Black holes in extra-dimensional theories

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ASM, Phys. Rev. Lett. **90**, 031303 (2003).
ASM, A. Mehta and J.-M. Luck, Phys. Lett. B**607**, 219 (2005).
ASM and N. Mukherjee, Int. J. Mod. Phys. D**14**, 1095 (2005) (Review).
ASM and N. Mukherjee, Mod. Phys. Lett. A**20**, 2487 (2005).
ASM and N. Mukherjee, Gen. Rel. Grav. **39**, 583 (2007).
ASM, L. P. Singh and D. Gangopadhyay, arXiv:0709.319 to appear in MNRAS (2008).

Black Holes in Modified Gravity and Cosmology

$*$ Modified (String, Braneworld, JBD) black hole solutions

$*$ Modified cosmological evolution (e.g., in inflation, braneworld scenario)

$*$ Existence of Primordial black holes: formation and evolution; observational constraints from end products of Hawking evaporation

$*$ Structure formation through black holes; coalescence of binaries and gravitational waves

$*$ Astrophysical (massive) black holes: modified geometry due to modified gravity

$*$ Probe through Gravitational lensing (weak- and strong-field) observables

Braneworld Black Holes

Modified gravity on the brane: Corrections to the Newtonian potential at large distances:

$$V(r) = \frac{2M}{M_4^2 r}\left(1 + \frac{2l^2}{3r^2}\right)$$

Curvature radius of 5-th dimension, $l \leq 0.2$mm.

Effect of **Kaluza-Klein modes** on metric exterior to static and spherically symmetric matter distribution on brane in the **weak field** limit:

$$dS_4^2 = -\left(1 - \frac{2M}{M_4^2 r} + \frac{4Ml^2}{3M_4^2 r^3}\right)dt^2 + \left(1 + \frac{2M}{M_4^2 r} + \frac{2Ml^2}{3M_4^2 r^3}\right)(dr^2 +$$

Project Weyl term $E_{\mu\nu}$ on brane; Map 4-d Einstein-Maxwell to vacuum braneworld:

$$R_{\mu\nu} = -E_{\mu\nu}; R^\mu_\mu = 0; \nabla^\mu E_{\mu\nu} = 0; \kappa^2 T_{\mu\nu} \leftrightarrow -E_{\mu\nu}$$

Exact black hole solution to effective brane field eqns.:

$$dS_4^2 = -\left(1 - \frac{2M}{M_4^2 r} + \frac{Q}{r^2}\right)dt^2 + \left(1 - \frac{2M}{M_4^2 r} + \frac{Q}{r^2}\right)^{-1} dr^2 + r^2$$

Reissner-Nordstrom type metric with **tidal charge** $Q = -\frac{Ml}{M_4^2}$

Black hole solutions

Spherically symmetric, static solutions to field equations with 5-d cosmological constant:

$$ds_4^2 = -(1 - \frac{2M}{M_4^2 r})dt^2 + \frac{1 - \frac{3M}{2M_4^2 r}}{(1 - \frac{2M}{M_4^2 r})\left(1 - \frac{M(4\beta - 1)}{2M_4^2 r}\right)} + r^2 d\Omega^2$$

in terms of PPN parameter β.

Rotating braneworld black holes:
(in Boyer-Lindquist coordinates)

$$ds_4^2 = -(1 - \frac{2Mr - \beta}{\Sigma})dt^2 - \frac{2a(2Mr - \beta)}{\Sigma}\mathrm{s}in^2\theta dt d\phi$$

$$+ \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + (r^2 + a^2 + \frac{2Mr - \beta}{\Sigma}a^2 \mathrm{s}in^2\theta)\mathrm{s}in^2\theta d\phi^2$$

tidal charge β. Kerr black hole when $\beta \to 0$.
$\Delta = r^2 a + a^2 - 2Mr + \beta \qquad \Sigma = r^2 + a^2 \mathrm{c}os^2\theta$
Other solutions, e.g., black strings, wormholes, etc.

Arbitrariness of projected **Weyl term** $E_{\mu\nu}$ and its geometric origin responsible for a **variety** of black hole solutions.

