Cosmological consequences of generalised RS II braneworlds

Supratik Pal

Indian Statistical Institute, Kolkata

PWED 08, IIT Kharagpur

Outline

- Brane cosmology: generalised RS II ?
- Brane-based analysis of cosmology
- Effective Friedmann equations
- Perturbations on the brane
- Confrontation with observations
- Open issues

The warm-up

Brane : FRW $\Longleftrightarrow$ Bulk : Schwarzschild-AdS / Vaidya-AdS

	Brane metric	Bulk source	Bulk metric	Comments
1	Minkowski	$\Lambda_5 < 0$	warped, AdS_5	–
2	Minkowski	bulk field	warped	–
3	FRW	black hole (empty)	Schwarzschild-AdS_5	$m = 0 \Rightarrow 1$
4	FRW	radiative black hole(bulk field)	Vaidya-AdS_5	$m(v) = \mathrm{m} \Rightarrow 3$ $m(v) = 0 \Rightarrow 1$

Put $m = 0$ and get back warped (AdS) geometry in the bulk

This is a generalisation of RS II scenario

How to visualise ?

Schwarzschild-AdS bulk

$$dS_5^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Sigma_3^2 \quad \text{with} \quad f(r) = k - \frac{\Lambda_5}{6}r^2 - \frac{m}{r^2}$$

$*$ For embedding : tangents $u^\mu = (\dot{t},\ \dot{r})$, normals $n^\mu = (\dot{r},\ -\frac{\sqrt{f+\dot{r}^2}}{f})$

$*$ Induced metric on the brane

$$\boxed{ds^2 = -d\tau^2 + r^2(\tau)d\Omega_3^2}$$

$*$ Identify $r(\tau)$ with the scale factor $a(\tau) \Longrightarrow$ **FRW**

$$\boxed{\textbf{Expanding 4D universe} \equiv \textbf{Moving brane in the bulk}}$$

$\Uparrow$ Brane-based observer

$\Uparrow$ Bulk-based observer

J.Garriga et.al., PRD(2000)

Brane-based analysis: effective cosmology

The effective Einstein equation on the brane

$$G_{\mu\nu} = \underbrace{-\Lambda g_{\mu\nu} + \kappa_4^2 T_{\mu\nu}}_{\substack{\Downarrow \\ \text{4D GR}}} + \underbrace{\kappa_5^4 S_{\mu\nu}}_{\substack{\Downarrow \\ \text{Quadratic} \\ T_{\mu\nu}}} - \underbrace{\mathcal{E}_{\mu\nu}}_{\substack{\Downarrow \\ \text{Weyl} \\ \text{term}}} + \underbrace{\mathcal{F}_{\mu\nu}}_{\substack{\Downarrow \\ \text{Bulk} \\ \text{matter}}}$$

For convenience, express it in terms of an **effective EM tensor** as

$$\boxed{G_{\mu\nu} = -\Lambda g_{\mu\nu} + \kappa_4^2 T^{\text{eff}}_{\mu\nu}}$$

For a **perfect fluid on the brane**, components of $T^{\text{eff}}_{\mu\nu}$

$$\begin{aligned} \rho^{\text{eff}} &= \rho + \tfrac{\rho^2}{2\lambda_b} + \rho^* \\ p^{\text{eff}} &= p + \tfrac{\rho}{2\lambda_b}(\rho + 2p) + \tfrac{\rho^*}{3} \\ q^{\text{eff}}_\mu &= q^*_\mu \\ \pi^{\text{eff}}_{\mu\nu} &= \pi^*_{\mu\nu} \end{aligned}$$

For a bulk compatible to FRW geometry on the brane, $q^{\text{eff}}_\mu = 0 = \pi^{\text{eff}}_{\mu\nu}$

