Unparticle physics at hadron Colider

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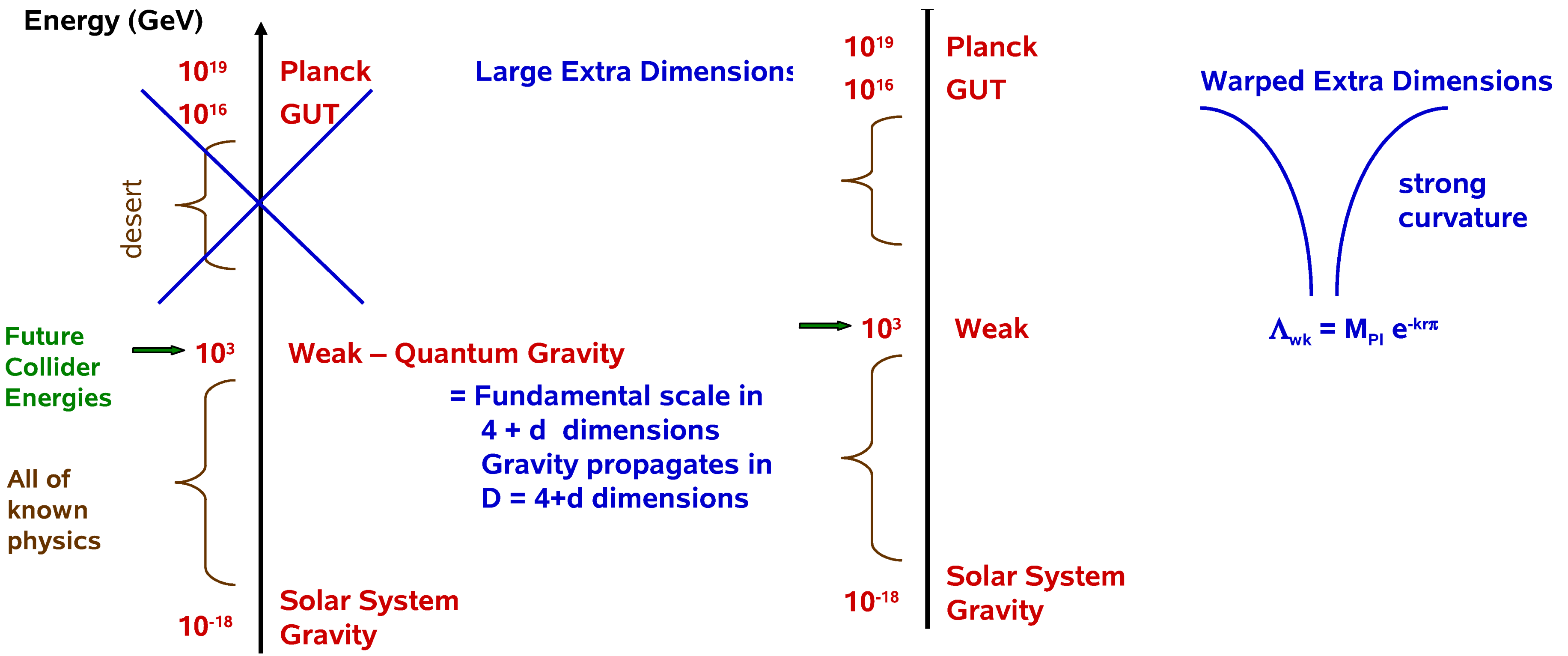
- Banks-Zaks & Georgi's $\mathcal{U}$-particle proposal
- Necessary $\mathcal{U}$-particle ingredients
- Di-lepton & di-photon production at hadron collider
- Unparticle <===> extra dimensions ?
- Summary

arXiv:0705.4599[hep-ph] with V Ravindran

arXiv:0709.2478[hep-ph] with MC Kumar, V Ravindran & A Tripathi

Some Non SUSY avenues beyond the SM

ADD was the first proposal to address the hierarchy problem in the context of extra dimensions. They provide an alternate view of the hierarchy between the EW and the Planck scale— additional structure in the gravity sector in contrast to previous approaches which introduced new structure in the particle physics



Geometry of extra spacial dimensions is responsible for the Hierarchy. These theories should have a viable mechanism (Brane world scenarios) to hide the extra dim such that space time is effectively four consistent with known physics

Banks-Zaks Perturbative IR fixed point

Banks & Zaks Nucl. Phys. B196 (1982) 189

- QCD β function calculated up to 4-loops in the $\overline{\mathrm{MS}}$ scheme

$$\frac{\partial a_s}{\partial \ln \mu^2} = \beta(a_s) = -\beta_0 a_s^2 - \beta_1 a_s^3 - \beta_2 a_s^4 - \beta_3 a_s^5 + \mathcal{O}(a_s^6)$$

$$a_s = \alpha_s/4\pi = g_s^2/16\pi^2, \; g_s = g_s(\mu^2)$$

- 4-loops β function for $N_c = 3$

$$\beta_0 = 11 - \tfrac{2}{3} N_f \qquad \beta_1 = 102 - \tfrac{38}{3} N_f \qquad \beta_2 = \tfrac{2857}{2} - \tfrac{5033}{18} N_f + \tfrac{325}{54} N_f^2$$

$$\beta_3 = \left(\tfrac{149753}{6} + 3564\zeta_3\right) - \left(\tfrac{1078361}{162} + \tfrac{6503}{27}\zeta_3\right) N_f + \left(\tfrac{50065}{162} + \tfrac{6472}{81}\zeta_3\right) N_f^2 + \tfrac{1093}{729} N_f^3$$

Ritbergen, Vermaseren, Larin Phys. Lett. B400 (1997) 379

- Varying N_f or N_c affects asymptotic freedom

- N_f dependence
  - $\beta_3 > 0$ for $N_f > 0$
  - β_2 changes sign before β_1

    $\beta_1(N_f = 8.05) = 0$

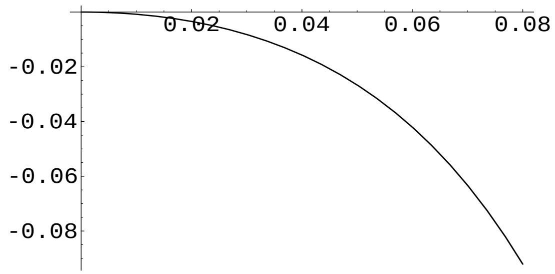
    $N' = 8.05$
  - β_1 changes sign before β_0

