# Particle Creation in presence of A Warped Extra Dimension

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# Plan of Talk

- Introduction
- Quantum field coupled to a background metric giving rise to particle creation and constraints on the field modes
- Production of particles in presence of different kinds of extra dimension and their generic features
- Discussions

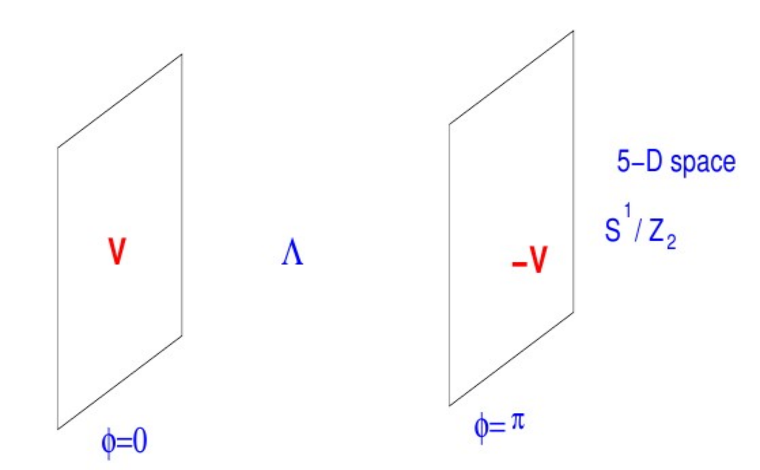
# Introduction

## The Braneworld scenario

*L. Randall & R. Sundrum*, 1999

$$ds^2 = e^{2f(\sigma)} \eta_{\mu\nu} dx^\mu dx^\nu + d\sigma^2$$

- Our world is a four dimensional hypersurface (3-brane) embedded in higher dimensional universe.
- The warping factor.




RS II scenario

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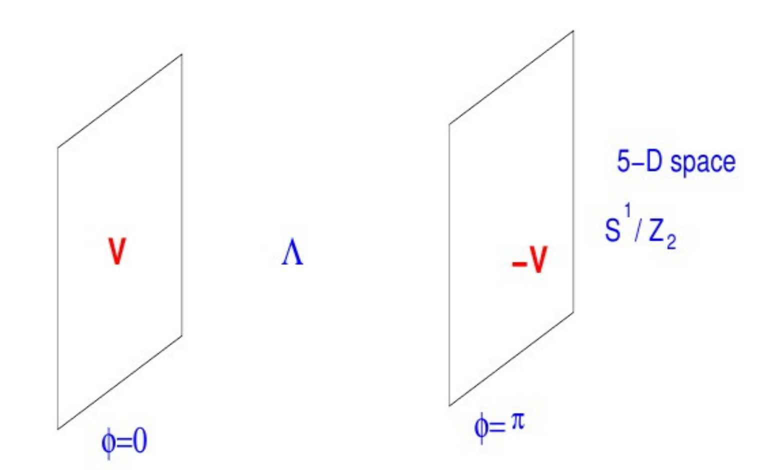

RS II scenario

These warped Braneworld models propose:

achievable experiments in search for signature of extra dimensions,
a solution to long standing hierarchy problem in high energy physics,
a dynamic way of compactification by localisation of fields on Brane
and many more interesting features to be explored.

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- Quantum fields in warped spacetimes
- Particle creation can be viewed as a quantum consequence of extra dimension.

The questions arise are:

- How do the extra dimension effect particle production ?
- What is the role of warping factor, cosmological evolution and time dependence of extra dimension?
- Is there any new feature ?

# Quantum field coupled to a spacetime with an extra dimension

## A formalism

A scalar field $\psi(\eta, \mathbf{x}, \sigma)$ Conformally coupled with

The metric $ds^2 = e^{2f(\sigma)}a^2(\eta)[-d\eta^2 + dx^2 + dy^2 + dz^2] + \phi^2(\eta)d\sigma^2$