Schwarzschild-AdS bulk, FRW brane

Empty bulk ($T_{AB}^{\text{bulk}} = 0$) $\Rightarrow \mathcal{F}_{\mu\nu} = 0$

Bianchi identity on the brane $\Rightarrow$ **conservation equations**

For brane matter/radiation :

$$\dot{\rho} + 3H(\rho + p) = 0$$

For Weyl fluid :

$$\dot{\rho}^* + 4H\rho^* = 0$$

$$\Longrightarrow \boxed{\rho^* = \frac{C}{a^4}}$$

$C \sim$ (constant) mass of the bulk Schwarzschild-AdS black hole

R.Maartens, LRR(2004)

Vaidya-AdS bulk, FRW brane

Radiative bulk ($T_{AB}^{\rm bulk} = \psi\ q_A q_B$) $\Rightarrow \mathcal{F}_{\mu\nu} = \frac{2}{3}\kappa_5^2\ \psi h_{\mu\nu}$

Bianchi identity on the brane $\Rightarrow$ **(non)conservation equations**

For brane matter/radiation :

$$\dot\rho + 3H(\rho + p) = -2\psi$$

For Weyl fluid :

$$\dot\rho^* + 4H\rho^* = 2\psi - \frac{2\kappa_5^2}{3\kappa_4^2}\left[\dot\psi + 3H\psi\right]$$

$$\Longrightarrow \boxed{\rho^* = \frac{C(\tau)}{a^4}}$$

$C(\tau) \sim$ (variable) mass of the bulk Vaidya-AdS black hole
Put $\psi = 0$ and get back Schwarzschild-AdS bulk scenario

Vaidya-AdS bulk is so far the most general scenario

Brane Friedmann equations

$$H^2 = \frac{\kappa_4^2}{3}\left[\rho + \frac{\rho^2}{2\lambda_b} + \frac{C(\tau)}{a^4}\right] + \frac{\Lambda}{3} - \frac{k}{a^2}$$

$$\dot{H} = -\frac{\kappa_4^2}{2}\left[(\rho + p)\left(1 + \frac{\rho}{\lambda_b}\right) + \frac{4C(\tau)}{3a^4}\right] + \frac{k}{a^2} - \frac{\kappa_5^2}{3}\psi$$

Look too complicated !

Rewrite in terms of the effective perfect fluid $T_{\mu\nu}^{\text{eff}}$

$$H^2 = \frac{\kappa_4^2}{3}\rho^{\text{eff}} + \frac{\Lambda}{3} - \frac{k}{a^2}$$

$$\dot{H} = -\frac{\kappa_4^2}{2}(\rho^{\text{eff}} + p^{\text{eff}}) + \frac{k}{a^2} - \frac{\kappa_5^2}{3}\psi$$

Now they look much familiar

As if there is an effective perfect fluid that governs cosmological dynamics

D.Langlois et.al., PRL(2002)

Just for the sake of completeness...

Effective equation of state

$$w^{\rm eff} = \frac{p^{\rm eff}}{\rho^{\rm eff}} = \frac{p+\frac{\rho}{2\lambda_b}(\rho+2p)+\frac{\rho^*}{3}}{\rho+\frac{\rho^2}{2\lambda_b}+\rho^*}$$

Effective sound speed

$$c_{\rm eff}^2 = \frac{\dot{p}^{\rm eff}}{\dot{\rho}^{\rm eff}} = \left[c_s^2 + \frac{\rho+p}{\rho+\lambda_b} + \frac{4\rho^*}{9(\rho+p)(1+\rho/\lambda_b)}\right]\left[1 + \frac{4\rho^*}{3(\rho+p)(1+\rho/\lambda_b)}\right]^{-1}$$

Role of the quadratic term

Early universe :

$$\rho^2 \gg \lambda_b \Rightarrow \textbf{significant}$$

- Faster Hubble expansion implies inflation at a much faster rate than standard cosmology