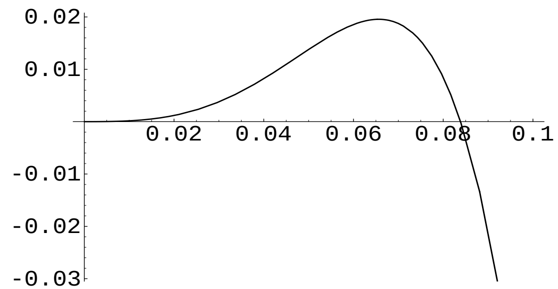
    $\beta_0(N_f = 16.5) = 0$

    $N^* = 16.5$

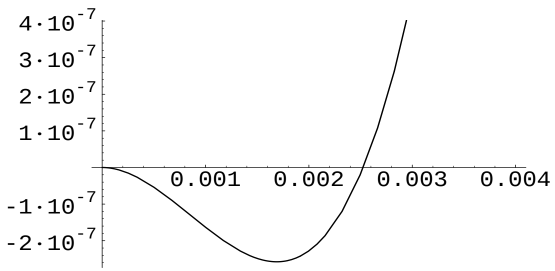
β-function

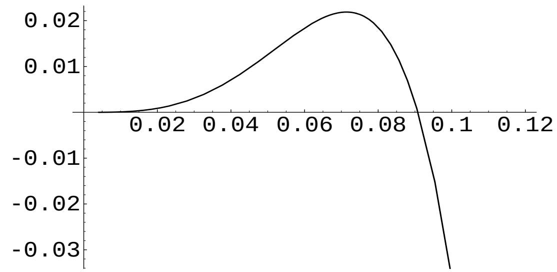
- Implies the presence of a non-trivial zero of the β function





$N_f = 6 \;\; (N_f < N')$ $\qquad$ $N_f = 17 \;\; (N_f > N^*)$





$$N' < (N_f = 16.1) < N^*$$

- As $N_f \longrightarrow N^*$, zeros of the $\beta(\alpha_s)$ occurs at lower values of a_s— the zero of the β function is in the perturbation region

$\beta(a_s; N_f)$-function

- The theory can in principle be studied as an expansion in powers of $(N_f - N^*)$
- If $N' < N_f < N^*$ the β function has a non trivial zero & if $N_f = N^*(1 - 11\,\varepsilon)$, then the zero is at

$$a_s^* = \frac{\varepsilon/3}{21 + 11C_F - (5 + C_F)\varepsilon}$$

- Close to N^*, is highly perturbative region (a_s is small), hence two loop is sufficient to extract the IR fixed point
- $(N_f - N^*)$ expansion suggests that for N_f in the range $N' < N_f < N^*$, close to N^* the theory has an:
  - Exact scale invariant sector
  - No particle interpretation
  - No mass gaps

Scenario Proposed by Georgi: Unparticles

A SCALE INVARIANT SECTOR WEAKLY COUPLED TO THE SM

H. Georgi, Phys. Rev. Lett. 98 (2007) 221601

- Scale invariant sector (Banks-Zaks) which is hidden from the SM at low energies
- In the UV theory the hidden sector could couple to the SM fields through some non-renormalisable interactions by the exchange of massive particles M

$$\mathcal{L}_{UV} = \frac{O_{BZ} O_{SM}}{M^{d_{BZ}+d_{SM}-4}}$$

- The hidden sector has a non-trivial IR fixed point, $\Lambda_{\mathcal{U}}$, below which the hidden sector exhibits scale invariance and the operators $O_{BZ} \rightarrow O_{\mathcal{U}}$ unparticle operator with scaling dimension $d_{\mathcal{U}}$

$$\begin{aligned} \mathcal{L}_{\mathcal{U}} &= C_{\mathcal{U}} \frac{\Lambda_{\mathcal{U}}^{d_{BZ}-d_{\mathcal{U}}}}{M^{d_{BZ}+d_{SM}-4}} O_{\mathcal{U}} O_{SM} \\ &= \frac{\lambda}{\Lambda_{U}^{d_{\mathcal{U}}}} O_{\mathcal{U}} O_{SM} \qquad\qquad d_{SM} = 4 \\ \lambda &= C_{\mathcal{U}} \left(\frac{\Lambda_{\mathcal{U}}}{M} \right)^{d_{BZ}} \end{aligned}$$

LOW ENERGY EFFECTIVE THEORY WHICH IS SCALE INVARIANT BELOW THE CUT OFF SCALE $\Lambda_{\mathcal{U}}$

General $\mathcal{U}$-particle coupling to the SM

- Unparticle operators with different Lorentz structure $O_{\mathcal{U}}$, $O_{\mathcal{U}}^{\mu}$, $O_{\mathcal{U}}^{\mu\nu}$ has been considered

$$\frac{\lambda_s}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} T_{\mu}^{\mu} O_{\mathcal{U}} \qquad \frac{\lambda_v}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \overline{\psi}\gamma_{\mu}\psi O_{\mathcal{U}}^{\mu} \qquad \frac{\lambda_t}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} T_{\mu\nu} O_{\mathcal{U}}^{\mu\nu}$$

Scalar | Vector | Tensor

- λ_{κ} dimensionless coupling corresponding $\mathcal{U}$-particle operator $O_{\mathcal{U}}^{\kappa}$
- $T_{\mu\nu}$ energy momentum tensor of the SM
- $d_{\mathcal{U}}$ scaling dimension of $O_{\mathcal{U}}^{\kappa}$

Phase space for real emission

- Scale invariance can be used to fix the 2-point function of the unparticle

$$\langle 0|O_{\mathcal{U}}(x)O_{\mathcal{U}}^{\dagger}(0)|0\rangle = \int \frac{d^4P}{(2\pi)^4} \exp(-iP.x)\rho_{\mathcal{U}}(P^2)$$

Spectral density $\rho_{\mathcal{U}}$

$$\rho_{\mathcal{U}} = (2\pi)^4 \int d\tilde{k} |\langle 0|O_U(0)|k\rangle|^2 \delta^4(p-k)$$

$|k\rangle$ is an unparticle state with 4-momentum k^{μ} produced from vacuum by O_U

$$\begin{aligned} \rho_{\mathcal{U}}(P^2) &= \int d^4x \exp(iP.x)\langle 0|O_{\mathcal{U}}(x)O_{\mathcal{U}}^{\dagger}|0\rangle \\ &= A_{d_U}\Theta(P^0)\Theta(P^2)(P^2)^{\alpha} \end{aligned}$$

- Scale invariance is used to fix the *exponent* α, while the *normalisation* A_{d_U} is fixed by comparing with the n-particle phase space

Scale Invariance

- Scale transformation: $x \longrightarrow \lambda x$ $\qquad O_U(\lambda x) \longrightarrow \lambda^{-d_U} O_U(x)$

$$
\begin{aligned}
A_{d_U}\Theta(P^0)\Theta(P^2)(P^2)^\alpha &= \lambda^{-2(d_U-2)} \int d^4x \exp(i\lambda P.x) < 0|O_U(x)O_U^\dagger(0)|0 > \\
&= \lambda^{-2(d_U-2)} A_{d_U}\Theta(\lambda P^0)\Theta(\lambda^2 P^2)(\lambda^2 P^2)^\alpha
\end{aligned}
$$

  Scale invariance fixes $\alpha = (d_U - 2)$

- Phase space of n massless particles

$$
(2\pi)^4\delta^4\left(P - \sum_{j=1}^{n} p_j\right)\prod_{j=1}^{n}\delta(p_j^2)\theta(p_j^0)\frac{d^4p_j}{(2\pi)^4} = A_n\Theta(P^0)\Theta(P^2)(P^2)^{n-2}
$$

$$
A_n = \frac{16\pi^{5/2}}{(2\pi)^{2n}}\frac{\Gamma(n+1/2)}{\Gamma(n-1)\Gamma(2n)}
$$

- Comparing the spectral density & n-particle phase space, Georgi adopted the normalisation

$$
A_{d_U} \equiv A_n
$$

  with $n \longrightarrow d_U$ now taking *non-integral* values, just a convention as an alternate definition could be absorbed in the Wilson coefficient $C_{\mathcal{U}}$

