BLACK HOLES IN A BLACK STRING BRANEWORLD

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PLAN

- *Brane World Gravity*
- *Schwarzschild Black Holes*
- *Kerr Black Holes*
- *Myers Perry Black Holes*
- *Summary and Outlook*

- *Rotating Braneworld Black Holes; M.Singh,S.Panda and GS;* **Mod. Phys. Lett. A17:1479 (2002).**

- *Black Holes in Brane Worlds, M.Singh,S.Panda and GS; Proccedings of PASCOS 2003, TIFR,* **Pramana 62, 3, 707 (2004).**

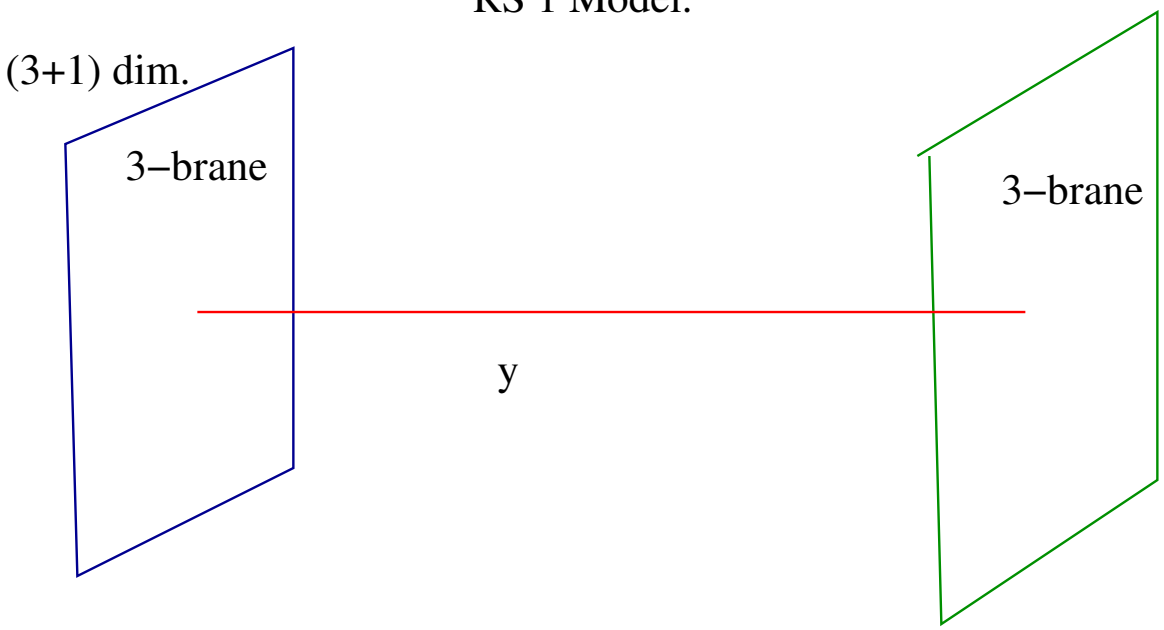
- *Rotating Black Holes in Higher Dimensional Brane Worlds; GS,* **IJMP D 15 171, (2006)**

• *Brane worlds as low energy effective higher dimensional scenario for the universe inspired by String Theory.*