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gives rise to the following klein-Gordon equation

$$-\frac{e^{-2f(\sigma)}}{a^2}\frac{\partial^2\psi}{\partial\eta^2} + \frac{e^{-2f(\sigma)}}{a^2}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) + \frac{1}{\phi^2}\frac{\partial^2\psi}{\partial\sigma^2} - \frac{e^{-2f(\sigma)}}{a^2}\left(\frac{2\dot{a}}{a} + \frac{\dot{\phi}}{\phi}\right)\frac{\partial\psi}{\partial\eta} + \frac{4f'}{\phi^2}\frac{\partial\psi}{\partial\sigma} - (m^2 + \xi R)\psi = 0$$

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Assuming separability $\longrightarrow$ $\psi(\eta, \mathbf{x}, \sigma) = \frac{1}{e^f a\phi^{\frac{1}{2}}}\chi_l(\eta)F(\mathbf{x})G(\sigma)$

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$$\boxed{\ddot{\chi}_l(\eta) + \left[\left(\mathbf{k}^2 + \frac{a^2}{\phi^2}k_\sigma^2\right) + \frac{\ddot{a}}{8a} - \frac{\ddot{\phi}}{8\phi} + \frac{\dot{\phi}^2}{4\phi^2} - \frac{\dot{a}\dot{\phi}}{4a\phi}\right]\chi_l(\eta) = 0}$$

$m = 0$ and $\xi = \dfrac{3}{16}$

$$\frac{1}{F(\mathbf{x})}\left\{\frac{d^2F(\mathbf{x})}{dx^2} + \frac{d^2F(\mathbf{x})}{dy^2} + \frac{d^2F(\mathbf{x})}{dz^2}\right\} = -\mathbf{k}^2$$

$$e^{2f}\left\{\frac{G''(\sigma)}{G(\sigma)} + 2f'\frac{G'(\sigma)}{G(\sigma)} + \left(\frac{f''}{2} + \frac{3f'^2}{4}\right)\right\} = -k_\sigma^2$$

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$$\Omega_l^2 = \left(\mathbf{k}^2 + \frac{a^2}{\phi^2} k_\sigma^2\right)$$

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In general $\longrightarrow$ $$\boxed{\ddot{\chi}_l(\eta) + \left[\Omega_l^2 + Q\right]\chi_l(\eta) = 0}$$

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WKB solutions → $\chi_l = \frac{\alpha_l}{\sqrt{2\Omega_l}}e^{-i\int^\eta \Omega_l d\eta} + \frac{\beta_l}{\sqrt{2\Omega_l}}e^{i\int^\eta \Omega_l d\eta}$

further restriction - $\dot{\chi}_l = -i\Omega_l\left[\frac{\alpha_l}{\sqrt{2\Omega_l}}e^{-i\int^\eta \Omega_l d\eta} - \frac{\beta_l}{\sqrt{2\Omega_l}}e^{i\int^\eta \Omega_l d\eta}\right]$

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$$v_l \equiv 2Re\left[\alpha_l \beta_l^* e^{-2i\int^\eta \Omega_l d\eta}\right]$$

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$$\boxed{N = \frac{1}{(2\pi a)^3}\int d^3l \; |\beta_l|^2}$$

Total particle number density

$$\boxed{\rho = \frac{1}{(2\pi a)^3 a}\int d^3l \; l|\beta_l|^2}$$

Total energy density

# Allowed values of $k_\sigma$

Eigenvalue equation $\longrightarrow$ $$e^{2f}\left\{\frac{G''(\sigma)}{G(\sigma)}+2f'\frac{G'(\sigma)}{G(\sigma)}+\left(\frac{f''}{2}+\frac{3f'^2}{4}\right)\right\}=-k_\sigma^2$$

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We get $\boxed{G_1''(\sigma)+\left[b\ \delta(\sigma)-\frac{b^2}{4}+k_\sigma^2\ e^{2b|\sigma|}\right]G_1(\sigma)=0}$ for $f(\sigma)=-\ b\ |\sigma|$

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For a two brane model:

$$\boxed{\tan\left[\theta+\tan^{-1}\left(-\frac{2}{3}\theta e^{-br_c\pi}\right)\right]=-\frac{2}{3}\theta} \quad \text{where} \quad k_\sigma=\theta\ b\ e^{-br_c\pi}$$

$\theta=0,\pm2.17463,\pm5.00365,\pm8.03846,\pm11.1295,\pm14.2421,\pm17.3649$ and so on

In case of an infinite extra dimension there are no discrete values of the modes and we have a continuum.