1-particle phase space $d_U \to 1$

- $d_U = 1 + \varepsilon$

$$\lim_{\varepsilon \to 0_+} \begin{pmatrix} A_{d_U} = 2\pi\varepsilon \\ (P^2)^{n-2} = \frac{1}{(P^2)^{1-\varepsilon}} \end{pmatrix} \qquad \lim_{\varepsilon \to 0_+} \frac{\varepsilon}{a^{1-\varepsilon}} = \delta(a)$$

$$\rho(P^2) \longrightarrow 2\pi\theta(P^0)\delta(P^2)$$

reproduces the 1-particle phase space

- Phase of A + B $\longrightarrow \mathcal{U}$ + 1 + 2 + 3 + $\cdots\ n-1$

$$d\Phi(P) = (2\pi)^4 \delta^4 \left(P - \sum_j^n p_j \right) \prod_j^n d\tilde{\phi}(p_j) \frac{d^4 p_j}{(2\pi)^4}$$

Particle $\quad d\tilde{\phi} = 2\pi\theta(p^0)\delta(p^2)$

Unparticle $\quad d\tilde{\phi}_U = A_{d_U}\theta(P^0)\theta(P^2)(P^2)^{d_U-2}$

- $\mathcal{U}$-particles does not have a definite invariant mass, instead a continuous mass spectrum

UNPARTICLE STUFF WITH SCALE DIMENSION d_U LOOKS LIKE A NON-INTEGRAL NUMBER d_U OF INVISIBLE MASSLESS PARTICLES

$\mathcal{U}$-particle propagator

H. Georgi, Phys. Lett. B650 (2007) 275

- Exchange of a virtual $\mathcal{U}$-particle corresponding to operator O_U^κ between the SM particles would need the propagator. Using Källen-Lehmann spectral representation

$$\int d^4x \, \exp(iP.x)\langle 0|T(O_U(x)O_U(0)|0\rangle = \frac{i}{2\pi}\int_0^\infty dM^2\, \rho_U(M^2)\frac{1}{P^2-M^2+i\varepsilon}$$

$\rho_{\mathcal{U}}(P^2) = A_{d_U}\Theta(P^0)\Theta(P^2)(P^2)^{d_U-2}$ the integral convergent for $1 < d_U < 2$

- The unparticle propagator is

$$\Delta_F^\kappa(P^2) = \frac{iA_{d_\mathcal{U}}}{2\sin(d_\mathcal{U}\pi)}\frac{B_\kappa}{(-P^2-i\varepsilon)^{2-d_\mathcal{U}}}$$

$O_\mathcal{U}$	1
$O_\mathcal{U}^\rho$	$\eta_{\mu\nu}(P) = -g_{\mu\nu} + \frac{P_\mu P_\nu}{P^2}$
$O_\mathcal{U}^{\rho\sigma}$	$B_{\mu\nu\alpha\beta}(P) = \frac{1}{2}\left(\eta_{\mu\alpha}\eta_{\nu\beta}+\eta_{\mu\beta}\eta_{\nu\alpha}-\frac{2}{3}\eta_{\mu\nu}\eta_{\alpha\beta}\right)$

- Scale invariance and the transverse properties of the vector and tensor operators fixes the $\mathcal{U}$-particle propagator

Complex phase

- Peculiar propagator of scale invariant $\mathcal{U}$-particle

$$(-P^2 - i\varepsilon)^{d_U - 2} = \begin{cases} |P^2|^{d_U - 2} & P^2 < 0 \quad \text{No Complex Phase} \\ |P^2|^{d_U - 2} \exp(-i d_U \pi) & P^2 > 0 \quad \text{Complex Phase} \end{cases}$$

- The $\mathcal{U}$-particle propagator has an unusual phase in the time-like region, which can produce interesting interference patterns of s-channel $\mathcal{U}$-particle exchange and the SM processes
- $d_U \to 1^+$ the standard results are retrieved

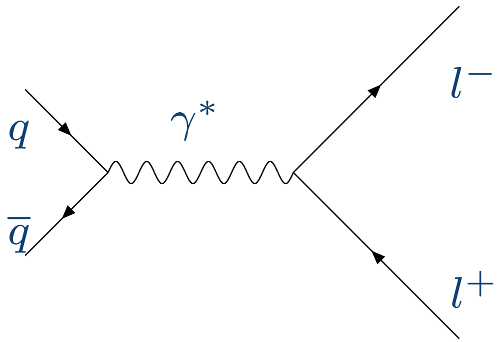
$$\lim_{d_U \to 1^+} \Delta_F^{\kappa}(P^2) = \frac{B_\kappa}{P^2}$$

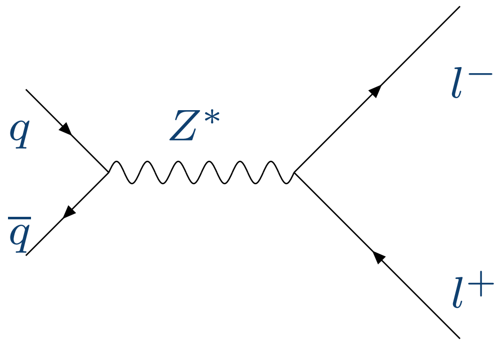
UNPARTICIE PHYSICS ASSOCIATED WITH HIDDEN SCALE INVARIANT SECTOR WITH NONTRIVIAL INFRARED FIXED POINT AT A HIGH ENERGY SCALE HAS INTERESTING PHENOMENOLOGICAL CONSEQUENCES AT PRESENT AND FUTURE COLLIDERS

Drell-Yan Process

$$\boldsymbol{P_1(p_1) + P_2(p_2) \longrightarrow [\gamma, Z, \mathcal{U}] + \text{hadronic states}(X)}$$
$$\boldsymbol{\hookrightarrow l^+(k_1) + l^-(k_2) \qquad (k_1 + k_2)^2 = Q^2}$$

To Leading Order in QCD





SM

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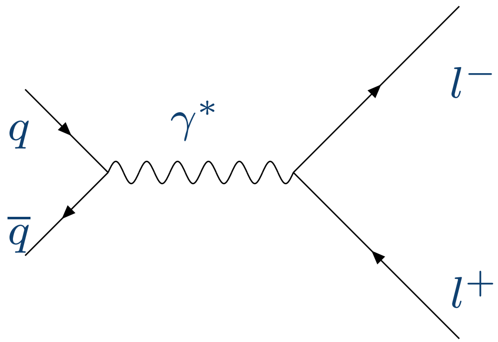

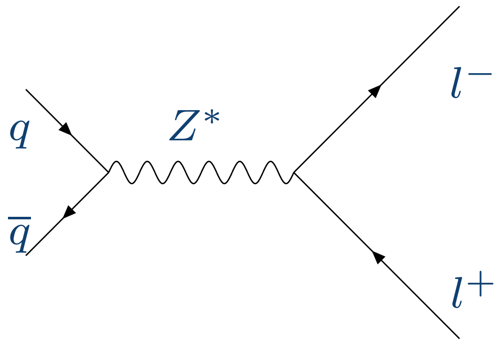

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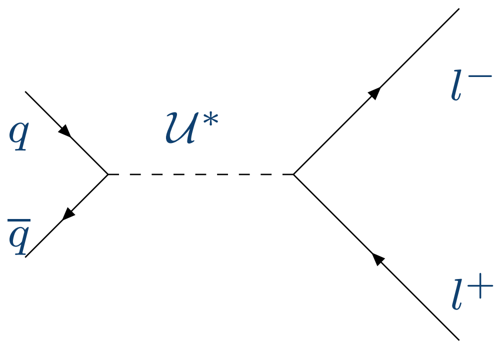