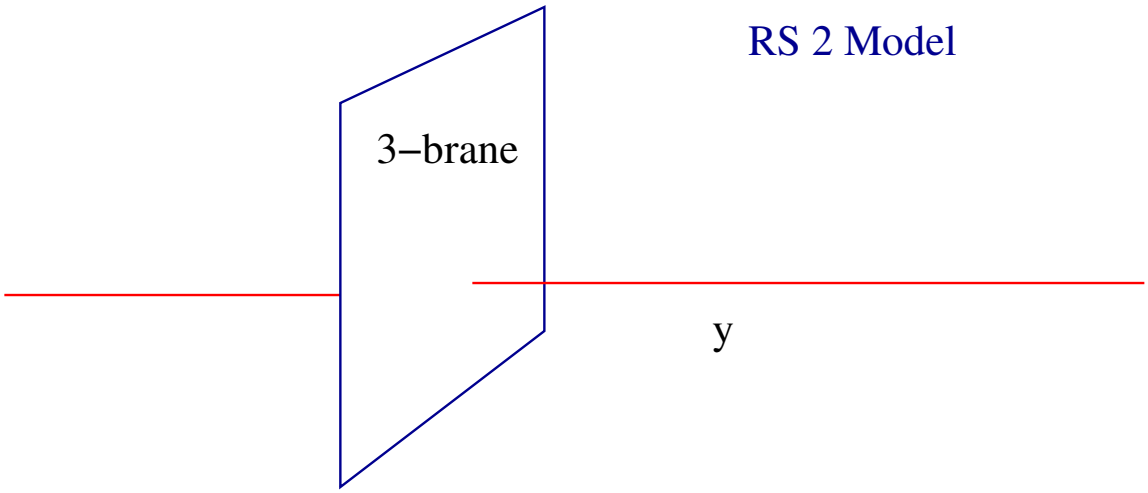
• *1. Kaluza Klein compactifications in string theory. Higher (D=10)dimensional universe factorizes to* $R^{(3,1)} \times X$ *where* X *is an internal compact 6 dimensional Calabi Yau space. Warped compactifications. ADD, RS1 and RS2.*

• *2. D branes support susy gauge theories on their world volumes. D brane configuration may appear as a space time topological defect (brane) at low energies.*

RS 1 Model.



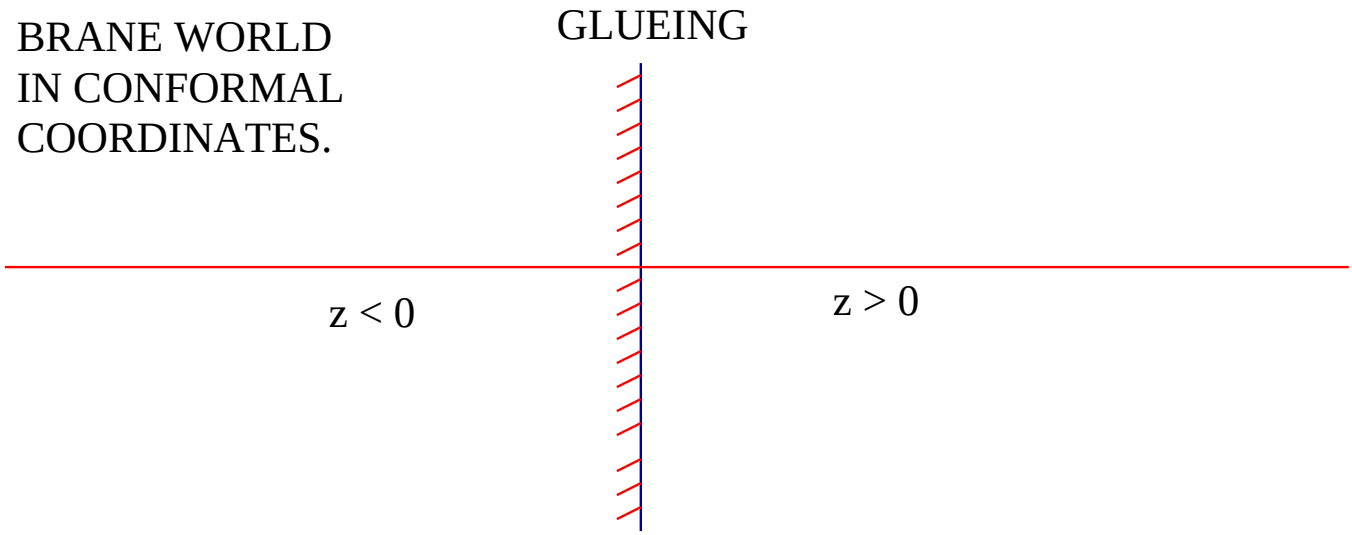
RS 2 Model



• *Focus on RS2 (single brane). Conformal coordinates* $z = lexp(\frac{y}{l})$. *Remove small* z *region. Glue mirror copy of large* z *region along* $z = z_0$ *location of the brane. Ensures* Z_2 *reflection symmetry on the brane.*

- $ds^2 = \frac{l^2}{z^2}[\eta_{\mu\nu}dx^\mu dx^\nu + dz^2]$

BRANE WORLD
IN CONFORMAL
COORDINATES.