Primordial black holes

Form through various processes in the early universe with typical size of the hubble volume

At **short distances** braneworld gravity is truly 5-dimensional: $1/r^2$ corrections may dominate over $1/r$ or weak field $1/r^3$. Natural to consider 5-d **Schwarzschild soln.**:

$$ds_5^2 = -\left(1 - \frac{r_{BH}^2}{r^2}\right) dt^2 + \left(1 - \frac{r_{BH}^2}{r^2}\right)^{-1} dr^2 + r^2 \left(d\Omega_3^2\right)$$

horizon size $r_0 \ll l$.

Induced 4-d metric near event horizon:

$$dS_4^2 = -\left(1 - \frac{r_{BH}^2}{r^2}\right) dt^2 + \left(1 - \frac{r_{BH}^2}{r^2}\right)^{-1} dr^2 + r^2 \left(d\Omega^2\right)$$

reflects 5-d character of **strong gravitational field** near horizon.

Dilaton black holes in accelerating universe

Gravity coupled to *Dilaton* and *Maxwell* field (Higher dimensional *String* action) $\rightarrow$ Compatification:

$$S = \int d^4x\sqrt{-g}\left[R - 2\partial_\mu\phi\partial^\mu\phi - V(\phi) - e^{-2\phi}F_{\mu\nu}F^{\mu\nu}\right]$$

Dilaton charge and potential:

$$D = \frac{Q^2 e^{2\phi_0}}{2M}$$

$$V(\phi) = \frac{4}{3}\lambda + \frac{\lambda}{3}\left[e^{2(\phi-\phi_0)} + e^{-2(\phi-\phi_0)}\right]$$

Accelerating de Sitter universe with *cosmological constant* λ

Dilaton-de Sitter black hole metric:

$$dS^2 = -\left(1 - \frac{2M}{r} - r(r-2D)H^2\right)dt^2$$

$$+\left(1 - \frac{2M}{r} - r(r-2D)H^2\right)^{-1} dr^2 r(r-2D)\left(d\Omega^2\right)$$

with $H^2 = \frac{\lambda}{3}$. For $H = 0$ the metric goes to GMGHS black hole. For both $D = 0$ and $H = 0$ it reduces to Schwarzschild metric. When $\phi = \phi_0 = 0$, RN metric is recovered.

Primordial black holes in JBD models

Generalized JBD (Scalar-tensor) models:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}[\phi R - \frac{\omega(\phi)}{\phi}(\partial_\mu \phi)^2] + S_{\text{matter}}$$

Solutions in radiation dominated era:

$$a(t) = a_i(\frac{t}{t_i})^{\frac{3}{\omega_i+6}};\ \phi(t) = \phi_i(\frac{t}{t_i})^{-\frac{3}{\omega_i+6}};\ \omega(t) = \omega_i(\frac{t}{t_i})^{\frac{2\omega_i+3}{\omega_i+6}}$$

Black holes with variable G

$$r = \frac{2M}{\phi};\ T = \frac{\phi}{8\pi M}$$

Evolving size and temperature.

Gravitational memory ?

Braneworld cosmology

$$H^2 = \frac{8\pi}{3M_4^2}\Big(\rho + \frac{\rho^2}{2\lambda} + \rho_{KK}\Big) + \frac{\Lambda_4}{3} - \frac{k}{a^2}$$

Cosmological Solutions for the Scale Factor and Energy Density:

$$\rho = \frac{3M_4^2}{32\pi t(t+t_c)}$$

$$a = a_0\Big[\frac{t(t+t_c)}{t_0(t_0+t_c)}\Big]^{1/4}$$

$t_c \equiv l/2$ is the **transition time**.

For $t < t_c$ (or $\rho >> \lambda$): Non-conventional **high energy regime**:

$a = a_0(\frac{t}{t_0})^{1/4}; \quad \rho = \frac{3M_4^2}{32\pi t_c t}$

Radiation temperature: $\rho = \frac{\pi^2 g T^4}{30}$

Modified temperature-time relation: $T_c(l_{\max}) \approx 10^3\mathrm{GeV}$

For $t >> t_c$ (or $\rho \leq \lambda$): **Standard low energy regime**:

$a = a_0(\frac{t}{(t_0 t_c)^{1/2}})^{1/2}; \ \rho = \frac{3M_4^2}{32\pi t^2}$

Accretion and evaporation by primordial black holes

BH **radius** and **temperature**:

$$r_{BH} = \left(\frac{8}{3\pi}\right)^{1/2}\left(\frac{l}{l_4}\right)^{1/2}\left(\frac{M}{M_4}\right)^{1/2} l_4; T_{BH} = \frac{1}{2\pi r_{BH}}$$

Rate of **Hawking evaporation**:

$$\frac{dM}{dt} \approx -g_{\text{brane}}\bar{\sigma}_4 A_{\text{eff},4} T_{BH}^4$$

Accretion of surrounding **radiation** effective in braneworld **high energy phase**. Radiation density scales as:

$$\rho_R = \frac{3M_4^2}{32\pi t_c t}$$

. Rate of accretion:

$$\left(\frac{dM}{dt}\right)_{\text{acr}} = f\rho_R A_{\text{eff},4}$$

Accretion efficiency: $0 < f < 1$.