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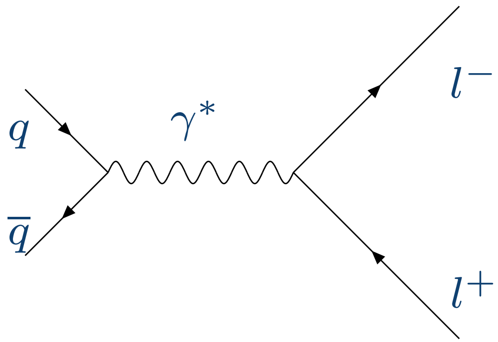

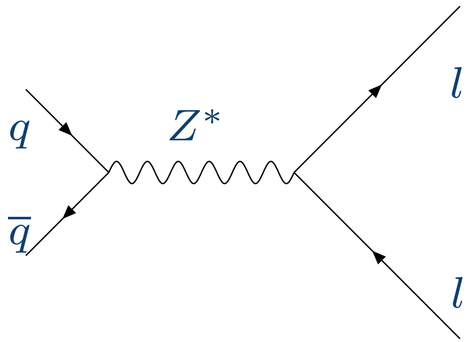

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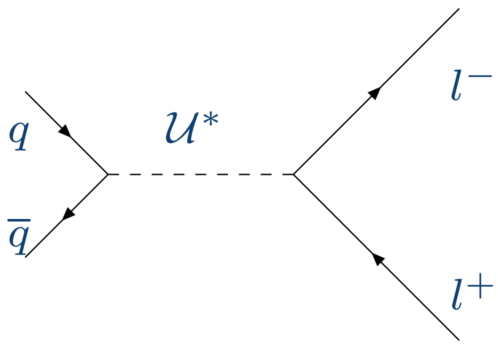
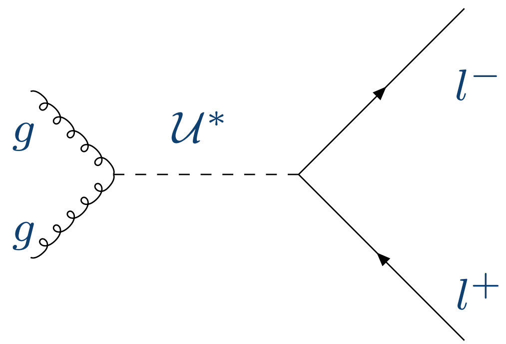


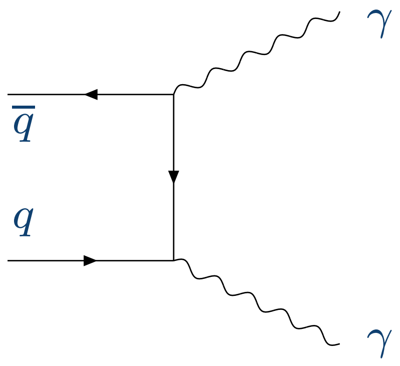


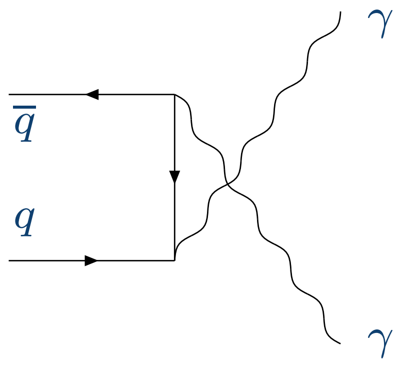
$Unparticle$

Di-photon Process

$$P_1(p_1) + P_2(p_2) \longrightarrow \gamma(k_1) + \gamma(k_2) + X$$

To Leading Order in QCD





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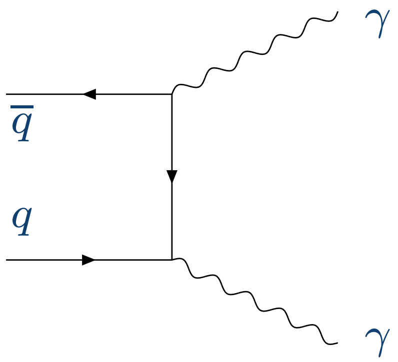

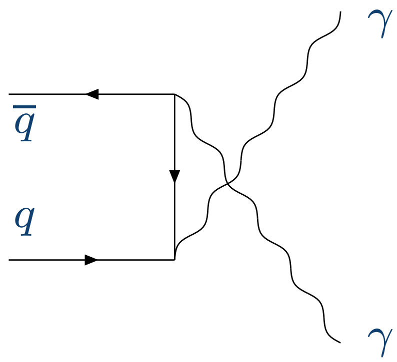

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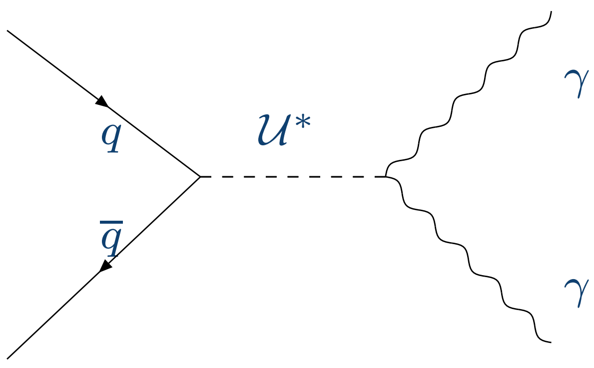

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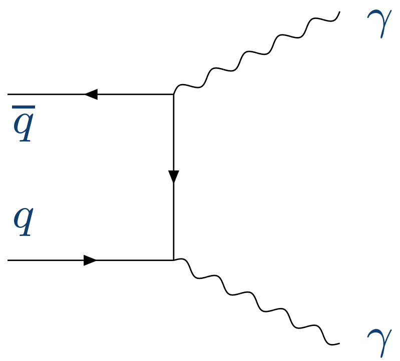