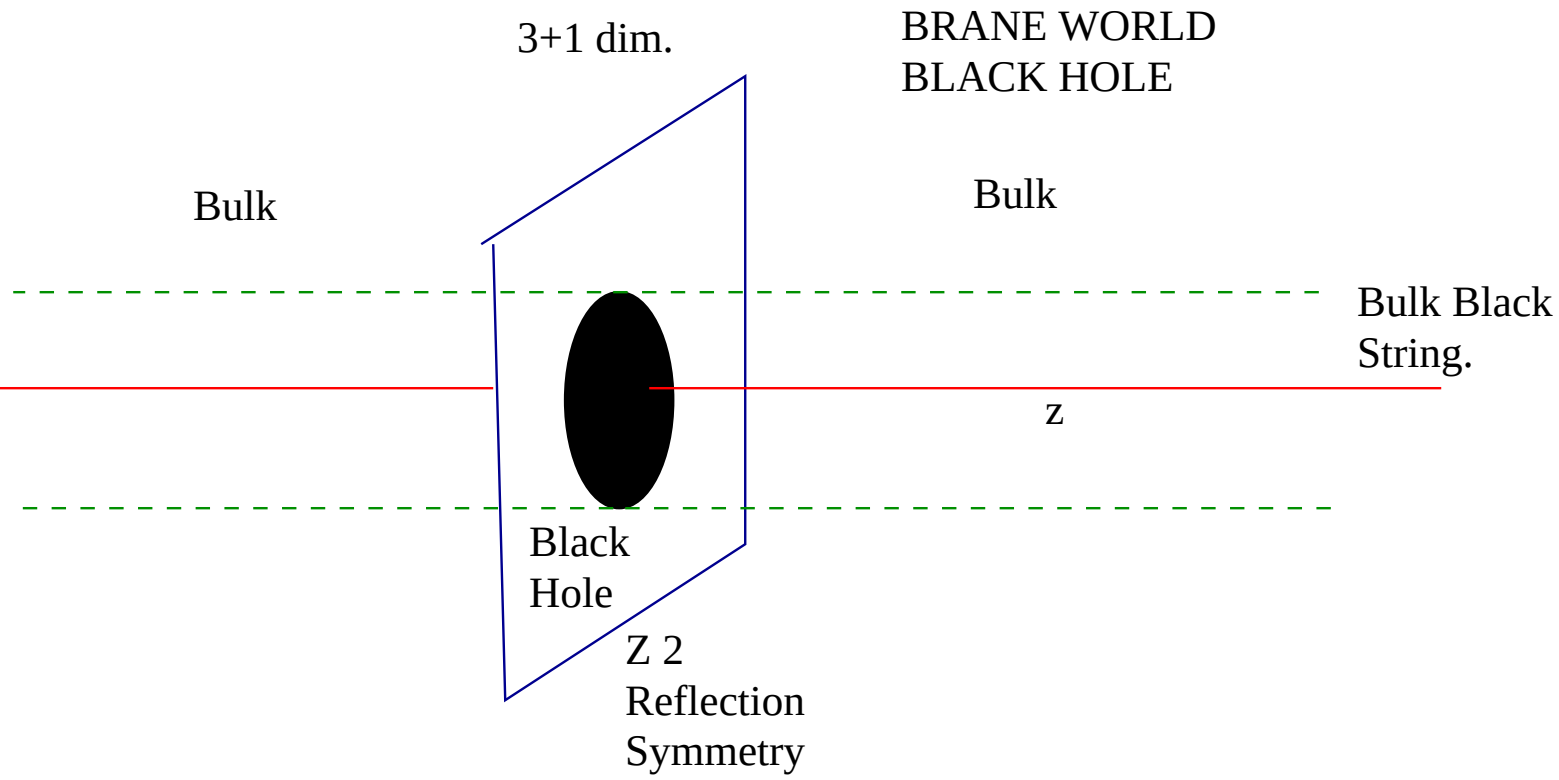


- *Finite discontinuity in the Extrinsic Curvature across the brane causes δ- function discontinuity in the Energy Momentum Tensor. Brane Tension.*