Cosmology with Black Holes

A **population** of primordial black holes with an initial **number density** $n_{BH}(t_0)$ and total **energy fraction** β exchange energy with the surrounding radiation by accretion and evaporation. $\rho_T(t_0) = \rho_R(t_0) + \rho_{BH}(t_0)$
$\rho_{BH}(t_0) = \beta\rho_T(t_0) = M_0 n_{BH}(t_0)$
Evolution of energy components:

$$\begin{aligned} \dot{M} &= -\frac{AM_4^2}{Mt_c} + \frac{BM}{t} \\ \frac{d}{dt}\Big(\rho_R(t)a^4(t)\Big) &= -\dot{M}(t)n_{BH}(t)a(t) \end{aligned}$$

BH's grow monotonically for a while

$$\frac{M(t)}{M_0} \simeq \Big(\frac{t}{t_0}\Big)^B$$

During this period $n_{BH}(t) \propto a(t)^{-3}$. Since $a(t) \propto t^{1/4}$, thus $(n_{BH}(t)/n_{BH}(t_0)) = (t_0/t)^{3/4}$.

The condition for the universe to remain radiation dominated (i.e., $\rho_{BH}(t) < \rho_R(t)$) at any instant t is

$$\beta < \frac{(t_0/t)^{B+1/4}}{1 + (t_0/t)^{B+1/4}}$$

Examples of BH lifetime enhancement

Numerical integration of the coupled system of the BH equation, radiation density equation, and the hubble expansion equation is required to determine the evolution exactly. Nevertheless:

$*$ For $(l/l_4) \simeq 10^{30}$, BHs with $M_0 = 10^8 M_4 \simeq 10^3$g survive up to the **present era** if $t_t \sim t_c$.

$*$ For $(l/l_4) \sim 10^{20}$, black holes formed with $M_0 = 10^{15} M_4 \simeq 10^{10}$g, will have $M_{max} \simeq 10^{15}$g, and will **evaporate now** completing their life cycle as $5D$ black holes.

$*$ For $(l/l_4) \sim 10^{20}$, BHs with $M_0 \leq M_4$ survive up to the era of **nucleosynthesis** and beyond if $t_t \geq 10^5 t_c$.

ASM, Phys. Rev. Lett. **90**, 031303 (2003)

Jordan-Brans-Dicke cosmology

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}[\phi R - \frac{\omega(\phi)}{\phi}(\partial_\mu\phi)^2] + S_{\text{matter}}$$

Friedman Equation:

$$\frac{\dot{a}^2}{a^2} + \frac{\dot{a}\dot{\phi}}{a\phi} - \frac{\omega\dot{\phi}^2}{6\phi^2} = \frac{\rho}{3\phi}$$

JBD field equation:

$$\ddot{\phi} + 3\frac{\dot{a}\dot{\phi}}{a} = \frac{\rho - 3p}{2\omega + 3} - \frac{\dot{\omega}\dot{\phi}}{2\omega + 3}$$

Energy conservation:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

Evolution of PBHs in JBD cosmology

Accretion and evaporation:

$\dot{M}_{acc} = 4\pi f r^2 \rho \qquad \dot{M}_{evap} = -4\pi g r^2 \sigma T^4$

Solutions in radiation dominated era:

$$a(t) = a_i(\frac{t}{t_i})^{\frac{3}{\omega_i+6}};\ \phi(t) = \phi_i(\frac{t}{t_i})^{-\frac{3}{\omega_i+6}};\ \omega(t) = \omega_i(\frac{t}{t_i})^{\frac{2\omega_i+3}{\omega_i+6}}$$

PBHs with $M_0 < M_c$ evaporate out in the R.D. era.

