Cosmological Constant, brane tension and large hierarchy in a generalised Randall-Sundrum scenario

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$S.Das, D.Maity,\ and\ S.SenGupta, \quad arXiv : 0711.1744(hep-th),$

Plan of my talk

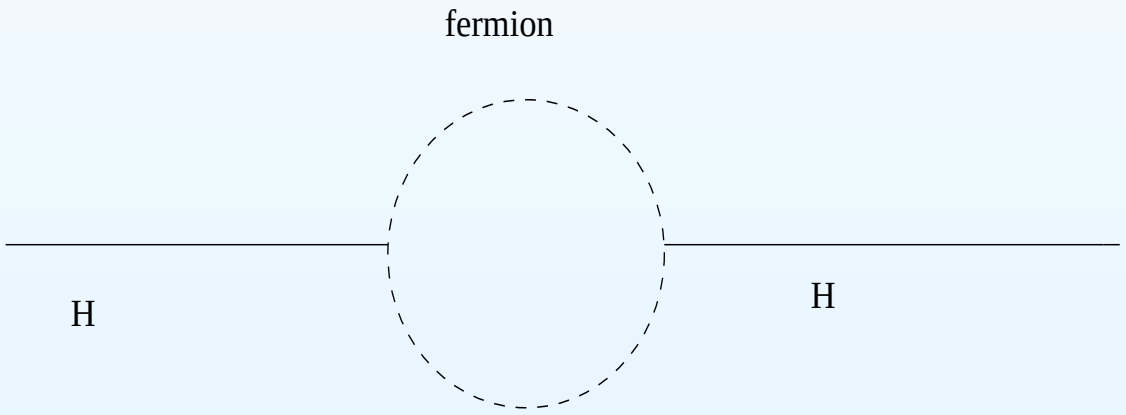
- Introduction and Motivation
- Randall-Sundrum(RS) model and it’s outcome
- Generalization of RS
- New results
- Conclusion

Introduction and Motivation

- Standard model of particle physics and cosmology have grate success in accounting various experimental observations.
- But few unanswered questions
  - Why is the ratio of the electroweak scale/Higgs mass (m) to the Planck mass (m_0) so tiny (10^{-16})? This gives rise to the fine tuning or naturalness or gauge hierarchy problem.
  - Why is the observed value of the cosmological constant extremely small (10^{-124}) (in Planck units)? This gives rise to the cosmological fine tuning problem

Introduction

- Gauge hierarchy refers to vast disparity between the Weak and Planck scale.
- In the context of SM, this hierarchy of scales is unnatural since it requires a fine tuning order by order in perturbation theory in order to stabilize the Higgs mass.



  - which gives $\delta m_H^2 = \alpha \Lambda^2$

Introduction

- Recent cosmological observation (high red-shift Supernova-Ia observation) shows: Universe is presently accelerating $\rightarrow$ Di-sitter universe with very small cosmological constant(10^{-124} in Planck units)?.
- Till now we don't have any explanation of the origin and also the smallness of this energy density.
- But Quantum field theoretic argument again gives rise to an another fine-tuning in cosmological sector.
- There exists no connection between these two finetunig problems.

Introduction

There are two important paths

- Supersymmetry
- Extra dimension

Introduction

In the context of extra dimension

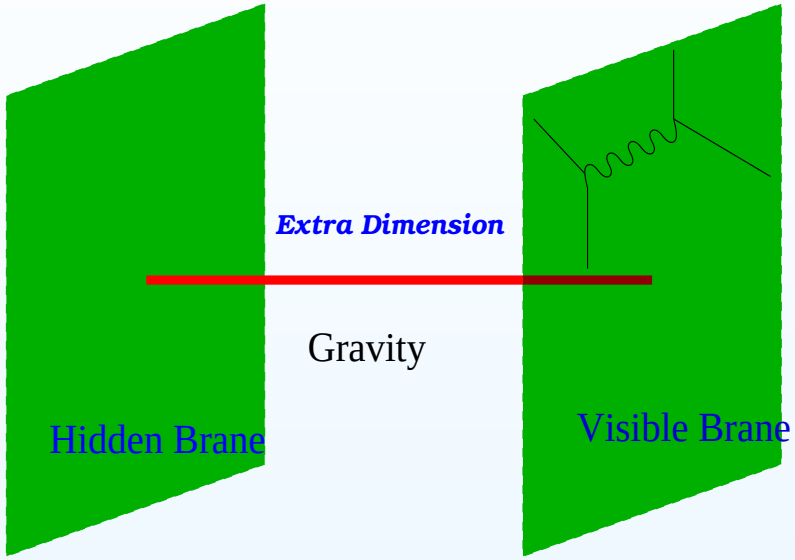
- Rubakov and Shaposnikov, Phys.Lett. 125B, 1983 had proposed a model of extra dimension solving cosmological constant problem.