- *Analysis holds also for Ricci flat backgrounds $R_{\mu\nu} = 0$. Extended to $(N+1)$ dimensions with a $(N-1)$ brane and a single AdS like direction.*

• *Chamblin, Hawking and Reall (CHR)construction. Schwarzschild black hole on the brane arises from a bulk black string in the RS2 braneworld.*

• $ds^2 = \frac{l^2}{z^2}[-f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2 + dz^2]$ and $f(r) = (1 - \frac{2M}{r})$.



• On the brane scaled mass $M^* = M\frac{l}{z_0}$ and $r = 2M^*$ is the 4D event horizon extended into the 5D bulk $S^2 \times R$ hypercylinder.

• Singularity at $r = 0$ where $R_{\lambda\mu\nu\kappa}R^{\lambda\mu\nu\kappa} \sim \frac{z^4}{r^6}$ for large z OR small r. Bulk 5D line singularity extended in z. Singularity also at $z \to \infty$ AdS horizon. Pathology

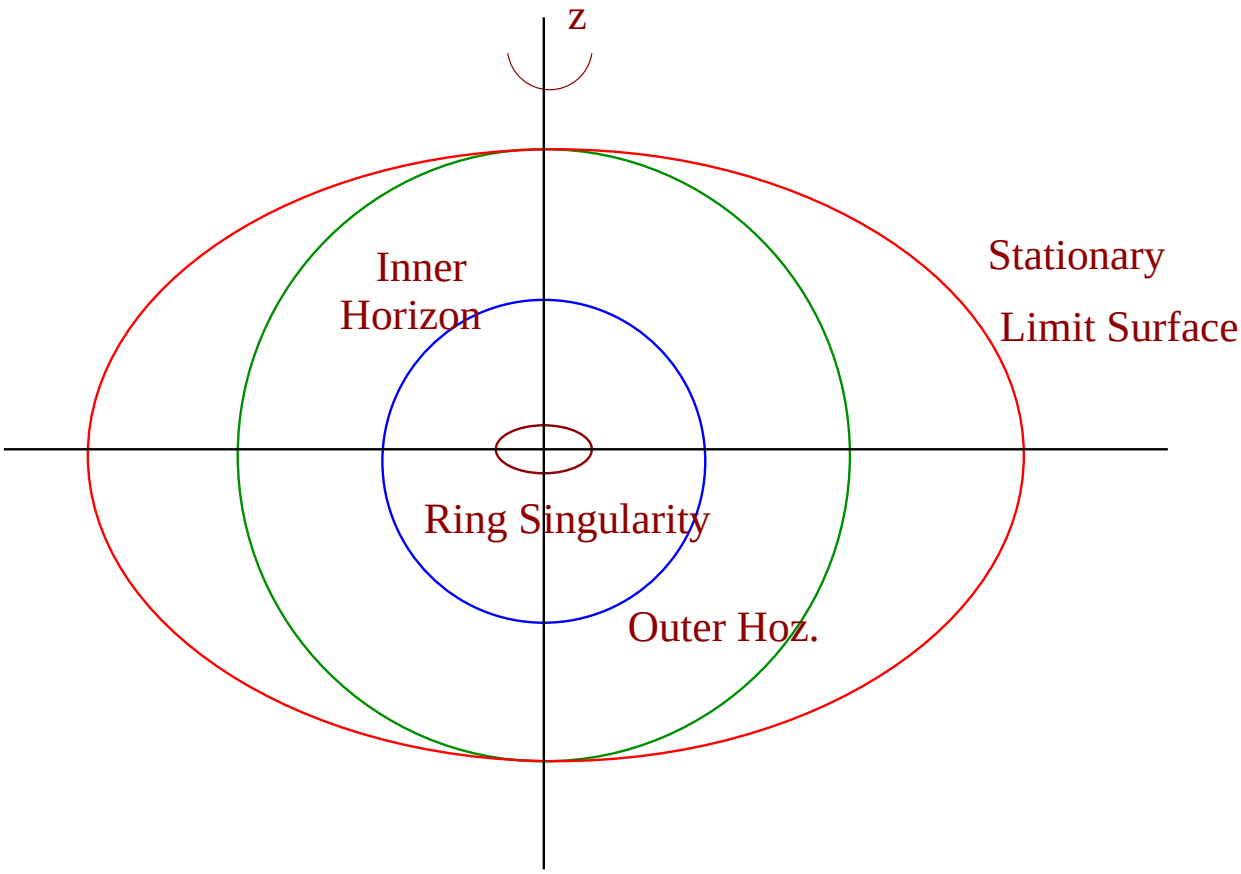
• Nature of pathology. Construction of 5D bulk Geodesics (causal structure) and project on 3- brane (4 dim.) through scaling. 5D bulk timelike/null radial geodesic equation