Survival and growth of PBHs:

$$\frac{M_{max}}{M_0} \simeq \frac{1}{1 - BM_0 ln(t_{eq}/t_0)}$$

PBH lifetime:

$$t_{evap} = t_0 \Big[1 - \frac{M_0^3}{f(M_0, t_0)}\Big]^{-3/5}$$

PBHs with $M_0 > M_c$ survive as candidates of dark matter.

ASM, L. P. Singh and D. Gangopadhyay, MNRAS (2008)

Observational constraints on mass distribution

Initial Mass fraction:

$$\alpha_{M_0}(t) = \frac{\rho_{BH}(t)}{\rho_T}; \;\; \alpha_0 = \frac{\beta}{1-\beta}$$

'Final' mass fraction: $\alpha(t_{\mathrm{evap}})$

Strategy: Consider Observational Constraints (on **end products of evaporation**) on $\alpha(t_{\mathrm{evap}})$ at different cosmological epochs: $\alpha(t) < L_{4D}(t)$ or $\alpha(t) < L_{5D}(t)$. Trace back to constrain **initial mass spectrum**.

$\frac{L^0_{5D}}{L^0_{4D}} = \frac{L_{5D}(t_{\mathrm{evap}})}{L_{4D}(t_{\mathrm{evap}})}(\frac{l}{l_{\mathrm{min}}})^{\frac{5-16B}{16-8B}}$; $\qquad l_{\mathrm{min}} \propto t_{\mathrm{evap}}^{1/3}$

Departure from standard constraints sensitive to accretion efficiency: Liddle et. al, PRD **68** (2003). Constraints from:

∗ Diffuse photon background: Sendouda et al, PRD **68** (2003).

∗ sub-GeV galactic antiprotons: Sato el al, PRD **71** (2005).

Gravitational waves from coalescing binaries

Binaries of primordial BHs with parameters in SULCO range emit gravitatonal waves in coalescing stage that may be detected in future interferometers (Inoue and Tanaka, PRL **91** (2003)).

Mechanism for formation of braneworld PBH binaries: interacting black holes in radiation.

$$\dot{m}_i = \frac{Bm_i}{t} - \frac{Am_4^2}{m_i t_c} - \frac{r_i^2 \dot{m}_j}{4d^2}$$

Effect of interaction: e.g. large BH in accreting phase suppresses the growth of smaller BH: **Divergence of mass differences**.

Formation of BH binaries by **three body gravitational interactions**:

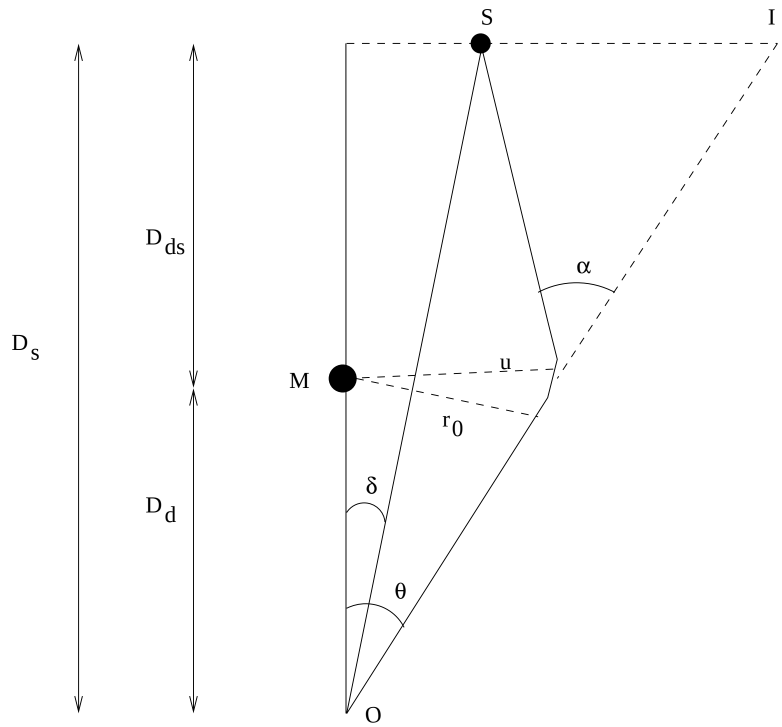
$m_1 = \delta_{12} m_2 \quad m_2 = \delta_{23} m_3 \quad \delta_{12}, \delta_{23} < 1$

Bound system of two black holes gets decoupled from the background expansion. Third neighbour provides tidal force to prevent headlong collision.