R. Maartens et.al., PRD(2001)

- Inflation without 4D inflatonic field on the brane

Y. Himemoto et.al., PRD(2001)

Late time :

$$\rho^2 \ll \lambda_b > (100GeV)^4 \Rightarrow \textbf{negligible}$$

Role of the Weyl fluid

Evolution equation :

$$\dot{\rho}^* + 4H\rho^* = 2\psi - \frac{2\kappa_5^2}{3\kappa_4^2}\left[\dot{\psi} + 3H\psi\right]$$

For empty bulk :

$$\psi = 0 \Rightarrow \rho^* = \frac{C}{a^4} \Longrightarrow \textbf{dark radiation}$$

Nucleosynthesis data $\Rightarrow < 3\%$ of radiation energy density
Since radiation-like, its effect negligible at late times

J.D.Barrow et.al., PLB(2002)

For radiative bulk :

$$\psi \neq 0 \Rightarrow \rho^* = \frac{C(\tau)}{a^4} \Longrightarrow \text{no longer radiation-like !}$$

$$\Downarrow$$

$\boxed{\textbf{may be significant at late times}}$

“Newtonian” perturbations on the brane

SP, PRD 74, 024005 (2006)

Hydrodynamic equations in terms of effective perfect fluid

$$\frac{\partial \rho^{\rm eff}}{\partial t} + \overrightarrow{\nabla}.(\rho^{\rm eff}\ \overrightarrow{v}^{\rm eff}) = 0$$

$$\frac{\partial \overrightarrow{v}^{\rm eff}}{\partial t} + (\overrightarrow{v}^{\rm eff}.\overrightarrow{\nabla})\overrightarrow{v}^{\rm eff} = -\frac{\overrightarrow{\nabla}p^{\rm eff}}{\rho^{\rm eff}} - \overrightarrow{\nabla}\Phi^{\rm eff}$$

$$\nabla^2\Phi^{\rm eff} = 4\pi G\rho^{\rm eff}$$

Consider **perturbations**

$$\rho^{\rm eff}(\overrightarrow{x},\ \tau) = \bar{\rho}^{\rm eff}(\tau)(1+\delta^{\rm eff}(\overrightarrow{x},\ \tau))$$

$$\Phi^{\rm eff}(\overrightarrow{x},\ \tau) = \Phi_0^{\rm eff} + \phi^{\rm eff}$$

$*$ Use comoving coordinates $v^{\text{eff}} = \dot{a}\ r + u^{\text{eff}}$

$*$ Express in terms of Fourier transform

$*$ Apply $p^{\text{eff}} = p(\rho^{\text{eff}})$

Perturbation equations for Fourier modes $\delta_k^{\text{eff}}(\tau)$

$$\frac{d^2\delta_k^{\text{eff}}}{d\tau^2} + 2\frac{\dot{a}}{a}\frac{d\delta_k^{\text{eff}}}{d\tau} - \left[4\pi G\bar{\rho}^{\text{eff}} - \left(c_{\text{eff}}^2 k/a\right)^2\right]\delta_k^{\text{eff}} = 0$$

$$\rho^{\text{eff}} = \rho + \underbrace{\frac{\rho^2}{2\lambda_b}}_{\substack{\Downarrow \\ 0}} + \rho^*$$

$$\Rightarrow \boxed{\rho^{\text{eff}} = \rho + \rho^*}$$

So, a two fluid system : baryonic matter + Weyl fluid

So, break the perturbation equation into :

- **Baryonic matter fluctuation**

$$\frac{d^2\delta_b}{d\tau^2} + 2\frac{\dot{a}}{a}\frac{d\delta_b}{d\tau} = 4\pi G\bar{\rho}_b\delta_b + 4\pi G\bar{\rho}^*\delta^*$$

- **Weyl fluid fluctuation**

$$\frac{d^2\delta^*}{d\tau^2} + 2\frac{\dot{a}}{a}\frac{d\delta^*}{d\tau} = 4\pi G\bar{\rho}^*\delta^* + 4\pi G\bar{\rho}_b\delta_b$$

For radiative bulk, evolution eqn for the Weyl fluid is, in general, governed by

$$\dot{\rho}^* + 4H\rho^* = \alpha H\rho^*$$

$$\Longrightarrow \rho^* = C_0 a^{-(4-\alpha)}$$

For $1 < \alpha < 4$, Weyl fluid can dominate over matter !