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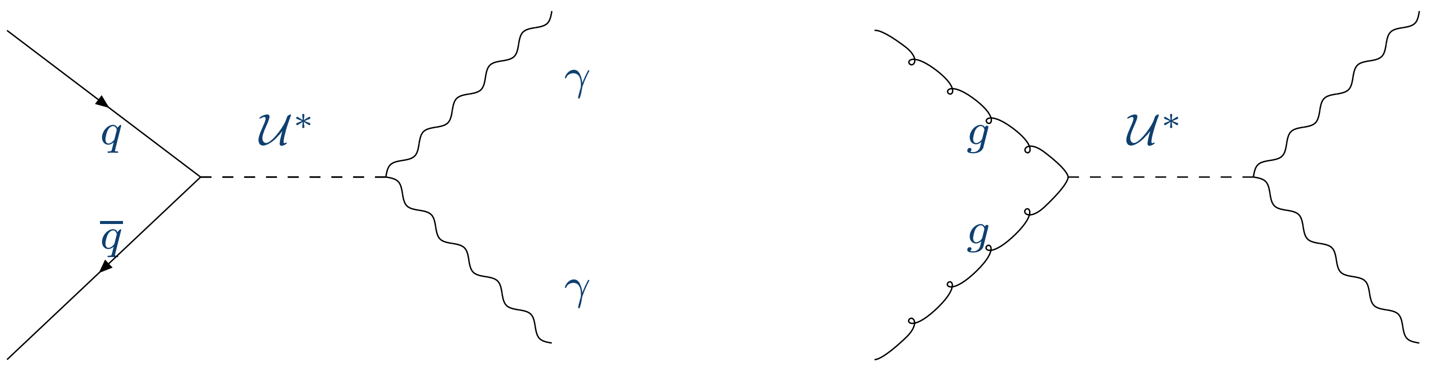


Unparticle

Observables

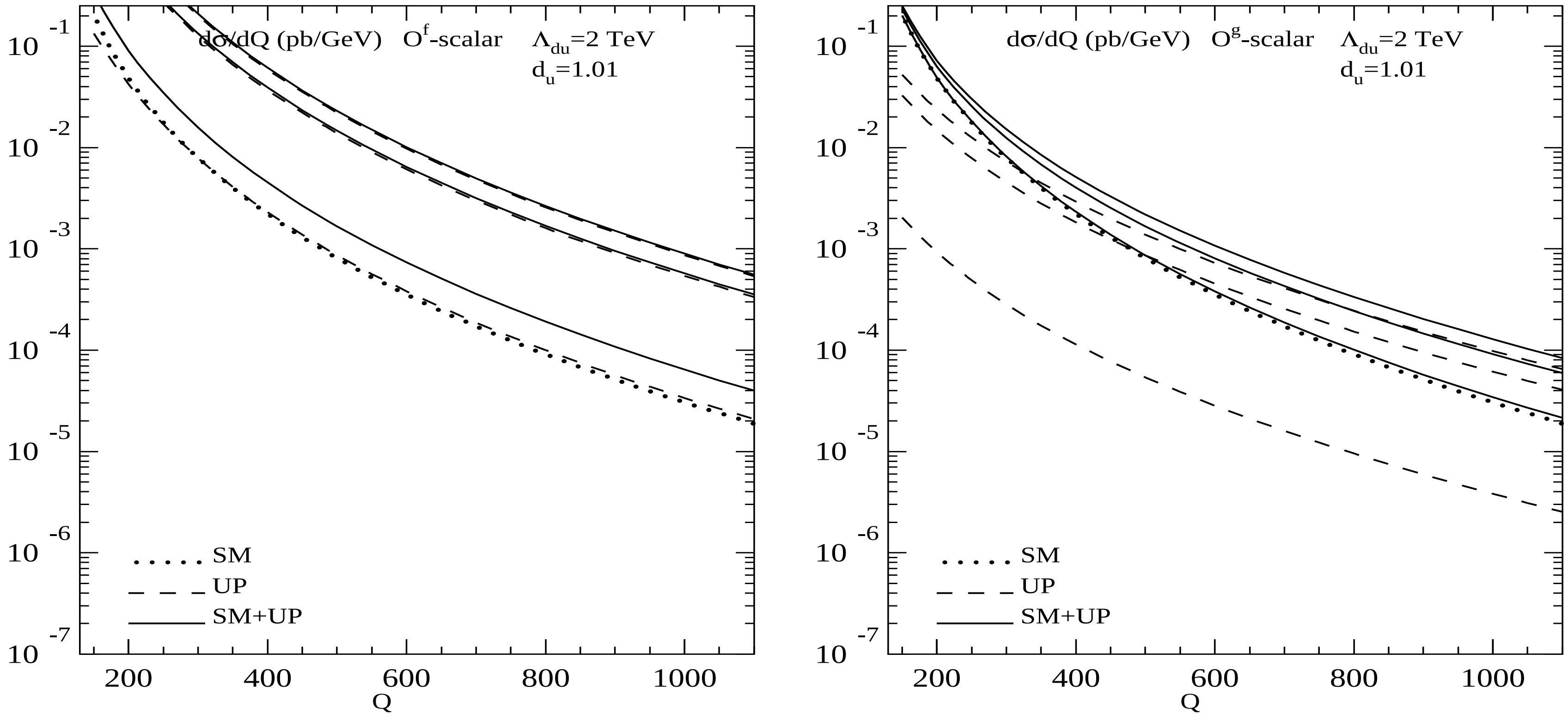
$\boxed{\mathcal{U}\text{-particle parameters } (\lambda,\ \Lambda,\ d_{\mathcal{U}})}$

Distributions:

$$\frac{d\sigma(Q)}{dQ} \qquad \frac{d\sigma(Y)}{dY} \qquad \frac{d\sigma(\cos\theta)}{d\cos\theta}$$

Invariant mass distribution of lepton pair: Scalar $\mathcal{U}$-particle

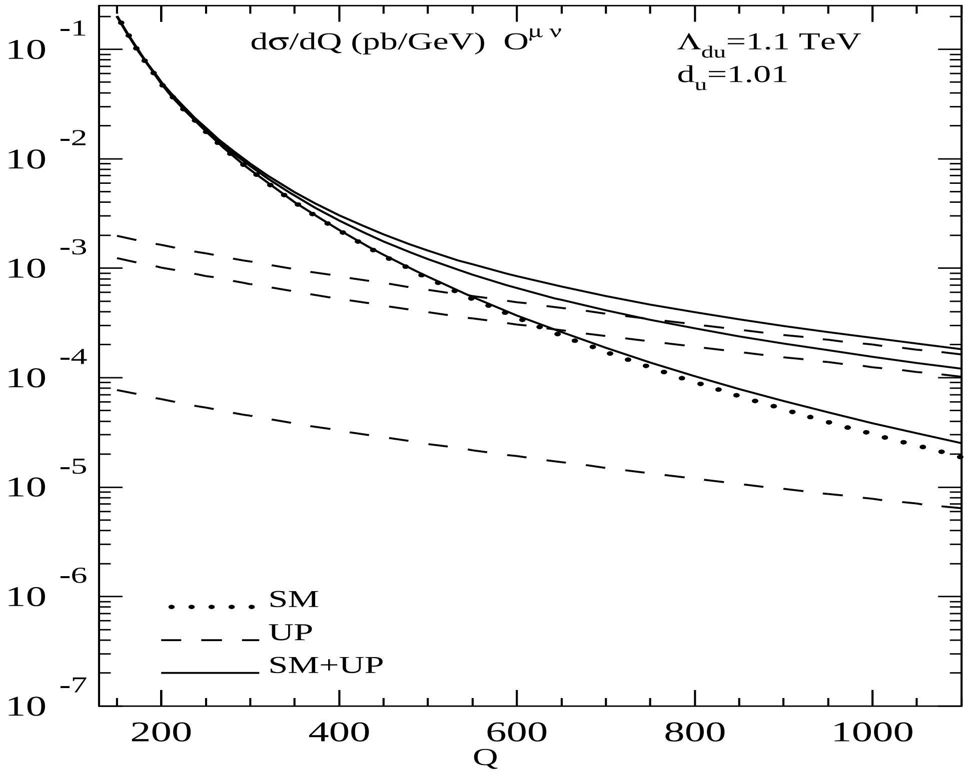
$$\frac{d\sigma^{DY}(Q)}{dQ}$$



- $q\bar{q} \longrightarrow \ell^+\ell^- \quad qg \longrightarrow q\,\ell^+\ell^- \quad \bar{q}g \longrightarrow \bar{q}\,\ell^+\ell^-$
- $gg \longrightarrow \ell^+\ell^-$
- Bottom set corresponds to $\lambda_s = 0.4$, middle $\lambda_s = 0.8$ and upper $\lambda_s = 0.9$