# Particle creation in a four dimensional universe

4d metric $ds^2 = a(\eta)^2(-d\eta^2 + dx^2 + dy^2 + dz^2)$ with $a^2(\eta) = b_1^2 + b_2^2\eta^2$

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K-G eq. $\ddot{\chi}_l(\eta) + \Omega_l^2(\eta)\chi_l(\eta) = 0$

where $\Omega_l^2(\eta) = k^2 + (b_1^2 + b_2^2\eta^2)m^2$

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$$|\beta_k|^2 = exp\left[-\pi\left(\frac{k^2}{mb_2} + \frac{mb_1^2}{b_2}\right)\right]$$

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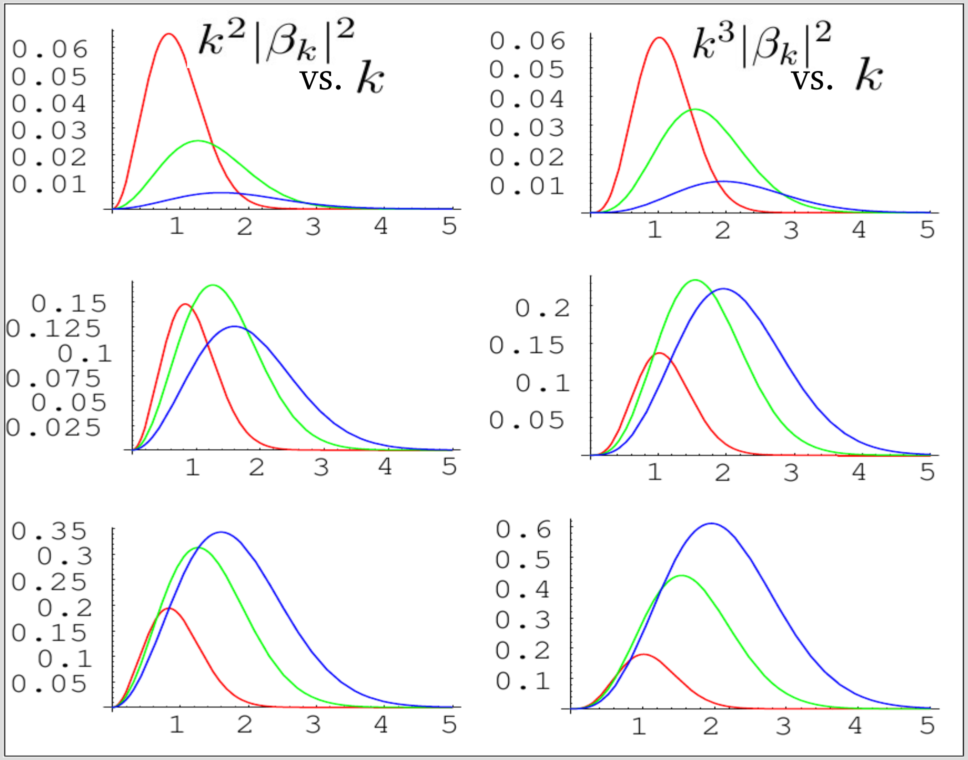