$*$ Apply $\Omega_b \ll \Omega_*$

$*$ Scale factor at late time $a(\tau) = \left(\frac{3}{2} H_0 \tau\right)^{2/3(w^{\text{eff}}+1)}$

And obtain a consistent solution for the fluctuation equations :

$$\boxed{\delta^*(z) = \delta^*(0)(1+z)^{-1}} \qquad \boxed{\delta_b(z) = \delta^*(z)\left(1 - \frac{1+z}{1+z_N}\right)}$$

So.....

- $z \ll z_N \implies \delta_b \simeq \delta^*$
- $z \to z_N \implies \delta_b \to 0$ but δ^* still finite

This is precisely what is required to explain structure formation

$\boxed{\textbf{Weyl fluid mimics dark matter}}$

Relativistic perturbations: a brief sketch

R.Koley & SP, in progress

- Acts like a two fluid system
- Material fluid : $\rho^{(b)}$ & Geometric fluid : ρ^*
- Interacting and Exchanging energy between them

Comoving fractional gradients of density and expansion as in GR

$$\Delta_\mu^{(i)} = \tfrac{a}{\rho^{(i)}} D_\mu \rho^{(i)} \quad \text{and} \quad Z_\mu = a D_\mu \Theta$$

(Non)conservation equations can be written composedly as

$$\boxed{\dot{\rho}^{(i)} + \Theta(\rho^{(i)} + p^{(i)}) = I^{(i)}}$$

For Einstein-de Sitter brane universe $(\Omega^* = 1, \Omega_\Lambda = 0)$, the linearised evolution equations :

$$\dot{\Delta}_\mu^{(i)} = \left(3Hw^{(i)} - \frac{I^{(i)}}{\rho^{(i)}}\right)\Delta_\mu^{(i)} - (1+w^{(i)})Z_\mu - \frac{c_s^2\ I^{(i)}}{\rho^{(i)}(1+w)}\Delta_\mu - \frac{3aHI_\mu^{(i)}}{\rho^{(i)}} + \frac{a}{\rho^{(i)}}D_\mu I^{(i)}$$

$$\dot{Z}_\mu + 2HZ_\mu = -\frac{\kappa^2}{2}\rho\Delta - \frac{c_s^2}{1+w}D_\mu D^\nu \Delta_\nu + \frac{\kappa_5^2\psi}{1+w}c_s^2\Delta_\mu - a\kappa_5^2 D_\mu\psi$$

Density perturbations are governed by the fluctuation of the **covariant projections**

$$\Delta^{(i)} = a\ D^\mu \Delta_\mu^{(i)} \quad \text{and} \quad Z = a\ D^\mu Z_\mu$$

Covariant density perturbation equations on the brane :

$$\dot{\Delta}^{(i)} = \left(3Hw^{(i)} - \frac{I^{(i)}}{\rho_{(i)}}\right)\Delta^{(i)} - (1+w^{(i)})Z - \frac{c_s^2\ I^{(i)}}{\rho^{(i)}(1+w)}\Delta - \frac{3a^2HD^\mu I_\mu^{(i)}}{\rho^{(i)}} + \frac{a^2}{\rho^{(i)}}D^2 I^{(i)}$$

$$\dot{Z} + 2HZ = -\frac{\kappa^2}{2}\rho\Delta - \frac{ac_s^2}{1+w}D^2\Delta + \frac{\kappa_5^2\psi}{1+w}c_s^2\Delta - a^2\kappa_5^2 D^2\psi$$

If ψ is function of time only and the energy exchange is in equilibrium

$$\Rightarrow \boxed{\alpha = \frac{5}{2} \quad ; \quad \rho^* \propto \frac{1}{a^{3/2}}}$$

Hence .....

