y|^3(E^2 - m^2), \tag{8}$$

- unbounded timelike geodesics only when $E^2 > m^2$



Effective potential plot.Three values of E are as indicated in the box. The constants $m = R = 1$*.

It is apparent that there are no stable bound orbits, as expected for central potentials in more than four dimensions.

*Anurag Sahay,Gautam Sengupta JHEP06(2007)006

Braneworld Black Ring

- AdS6 in conformally flat co-ordinates

$$ds^2 = \frac{l^2}{z^2}[\eta_{\mu\nu} dx^\mu dx^\nu + dz^2] \tag{9}$$

- $5+1$ dimensional RS2 braneworld with a single transverse AdS direction and a reflection symmetric 4-brane obtained from AdS6 by slicing, orbifolding and then pushing the regulator brane to AdS horizon.

- rotating black string in ADS6 as the bulk solution intercepting the 4-brane in a five dimensional black ring.

$$\begin{aligned} ds^2 &= \frac{l^2}{(z_0+|w|)^2}\left[dw^2 - \frac{F(y)}{F(x)}\left(dt - CR\frac{1+y}{F(y)}d\psi\right)^2\right. \\ &+ \left.\frac{R^2}{(x-y)^2}F(x)\left(-\frac{G(y)}{F(y)}d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\phi^2\right)\right] \end{aligned} \tag{10}$$

- ADM mass and angular momentum on the brane are scaled by the warp factor

$$M_* = \left(\frac{l}{z_0}\right)^2 M\,, \qquad J_* = \left(\frac{l}{z_0}\right)^3 J. \tag{11}$$

- curvature squared diverges at the black ring singularity on the brane and at the AdS horizon ($z \to -\infty$) as well

$$\begin{aligned} R_{jklm}R^{jklm} &= \frac{1}{l^4}\left[60 + \frac{6(1+\nu^2)^2\nu^2 Q(x,y)}{R^4(1+\nu^2+2\nu x)^6} z^4 (x-y)^4\right] \\ &\sim \frac{z^4}{r^8} + \text{const.} \end{aligned} \tag{12}$$

- horizon singularity artifact of linearization(?) check with the geodesics

Black 2-brane

- in the large R limit the rotating black string reduces to a bulk black 2-brane in AdS6 boosted along the extended direction on the 4-brane and identified periodically.

$$ds^2 = \frac{l^2}{z^2}\left[dz^2 + du^2 - (1 - \frac{r_0}{r})dt^2 + (1 - \frac{r_0}{r})^{-1}dr^2 + r^2 d\Omega_2^2\right] \tag{13}$$

Braneworld Geodesics

- bulk geodesics that reach the AdS horizon inform the nature of singularity there

- we consider geodesics which on the 4-brane are restricted to the equatorial plane of the black ring.

$$\left(\frac{dy}{d\rho}\right)^2 + \frac{z^4}{l^4} g^{yy} \left(\frac{l^2 m^2}{z_1^2} + g^{tt} E^2 - g^{t\psi} E \Psi + g^{\psi\psi} \Psi^2\right) = 0 \tag{14}$$

$$z \to -\infty \text{ as } \rho \to 0^-$$

- both timelike and null geodesics when projected onto the 4-brane by scaling out z dependence behave as timelike geodesics on the brane with proper time γ

$$\left(\frac{d\tilde{y}}{d\gamma}\right)^2 + V_{eff}(\tilde{y}; \tilde{E}, \tilde{\Psi}) = 0 \tag{15}$$

$$\gamma \to \infty \text{ as } \rho \to 0^-$$

- late time beahviour of projected geodesics on the brane equivalent to near horizon behaviour of bulk geodesics

- no stable bound geodesic on the brane so only the bulk geodesics with unbounded projections reach the horizon

- along such geodesics the curvature squared remains finite thus indicating the presence of p-p curvature singularity

- explicit determination necessitates obtaining curvature components in a pp othonormal frame.. intractable so far!

Conclusion

- we have described five dimensional black ring in a Randall-Sundrum braneworld perspective as the intercept on the 4-brane of a rotating black string solution in AdS6

- this is motivated by the fact that for consistency, gravitational configurations on the brane,in particular black holes, arise from some higher dimensional bulk solutions.

- black rings and their generalizations like black di-rings, black saturn, doubly spinning black rings, etc, make for an interesting configuration to study from a bulk brane world perspective.