$$(\frac{dr}{d\lambda})^2 + \frac{z^4}{l^4}[(\frac{l^2}{z_1^2} + \frac{L^2}{r^2})(1 - \frac{2M}{r}) - E^2] = 0$$

• Projected radial geodesic equation in 4D matches standard effective potential expression in terms of rescaled coordinates and parameters. $(\frac{dr}{d\lambda})^2 + \tilde{V}(r) = E^2(r)$. And $V(r) = (1 + \frac{L^2}{r^2})(1 - \frac{2M}{r})$

• Singularity $\sim \frac{z^4}{r^6}$ along bounded orbits r finite and $z \to \infty$. Along unbounded orbits $r \to \infty$ curvature squared is regular at $z \to \infty$ as it falls of in r^6. Signal of a p-p curvature singularity. Pathology artifact of linear approximation.

• $z \to \infty$ Stability of the solution. Black string is unstable against perturbations. Stability parameter is the size of the black string. Solution pinches off before reaching AdS horizon. Final state possibly a non uniform black string.



KERR ROTATING BLACK HOLE

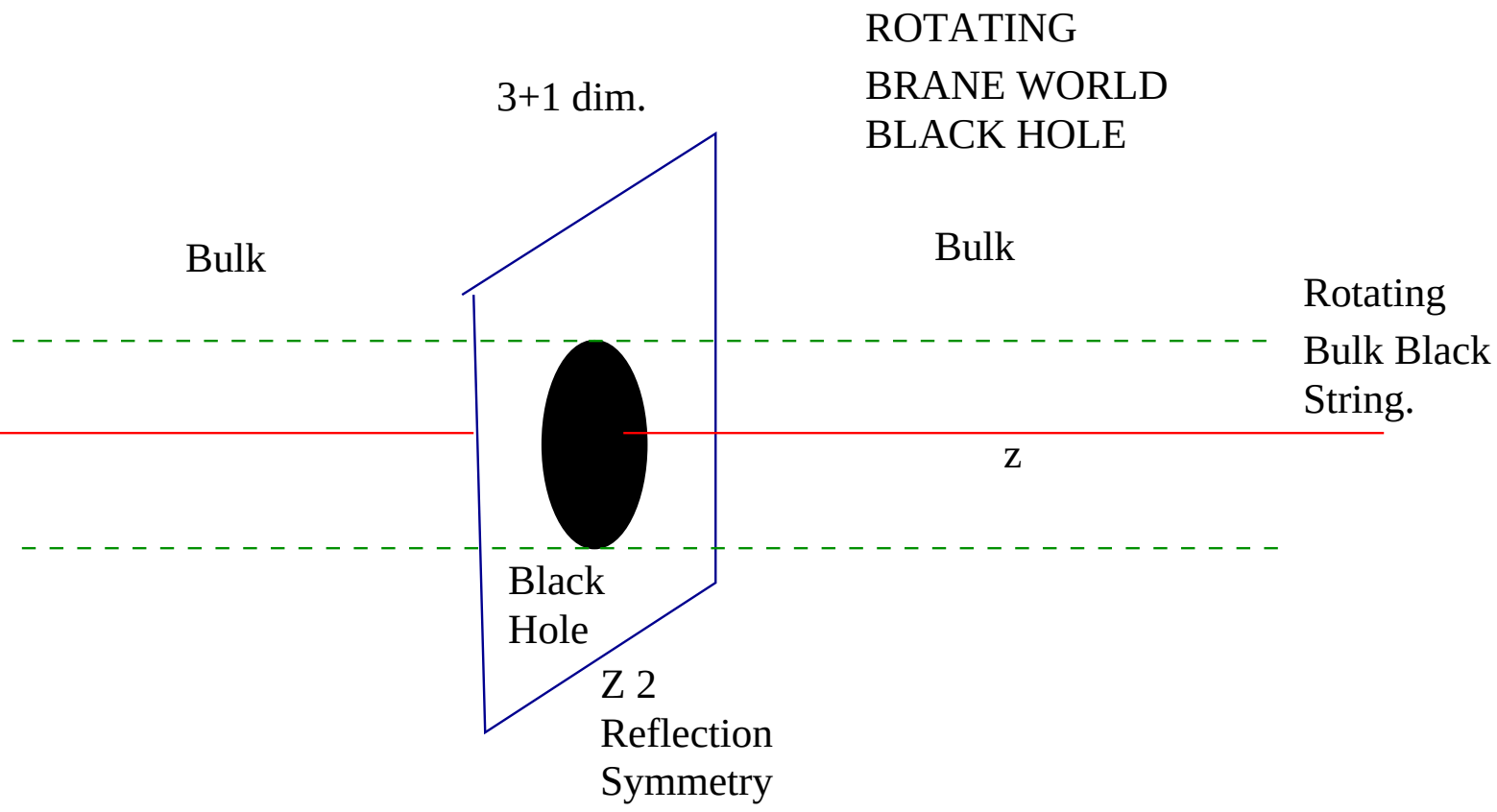
• Rotating black holes described by axialy symmetric Kerr metric. In spherical coordinates event horizon $\Delta = r^2 + a^2 - 2Mr = 0$. Two solutions for $a^2 \leq M^2$ as $r_\pm = M \pm \sqrt{M^2 - a^2}$. Rotating Inner and outer horizon. $a \to 0$ limit.

•Stationary limit surface $r = \tilde{r}_+ = M + \sqrt{M^2 - a^2 Cos^2\theta}$. $r > r_+$ Ergosphere.

•Curvature singularity $\sim \frac{1}{r^6}$ at $r = 0$ and $\theta = \frac{\pi}{2}$. In quasi cartesian Schild cordinates, singularity along boundary of disk $r = a$. Ring singularity.