Scalar perturbation equation for matter on brane

$$\ddot{\Delta}^{(b)} + 2H\dot{\Delta}^{(b)} = \frac{\kappa^2}{2}\rho\Delta - \frac{c_s^2\kappa_5^2\psi}{1+w}\Delta + \frac{4H\psi}{\rho^{(b)}}\left(\Delta^{(b)} + \frac{c_s^2\Delta}{1+w}\right) + \left(\frac{2\psi}{\rho^{(b)}}\left(\Delta^{(b)} + \frac{c_s^2\Delta}{1+w}\right)\right)^{\cdot}$$

Scalar perturbation equation for Weyl fluid on brane

$$\ddot{\Delta}^* + 2H\dot{\Delta}^* = \frac{4}{3}\frac{\kappa^2}{2}\rho\Delta - \frac{c_s^2\Delta}{1+w}\left(\frac{7H\psi}{\rho^*} + \frac{4\kappa_5^2\psi}{3} + \frac{2\dot{\psi}}{\rho^*}\right) - \frac{c_s^2\dot{\Delta}}{1+w}\frac{2\psi}{\rho^*}$$
$$+ \ \Delta^*\left(2H^2 - \frac{\kappa^2}{2} - \frac{7H\psi}{\rho^*} - \frac{2\dot{\psi}}{\rho^*}\right) + \dot{\Delta}^*\left(H - \frac{2\psi}{\rho^*} - \frac{\kappa_5^2\psi}{3}\right)$$

With $\rho^{(b)} << \rho^*$ and $\Delta^{(b)} << \Delta^*$

$$\ddot{\Delta}^* + 2H\dot{\Delta}^* - \frac{\kappa^2}{2}\rho^*\Delta^* = -\frac{A_1}{t}\Delta^* + \left(\frac{B_1}{t} + \frac{C_1}{t^2}\right)\Delta^*$$

$$\Rightarrow \quad \ddot{\Delta}^* + \frac{A}{t}\dot{\Delta}^* - \left(\frac{B}{t} + \frac{C}{t^2}\right)\Delta^* = 0$$

Solution for Δ^* :

$$\boxed{\Delta^* \sim t^{\frac{1}{2}-\frac{A}{2}} BesselI\left[\sqrt{1-2A+A^2+4C}, 2\sqrt{B}\sqrt{t}\right]}$$

Solution for Weyl fluid fluctuation Δ^* has a growing mode

Question: Is it compatible with observations ?

Confrontation with observations

In terms of dimensionless parameters

$$\Omega_\Lambda = \frac{\Lambda}{3H_0^2}, \Omega_\rho = \frac{\kappa^2 \rho_0}{3H_0^2}, \Omega_* = \frac{2C_0}{a_0^{4-\alpha} H_0^2}, \Omega_\lambda = \frac{\kappa^2 \rho_0^2}{6\lambda H_0^2}; \Omega_{\rm tot} = \sum_i \Omega_i = 1$$

Friedmann equations:

$$\frac{H^2}{H_0^2} = \Omega_\Lambda + \Omega_\rho \frac{a_0^3}{a^3} + \Omega_* \frac{a_0^{4-\alpha}}{a^{4-\alpha}} + \Omega_\lambda \frac{a_0^6}{a^6}$$