Tensor $\mathcal{U}$

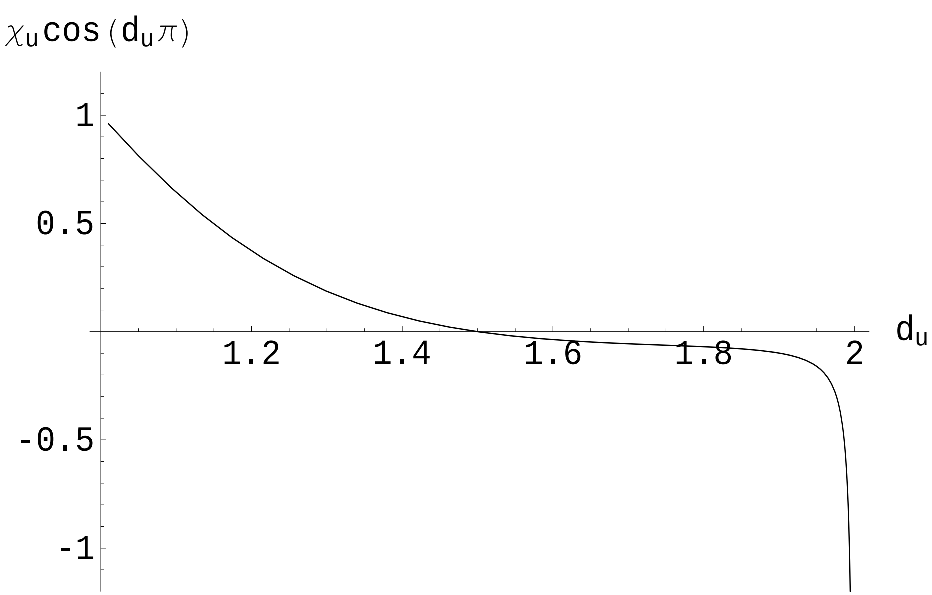
$$\frac{d\sigma^{DY}(Q)}{dQ}$$

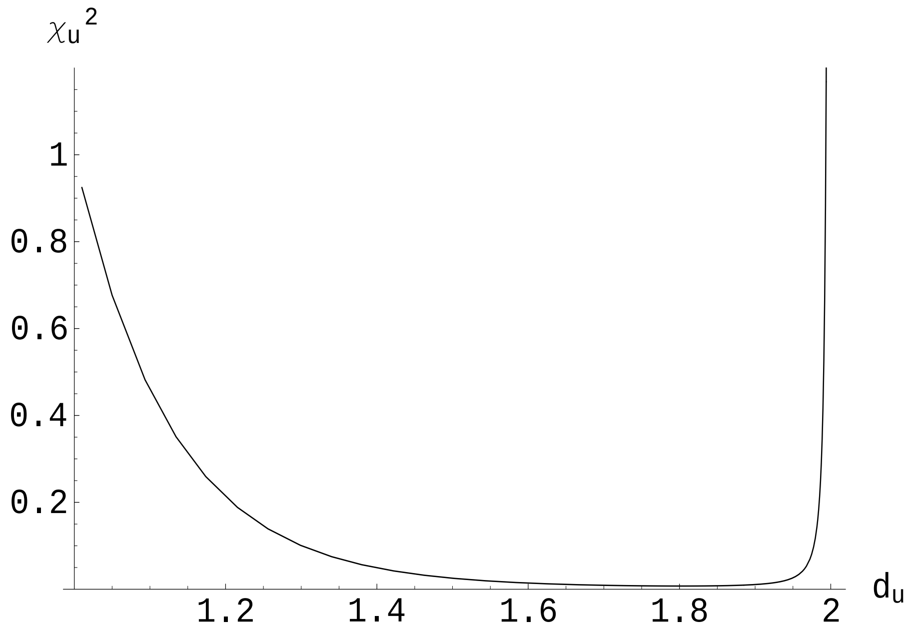


- Tensor $\mathcal{U}$-particles producing a di-lepton pair with invariant mass Q. $\lambda_t = 0.4,\ 0.8$ and 0.9

Di-photon production

$$P_1(p_1) + P_2(p_2) \to \gamma(p_3) + \gamma(p_4) + X(p_X)$$





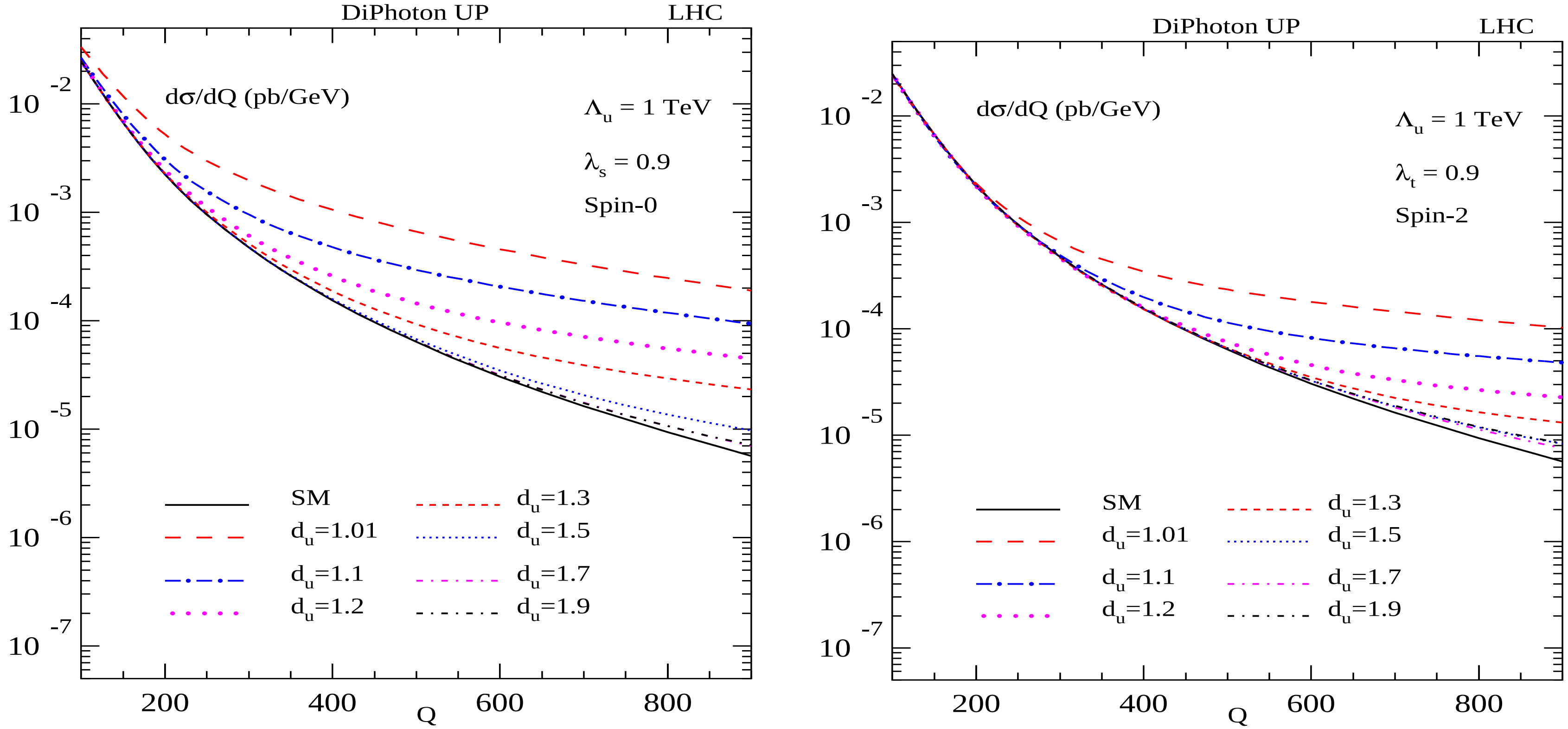
$$\chi_U = \frac{A_{d_U}}{2\sin(d_U \pi)}$$