• Geodesic structure: Isometries in t, ϕ leading to geodesic equations. $\theta = \frac{\pi}{2}$ equatorial plane geodesics. r equation defines a Central Force problem in the *Effective Potential* picture $\tilde{V}(r)$. Bound and Unbound r orbits

• We proposed that a 4D Kerr Black Hole on the brane arises from a bulk 5D rotating black string. Product of 4D Kerr Black Hole with a infinite direction.



• On the 3-brane we have the rescaled 4 D BH mass $M^* = \frac{Ml}{z_0}$ and angular momentum $J^* = \frac{l^2}{z_0^2}Ma$. The radius of the inner and outer horizons and the location of the stationary limit surfaces in 4D are also rescaled.

• The ring singularity in 4D is now extended in the z direction and the Curvature squared $\sim \frac{z^4}{r^6}$ blows up at $z \to \infty$ which is a pathology.

• Examine $z \to \infty$ pathology. Causal structure from 5 dim. bulk geodesics. Focus on class $\theta = \frac{\pi}{2}$. Projection of bulk geodesics to 4D on the 3-brane by scaling.

• The 4D geodesic equations in rescaled coordinates and parameters lead again to a standard Kerr 4D *effective potential*picture.

• Geodesic orbits. Bounded r finite and unbounded r infinite. Singularity $\sim \frac{z^4}{r^6}$ at $z \to \infty$ for bound orbits with r finite. Curvature squared finite on unbound orbits. Signals again a p-p curvature singularity at $z \to \infty$.