Decreasing $b_1$

# Particle creation in a universe with a static extra dimension

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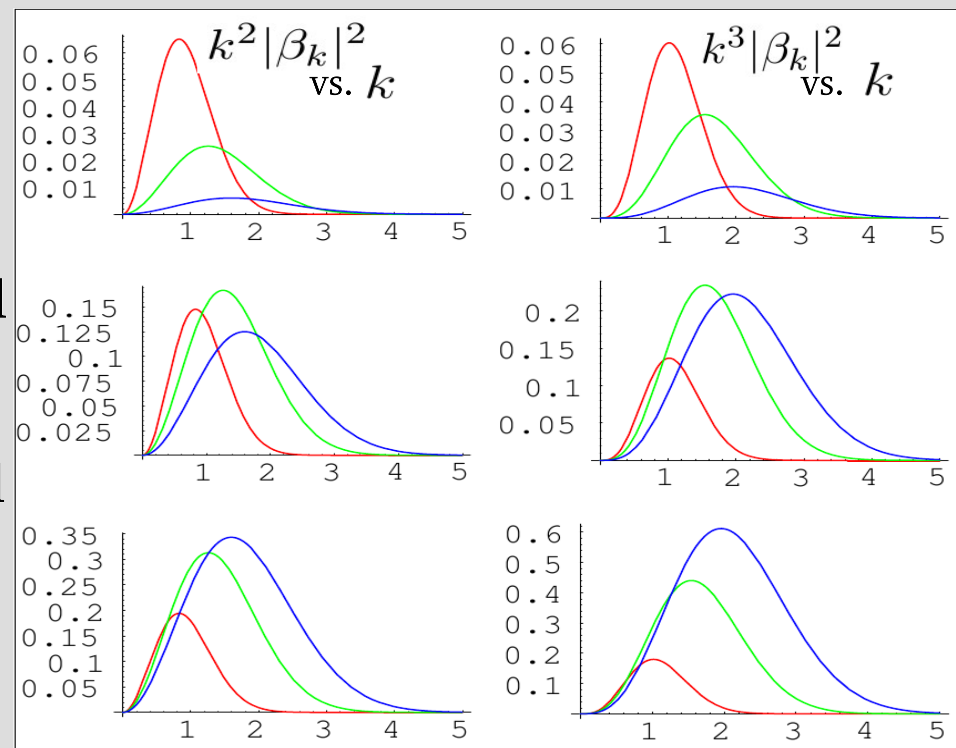


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$$Q(\eta) = 0 \Rightarrow m \equiv \frac{k_\sigma}{b_1}$$

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$$\phi(\eta) = b_1$$

$$\ddot{\chi}_l(\eta) + \left[\left(k^2 + \frac{b_1^2 + b_2^2\eta^2}{b_1^2} k_\sigma^2\right) + \frac{b_1^2 b_2^2}{8(b_1^2 + b_2^2\eta^2)}\right] \chi_l(\eta) = 0$$

$$Q(\eta) = 0 \Rightarrow m \equiv \frac{k_\sigma}{b_1}$$

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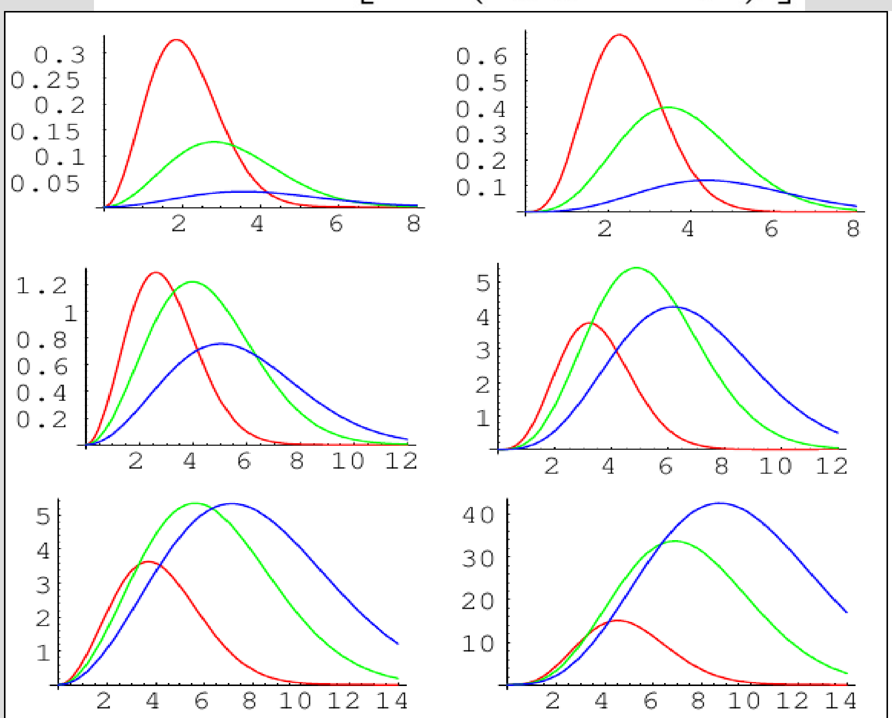