Luminosity distance for FRW branes

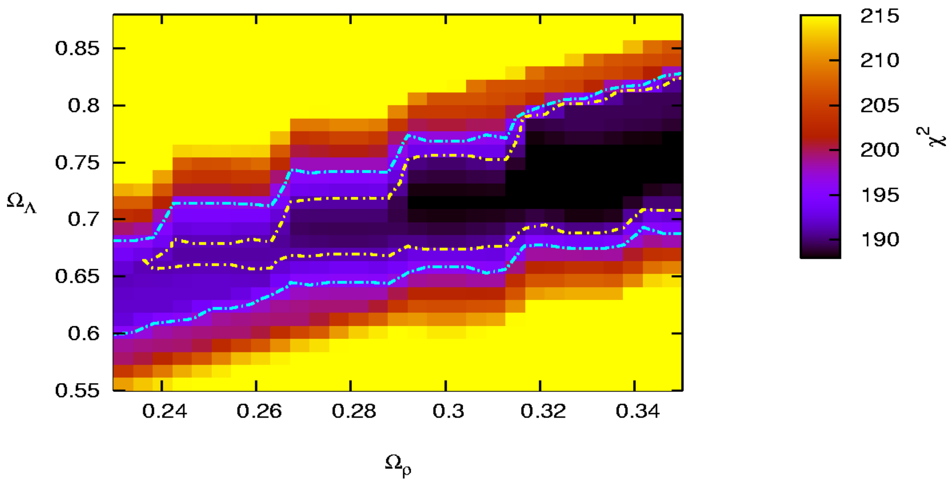
$$d_L(z) = \frac{(1+z)a_0}{H_0} \int_{a_{\rm em}}^{a_0} \frac{a\,da}{\left[\Omega_\Lambda a^6 + \Omega_\rho a_0^3 a^3 + \Omega_* a_0^{4-\alpha} a^{\alpha+2} + \Omega_\lambda a_0^6\right]^{1/2}}$$

For $\Omega_\lambda \to 0 \Longrightarrow$ $\boxed{d_L^{\Lambda\lambda *} = d_L^{\Lambda\rm CDM} + \Omega_* I_*}$

where I_* is a function having elliptic integrals of 1st and 2nd kind

Weyl fluid with $\alpha = 2 - 3$ is in nice agreement with SNe data

Results and Figure courtesy: L.A.Gergely et.al., 0709.0933[astro-ph]



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Ω_ρ
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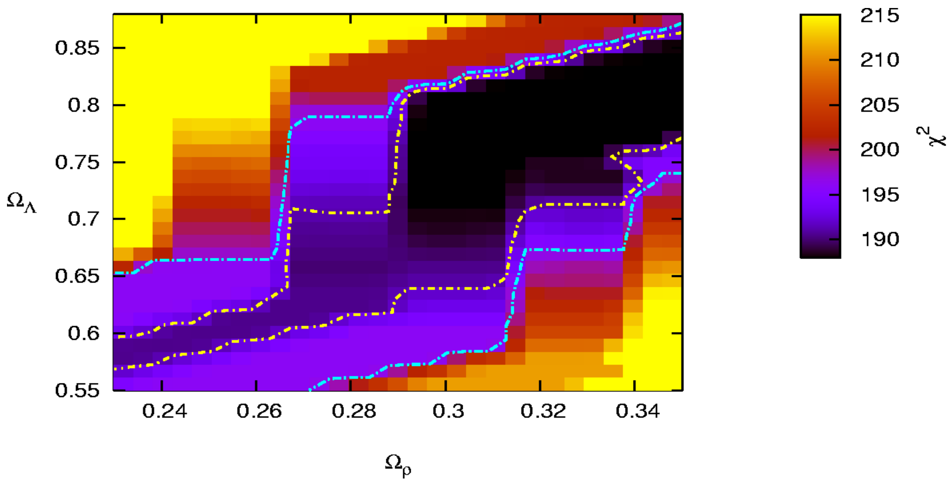
χ^2



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Open issues

- **Confront with observations** : Recent studies have shown some agreement with observations
- Calculate **power spectrum** etc and compare with ΛCDM
- **Metric-based perturbations** and allied phenomena
- **Stability** of bulk black hole under perturbations on Weyl fluid
- **Expansion history of the universe** : Studied to some extent in DGP, what about generalised RS II ?

  $\longrightarrow$ **Solve complicated Friedmann equations !**

And miles to go...