• Myers Perry metric: N dimensional rotating black hole. N (N-1, 1) odd/even. $[\frac{N-1}{2}]$ rotations in independent coord. planes. a_i rotation parameters. Angular momentum tensor in $N > 3$.

• Single angular momentum component simmilar to Kerr. Multiple angular momentum components. Structure depends on N and number of non-zero rotations in the (x^k, y^k) coord. planes. Angle ϕ_k. Quasi Spherical coords. (t, r, θ_i, ϕ_i)

• Event horizon is a *squashed (N-2) dim sphere* with rotational symmetry in every coord. plane. $r = 0$ surface is a (N -2p-2) ball for p indep. rotations. Where an M ball is a region of R^M bounded by a (M-1) sphere.

•Horizon equation $\Pi - \mu r^{2-\epsilon} = 0$ Π function of r and a_k. Solutions exist for N=4,5,6,7,9,11 ($\epsilon = 1, 0$ for even and odd N). Describes cases with 2 horizons and 1 degenerate horizon. Rich horizon and singularity structure.

•Extended singularity on the edge of a (N-2) ball i.e S^{N-3}. Higher dim. generalization of the 4D Ring Singularity of Kerr BH.

•Curvature tensor components diverge in an orthonormal frame as $R_{uvuv} = r^{-2p-\epsilon}$ where p is the number of non zero components of the angular momentum tensor.

•Geodesic structure: Exact solution for N=5 by Frolov et al. Equatorial geodesics for N arbitrary. Reduces to a central force problem in the effective potential picture. Form of potential allows only unbound r Orbits

•Metric independent of t and all the angles ϕ_k. Isometries. Conserved quantities. E and L_i.

Radial geodesic equation

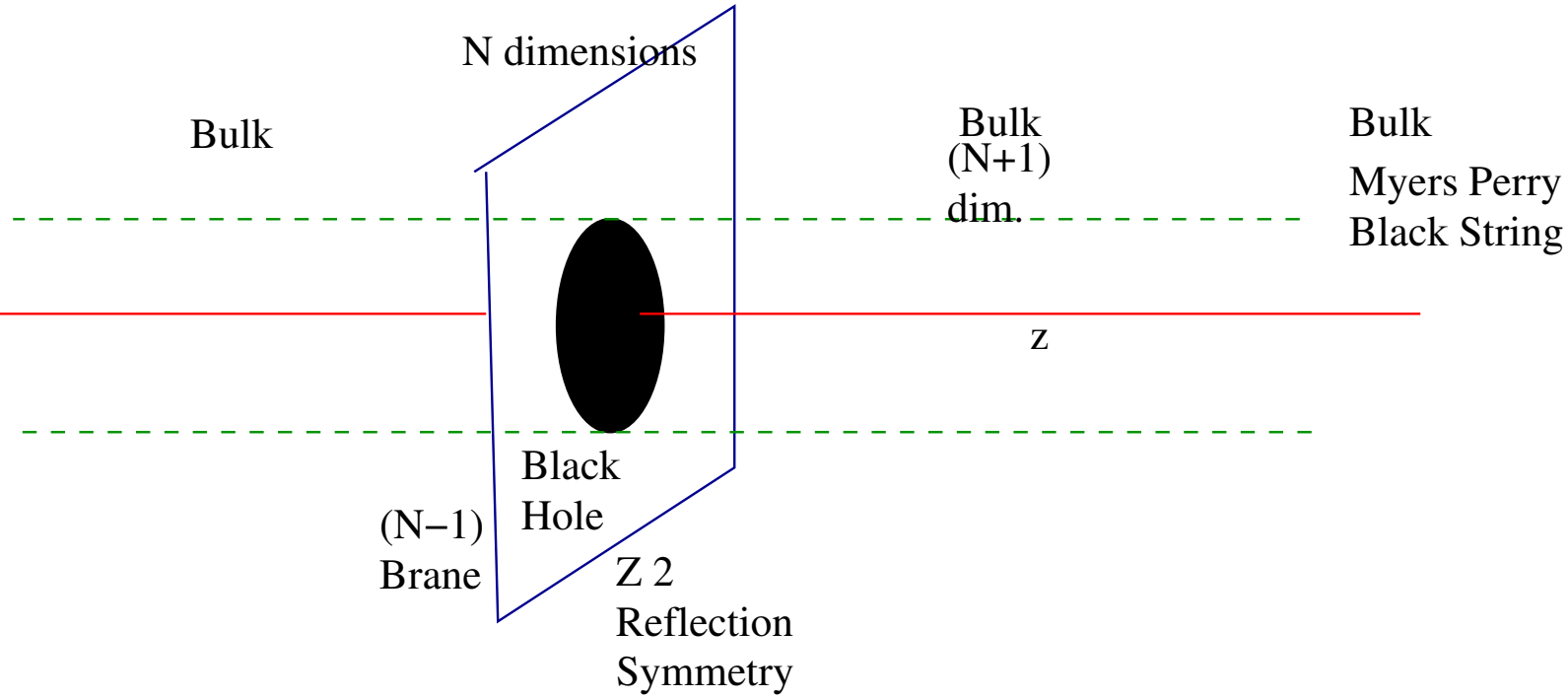
$$[\frac{\Pi F}{\Pi - \mu r}\dot{r}^2 + \sum_{\alpha}\sum_{\beta}(M_{\alpha\beta})^{-1}c_\alpha c_\beta] = 0$$