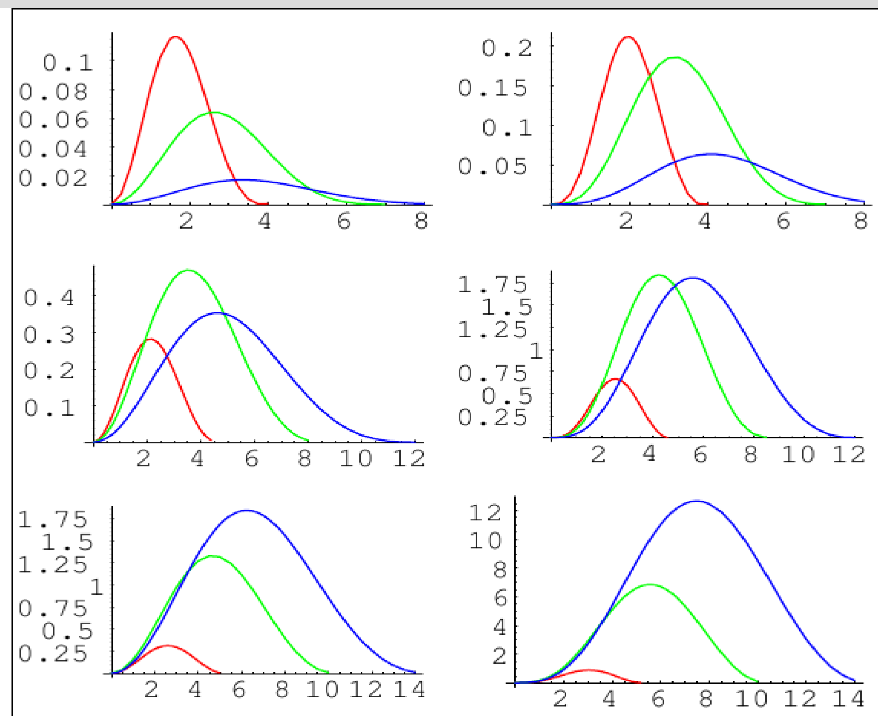


$k_\sigma = 2.17463, 5.00365, 8.03846$

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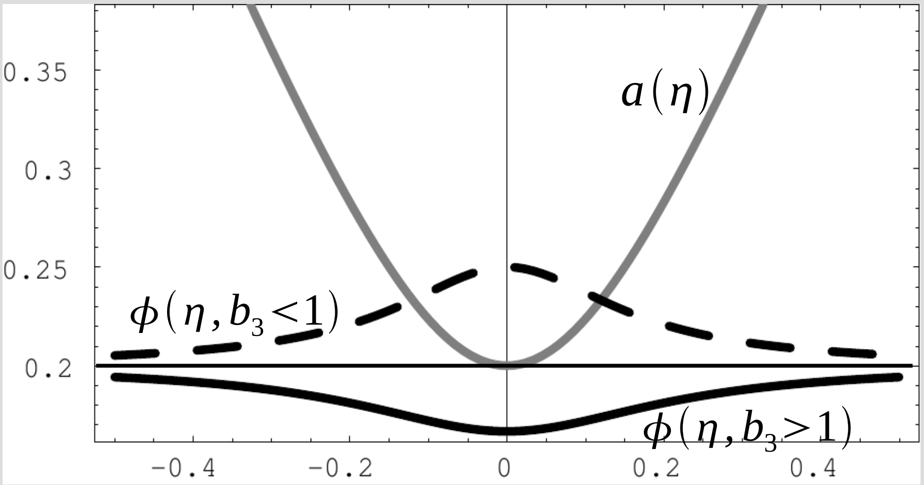