The function $\chi_{\mathcal{U}}\ \cos(\pi d_{\mathcal{U}})$ (left) and $\chi^2_{\mathcal{U}}$ (right) showing its variation with the scaling dimension $d_{\mathcal{U}}$ of the unparticle operator.

with MC Kumar, V Ravindran & A Tripati arXiv:0709.2478

Di-photon invariant mass distribution

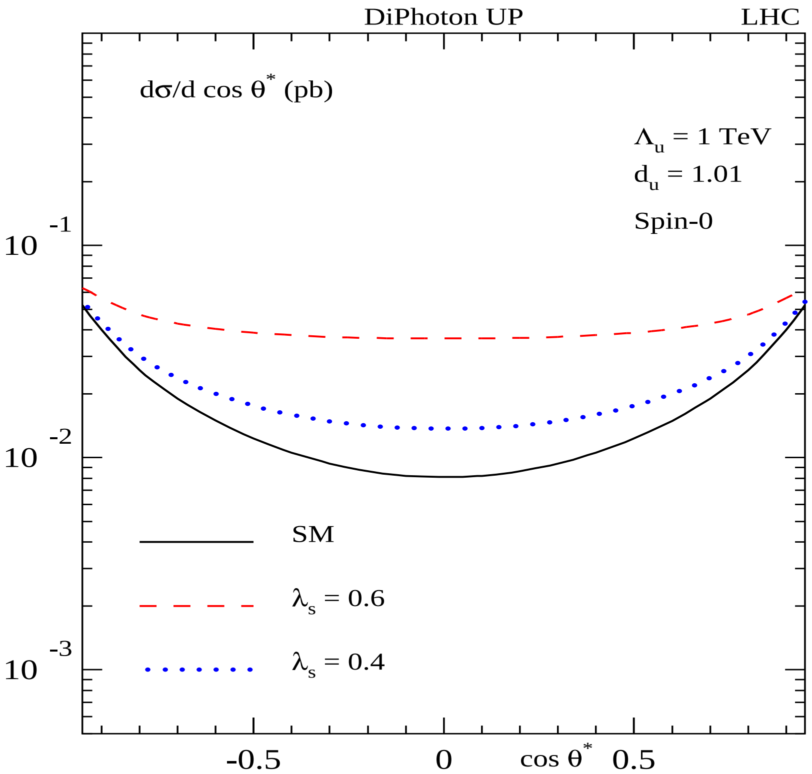
$$\frac{d\sigma(Q)}{dQ}$$

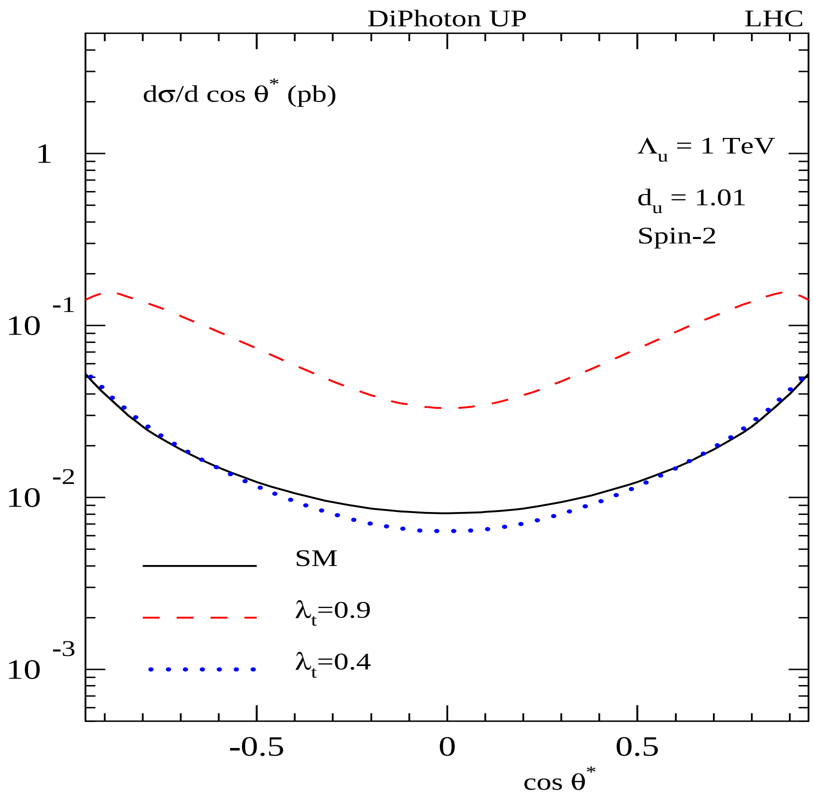


- Invariant mass distribution plotted for different values of $d_{\mathcal{U}}$ for spin-0 (left) and spin-2 (right) with $\Lambda_{\mathcal{U}} = 1$ TeV and $\lambda_s, t = 0.9$, with an angular cut on the photon $|\cos\theta_\gamma| < 0.8$.