. F is another function of r and a_k.

•Where c_α are the conserved quantities and Π and F are defined in terms of r and the rotation parameters a_k $M_{\alpha\beta}$ is a matrix function .

• Bulk perspective of a Myers-Perry rotating black hole. Consider a (N-1) brane in (N +1) dimensional brane world. Bulk AdS_{N+1}. Brane world volume is (N-1 +1)=N dim.

• We proposed that a N dimensional MP BH arises from a bulk (N+1) dimensional rotating Myers-Perry black string. Product of a N D MPBH with a flat direction.



BRANE WORLD MYERS−PERRY BLACK HOLE.

• Two cases odd and even (N-1) spatial directions on 3-brane and a rich horizon and singularity structure.

• As earlier we have N dim. masses and angular momenta on the (N-1) brane scaled by various powers of warp factor $\frac{l}{z_0}$. Horizons are given in terms of the rescaled Mass and rotation parameters.

•Singularities, curvature squared $\sim \frac{z^4}{r^{2(-2p-\epsilon)}}$. Singularity at $r = 0$ on the brane and as $z \to \infty$. Pathology

•Causal structure from $(N+1)$ dim. bulk geodesics projected to 4 dim. on the brane in rescaled variables. Matches the central force problem in the effective potential picture. Orbit equation in r.

•$\frac{\Pi F}{\Pi-\mu r}\dot{r}^2+\frac{z^4}{l^4}[\,\Sigma_\alpha\,\Sigma_\beta(M_{\alpha\beta})^{-1}c_\alpha c_\beta+\frac{l^2}{z_1^2}]=0$ $M_{\alpha\beta}$ are matrix functions.

•Specific solution for $N=6$. Lowest even N with a solution. 6 dim. Myers-Perry black hole on a 5-brane embedded in a $(6+1)$ dim. brane world. Bulk is a $(6+1)$ dim. rotating black string.

•Effective potential rules out stable bound geodesics. Unbounded geodesics allowed. Curvature squared finite along unbound orbits. Signal of p-p curvature singularity.

Summary and Conclusions.

•Black string approach of Chamblin, Hawking and Reall provides a method to study bulk structure of brane world black holes. Absence of exact solutions in $D > 4$.

•We have described 4D rotating Kerr black holes, N dim. rotating Myers Perry black holes in a RS single brane scenario.

•Pathological singularity at AdS horizon shown to be a mild p-p curvature singularity. Artifact of linearized approximation in the brane world. Stability.

•Higher dimensional black holes are of current interest in string theories. Myers-Perry and 5 dim. rotating black ring solutions lifted to 10 dim. string theories and D=11 M-theories. New solutions black ring and black saturns. Brane world black hole thermodynamics.