$\star$ Example of a braneworld BH binary:

Formation time: $t_f \sim 10^{12} t_c$; Major axis: $\sim 10^7$cm. [ASM, A. Mehta and J.-M. Luck, Phys. Lett. B**607**, 219 (2005)].

Gravitational lensing



Lense equation:

$$\tan\delta = \tan\theta - \frac{D_{ds}}{D_s}[\tan\theta + \tan(\alpha - \theta)]$$

Lensing by **spherically symmetric metrics**:

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + C(r)\left(d\Omega^2\right)$$

Impact parameter u; Distance of closest approach: r_0. $u = [C_0/A_0]^{1/2}$.

Deflection angle for strong gravitational lensing:

$$\alpha(r_0) = \int_{r_0}^{\infty} \frac{2\sqrt{B}dr}{\sqrt{C}\sqrt{\frac{C}{C_0}\frac{A_0}{A} - 1}} - \pi$$

Weak field lensing by braneworld black hole

$$dS_4^2 = -\left(1 - \frac{r_h^2}{r^2}\right)dt^2 + \left(1 - \frac{r_h^2}{r^2}\right)^{-1}dr^2 + r^2 d\Omega^2$$

Variational Principle $\rightarrow$ Bending angle of light:

$$\alpha = 2\left(\frac{l}{l_4}\right)\left(\frac{M}{M_4}\right)l_4^2 r_0^2$$

Comparison with Schwarzschild metric bending angle: $\alpha/\alpha^{Sch} = 2lu_0$. **Magnification**:

$$\mu = \frac{\triangle\theta}{\triangle\delta}\frac{\theta}{\delta} = \left|\frac{\theta^4}{\theta^4 - \alpha_0^4}\right|$$

Impact factor for perceptible magnification:

$$r_0^2 \simeq 2ll_4\frac{M}{M_4}$$

For **solar mass** BHs in the **galactic halo**, $r_0 \sim 1\text{c}m$

Absence of microlensing events do **NOT** rule out braneworld BHs.

ASM and N. Mukherjee,
Mod. Phys. Lett. A**20**, 2487 (2005).

Strong lensing in modified geometry

Lensing **observables:** n relativistic images: $\theta_1, ...\theta_\infty$.
Separation between first image and others:

$$\mathcal{S} = \theta_1 - \theta_\infty$$

Ratio of **fluxes**:

$$\mathcal{R} = \frac{\mu_1}{\sum\limits_{n=2}^{\infty} \mu_n}$$

Minimum impact parameter:

$$u_m = D_d \theta_\infty$$

Galactic Centre Black Hole
$M = 2.8 \times 10^6 M_\odot$; $D_d = 8.5$ kpc

Obser-vables	Schwzsld metric	Brane $Q = -0.1$	Metrics $\beta = 1 - 10^{-4}$
θ_∞	16.87	17.87	16.87
$\mathcal{S}$	0.0211	0.0142	0.01923
r_m	6.82	7.02	6.887
u_m/r_s	2.6	2.75	2.6
$\bar{a}$	1	0.9708	0.9999
$\bar{b}$	-0.4002	-0.612	-0.429

ASM and N. Mukherjee, IJMP (2005); Gen. Rel. Grav. (2007).

Black Holes in extra dimensions—Summary

ASM, Phys. Rev. Lett. **90**, 031303 (2003); ASM, A. Mehta and J.-M. Luck, Phys. Lett. B**607**, 219 (2005); ASM and N. Mukherjee, Mod. Phys. Lett. A**20**, 2487 (2005); ASM and N. Mukherjee, Int. J. Mod. Phys. D**14**, 1095 (2005); ASM and N. Mukherjee, Gen. Rel. Grav. **39**, 583 (2007) ASM, L. P. Singh and D. Gangopadhyay, MNRAS (2008).

∗ **Modified gravity** in string, brane models. **Several Black Hole Solutions**.

∗ **Primordial** BHs. modified geometry near horizon. Modified evaporation. Effective accretion of radiation.

Long-lived primordial BHs. Cosmological constraints modified.

∗ Interacting BHs. Divergence of mass differences. Formation of **BH-BH binaries**. Coalescence. **Gravitational waves**.

∗ **Gravitational lensing**. Reduced observability in microlensing. Candidates of **halo dark matter ?**

∗ Strong gravitational lensing. **Modified observables** for future probes.