Angular distribution

$$\frac{d\sigma}{d\cos\theta^*} \qquad\qquad \cos\theta^* = \frac{p_1.(p_3 - p_4)}{p_1.(p_3 + p_4)}$$

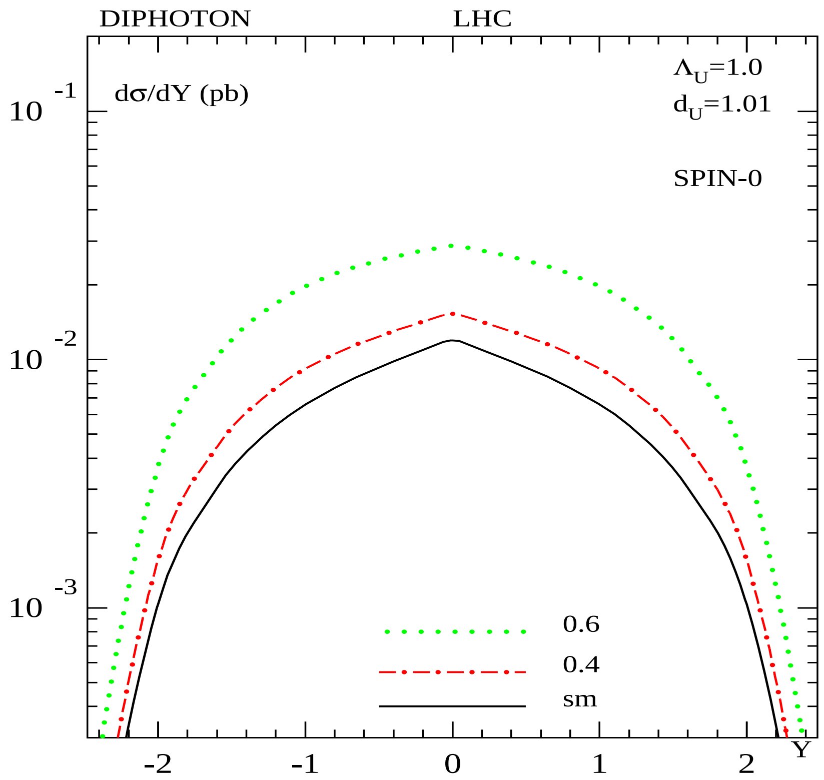


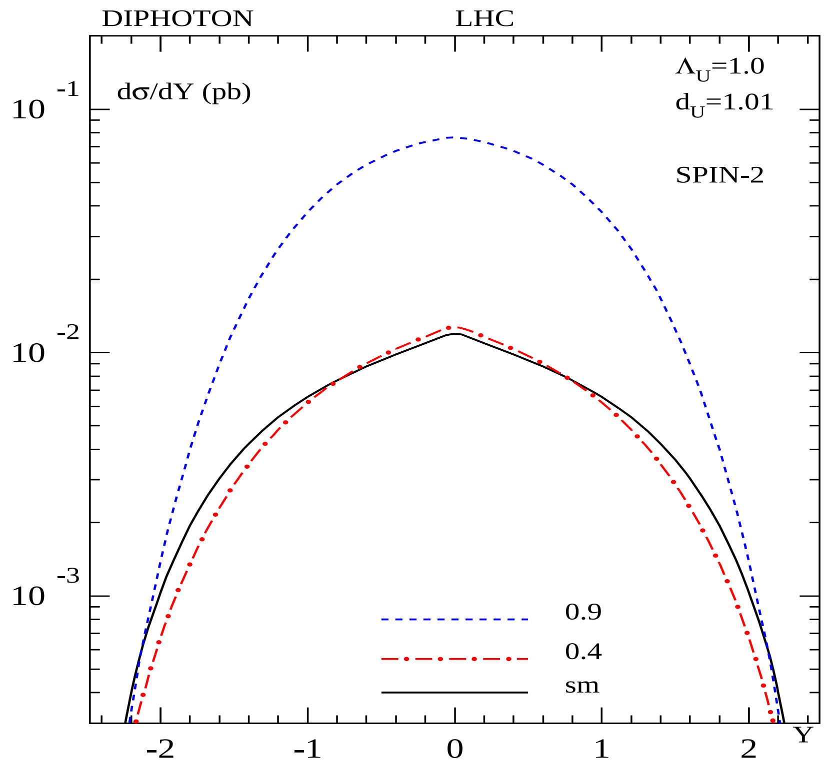


- Angular distribution of the photon for spin-0 (left) and spin-2 (right) with $\Lambda_{\mathcal{U}} = 1$ TeV and $d_{\mathcal{U}} = 1.01$ with $\lambda_s = 0.6, 0.4$ and $\lambda_t = 0.9, 0.4$. The invariant mass distribution has been integrated in the range 600 GeV < Q , 0.9 $\Lambda_{\mathcal{U}}$.

Rapidity distribution

$$\frac{d\sigma}{dY} \qquad\qquad Y = \frac{1}{2}\log\left(\frac{p_2.q}{p_1.q}\right)$$





- Rapidity distribution of the di-photon system for spin-0 (left) and spin-2 (right) $\Lambda_U = 1$ TeV, $d_U = 1.01$, $\lambda_s = 0.6, 0.4$ $\lambda_t = 0.9, 0.4$ and Q in the range 600 GeV <Q< 0.9 Λ_U

Deconstructing unparticles

Stephanov Phy. Rev. D76 (2007) 035008

- Using Källen-Lehmann spectral representation the unparticle propagator is

$$\int d^4x \, \exp(iP.x)\langle 0|T(O_U(x)O_U(0)|0\rangle = \frac{i}{2\pi}\int_0^\infty dM^2\,\rho_U(M^2)\frac{1}{P^2 - M^2 + i\varepsilon}$$

where

$$\rho_U(M^2) = 2\pi\sum_n \delta(M^2 - M_n^2)| < 0|O(0)|\lambda_n > |^2$$

- The propagator is

$$\int d^4x \, e^{iP.x}\langle 0|T(O_U(x)O_U(0)|0\rangle = \sum_n F_n^2 \frac{i}{P^2 - M_n^2 + i\epsilon}$$

where $F_n^2 = | < 0|O(0)|\lambda_n > |^2$

- If we imagine $O(x) = \sum_n F_n\lambda_n(x)$ then the

$$\int d^4x \, e^{iP.x}\langle 0|T(O_U(x)O_U(0)|0\rangle = \sum_n F_n^2 \int d^4x \, e^{iP.x}\langle 0|T[\lambda_n(x)\lambda_n(0)]|0\rangle$$

Unparticle propagator is a sum of propagtors of $\textbf{deconstructing particle fields}$ $\lambda_n(x)$

Deconstruction to Extra dimensions

- Consider a massive scalar field $\Phi(x, z)$ in 5-dim with AdS metric

$$ds^2 = \frac{1}{z^2}(dx_\mu dx^\mu - dz^2)$$

- the 4-dim Lagrangian density

$$\mathcal{L} = \frac{1}{2}\int dz\sqrt{g}[g^{MN}\partial_M\Phi\partial_N\Phi - m_5^2\Phi^2)$$

- Define

$$O(x) = \lim_{z\to 0} z^{-d_u}\Phi(x, z)$$

so that Φ couple to the SM fields on the boundary $z = 0$
If the size of the 5th dim is of order $1/\Delta$ then KK expansion gives

$$\Phi(x, z) = \sum_n \lambda_n(x)\phi_n(z)$$

- Since

$$O(x) = \sum_n \lambda_n(x)F_n \qquad ===> \qquad F_n = \lim_{z\to 0} z^{-d_u}\phi_n(z)$$

- $\lambda_n(x)$ are the 4-dim deconstructing particle fields and F_n are the normal modes on the boundary.

Unparticle is an infinite tower of massive particles with controllable mass-squared spacing Δ^2

Summary

- Di-lepton and di-photon production can be used to unravel various aspects of the unparticle physics and these signals could mimic the extra dimension scenarios
- Using deconstruction we can cast the interaction of the unparticle as a sum of interaction of the particles λ_n
- Field theory realisation of the deconstruction procedure is possible using a slice of AdS_5 space?