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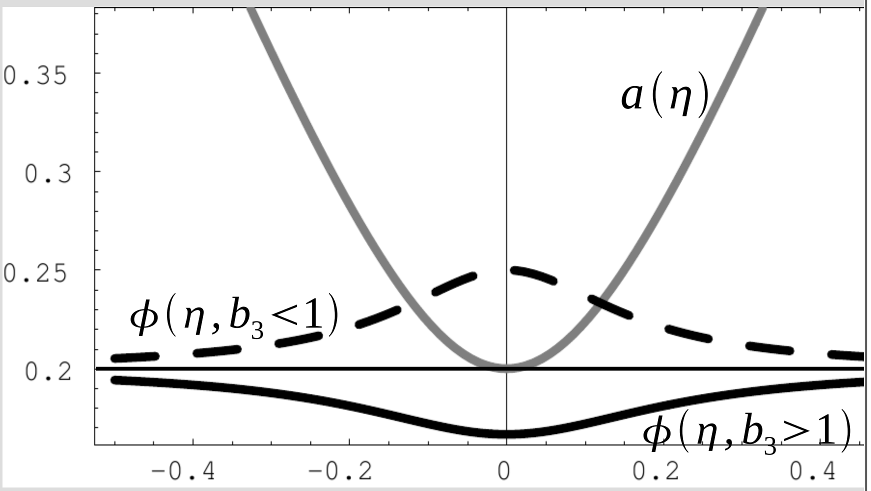
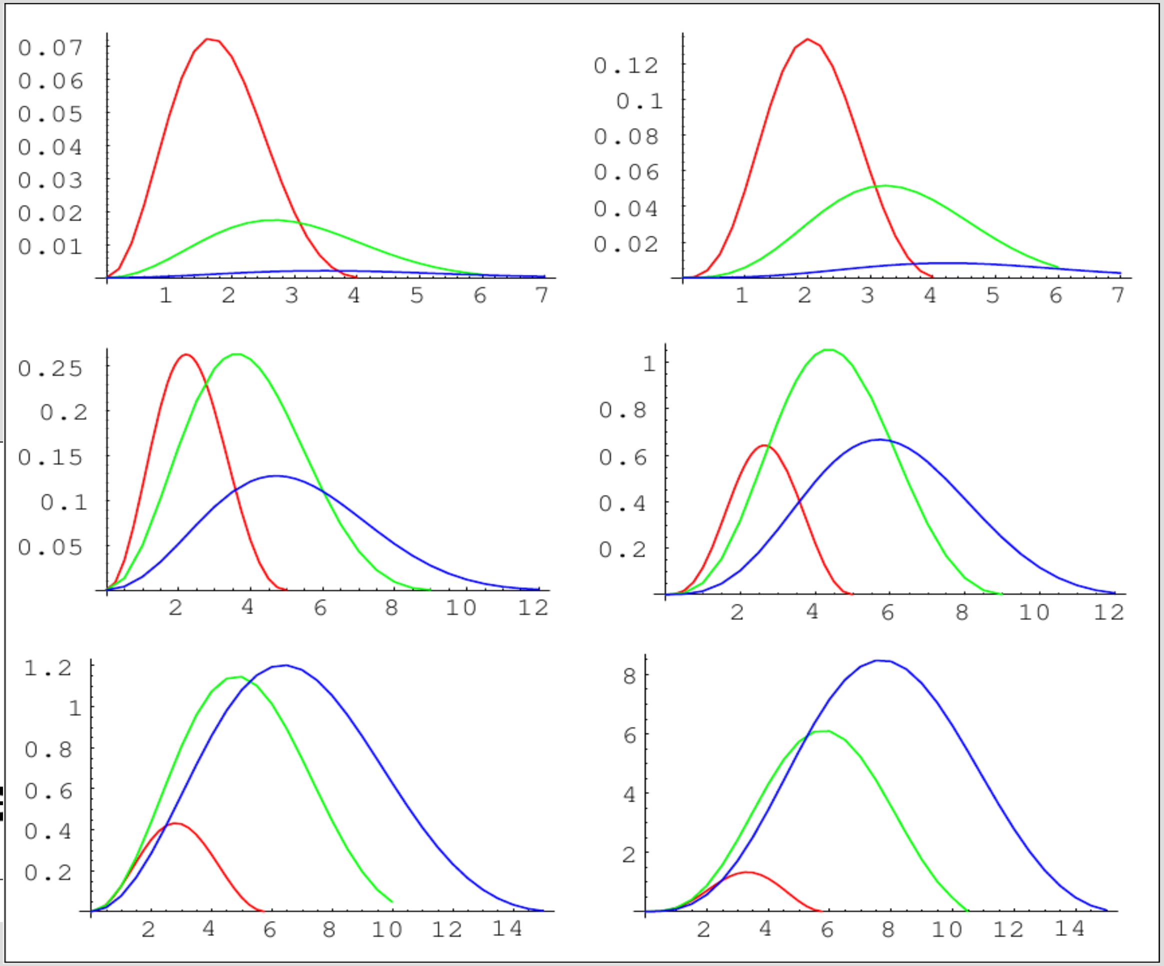
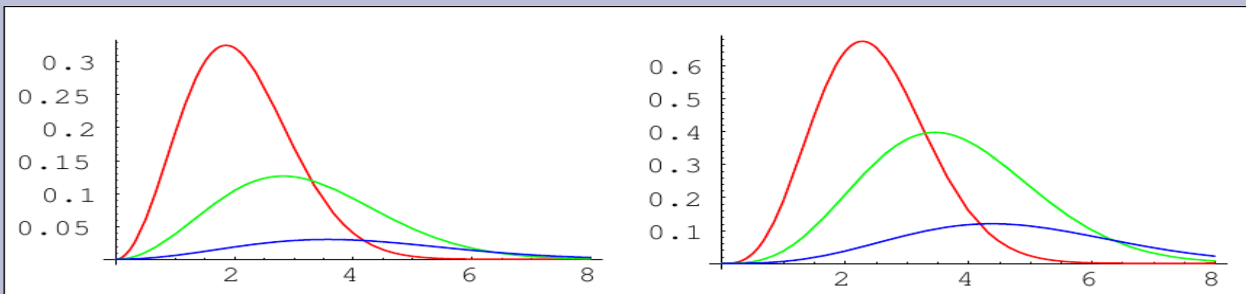


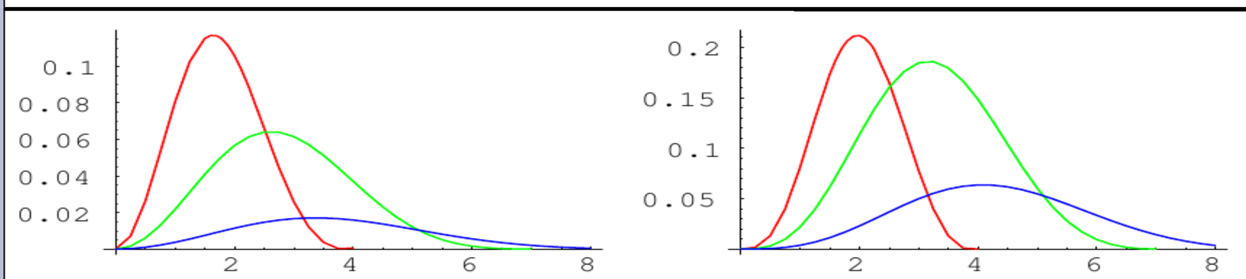

$\phi(\eta, b_3 > 1)$

# Comparison

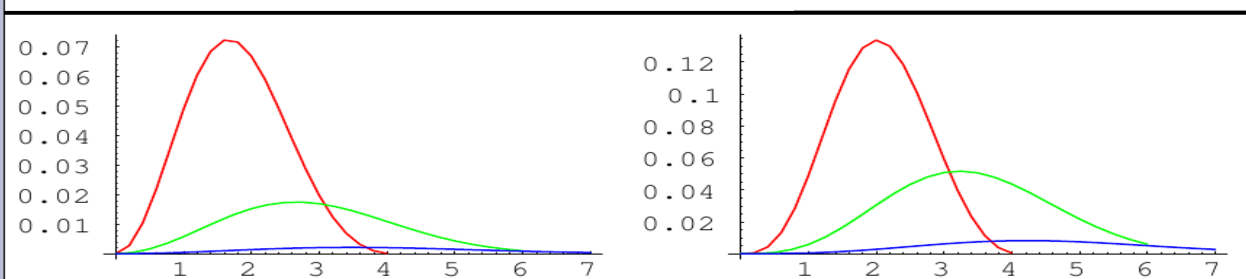
$$ds^2 = e^{2f(\sigma)}a^2(\eta)[-d\eta^2 + dx^2 + dy^2 + dz^2] + \phi^2(\eta)d\sigma^2 \qquad a^2(\eta) = b_1^2 + b_2^2\eta^2$$



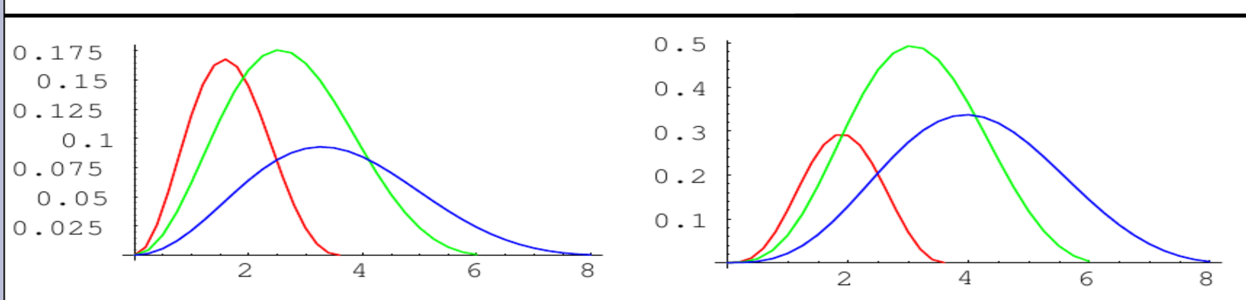

Static extra dimension
Analytic approximation
$Q(\eta)=0$




Static extra dimension
Numerical evaluation
$Q(\eta)\neq 0$




$\phi^2(\eta) = \frac{b_1^2 + b_2^2\eta^2}{b_3^2 + b_2^2\eta^2/b_1^2}$ and $b_3 > 1$

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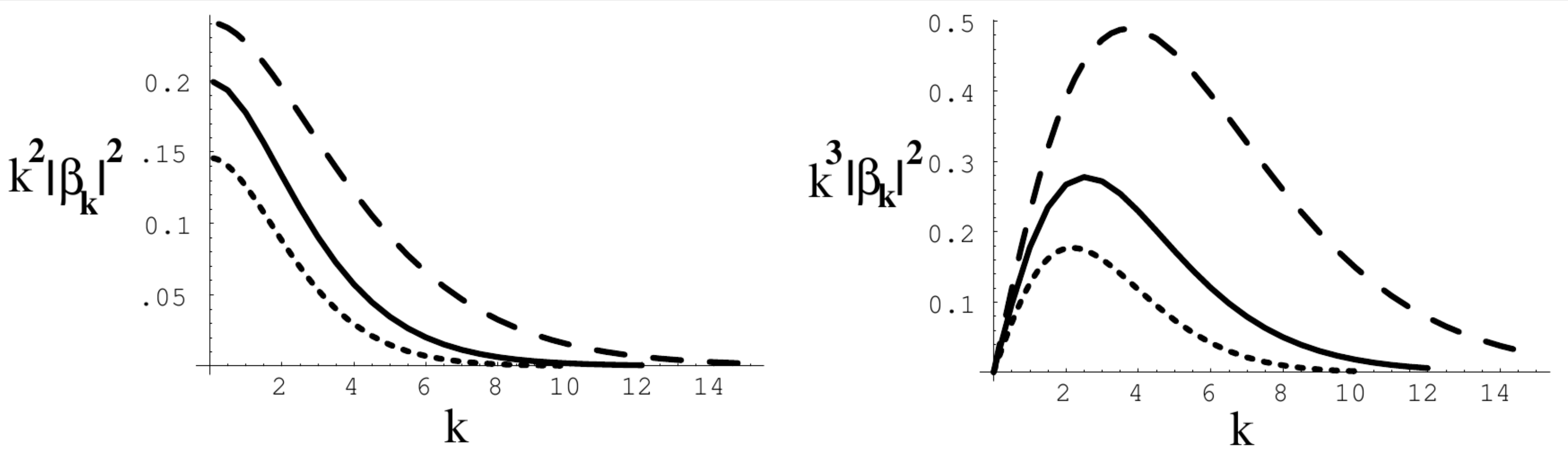


Continuous curve: Static extra dimension
Dashed curve: Dynamic extra dimension with $b_3 < 1$
Dotted curve: Dynamic extra dimension with $b_3 > 1$

# Summary & comments

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- ➢ One can try to derive such braneworld type solutions of Einsteins equation that we have assumed as our model.
- ➢ This same study can be done for different cosmological models.
- ➢ One can carry out the same study for different kinds of particles.

*Thank You*