Recently proposed higher dimensional models for hierarchy problem are

- ADD [N. A. Hamed, S. Dimopoulos and G. R. Dvali, PLB, 429,263 ,1998]
- RS [L. Randall and R. Sundrum, PRL ,83, 4690(1999) and PRL 83, 3370(1999)]
- It was shown that Hierarchy problem can be solved
  - ADD model resolves the hierarchy problem at the expense of bringing in new length hierarchy.
  - Randall-Sundrum two-brane warped model: resolve the fine tuning problem without bringing in any intermediate scale.

A brief outline on RS:

- It is a five dimensional model
- Model contains two 3-branes separated by compact fifth dimension
- Bulk is ADS
- Various standard model fields are expected to be localized on visible brane



The action corresponding to this model:

$$
\begin{aligned}
S_{tot} &= S_{bulk} + S_{vis} + S_{hid} \\
S_{tot} &= \int d^4x \int_{-\pi}^{\pi} \sqrt{-G}\{2M^3R - \Lambda\} \\
&+ \int d^4x \sqrt{-g_{vis}} V_{vis} + \int d^4x \sqrt{-g_{hid}} V_{hid}
\end{aligned}
$$

where

Λ is the bulk cosmological constant.

M is the five dimensional Planck mass.

$\mathbf{V}$,s are called brane tenson.

RS Scenario

The metric ansatz is

$$ds^2 = e^{-2A(\phi)}\eta_{\mu\nu}dx^\mu dx^\nu + rd\phi^2$$

Vacuum Solution: $\mathbf{A}(\phi) = \mathbf{kr}|\phi|$

where

$k = \sqrt{\frac{-\Lambda}{24M^3}}$: **Leading to negative cosmological constant(Ads bulk),**

$V_{hid} = -V_{vis} = 24M^3k.$

Important Results from this Solution:

- Four dimensional Planck Mass: $M_{pl}^2 = \frac{M^3}{k}[1 - e^{-2kr\pi}]$
- Consider any mass parameter m_0: $m_{phy} = e^{-kr\pi}m_0$ **(from the visible brane world point of view**

Outcome

(1) $$\left(\frac{m_H}{m_0}\right)^2 = e^{kr\pi} \sim 10^{-16}$$

So, 10^{19}GeV $\rightarrow$ 1TeV $\Rightarrow$ $kr \approx 12$.

Satisfactory feature

- Large hierarchy can be produced from small warp factor.

Unsatisfacoty features

- Introduction of negative tension visible brane.
- Problem of stabilizing the modulus r

Summary

There are two attempts using higher dimension

- Extra dimension $\longrightarrow$ cosmological constant
  Rubakov et al
- Extra dimension $\longrightarrow$ Gauge hierarchy
  ADD and RS
  Our aim
- Extra dimension $\longrightarrow$ cosmological constant and Gauge hierarchy

We extend such warped geometric model with non-zero cosmological constant on the visible 3-brane and look for possible occurrence of positive tension Tev brane when a large hierarchy exists between the two branes.

Generalization of RS

We start with a general metric ansatz,

$$ds^2 = e^{-2A(\phi)} g_{\mu\nu} dx^\mu dx^\nu + r d\phi^2$$

which extremises the action:

$$S = \int d^5x \sqrt{-G}(M^3\mathcal{R} - \Lambda) + \int d^4x \sqrt{-g_i}\mathcal{V}_i$$

$\mathcal{V}_i$ is tension of ith brane.

The resulting Einstein equations are:

$$\begin{aligned} {}^4G_{\mu\nu} - g_{\mu\nu}e^{-2A}[-6A'^2 + 3A''] &= -\frac{\Lambda}{2M^3} g_{\mu\nu} e^{-2A} \\ -\frac{1}{2}e^{2A}\ {}^4R + 6A'^2 &= -\frac{\Lambda}{2M^3} \end{aligned}$$

Generalized RS... Contd

with the boundary conditions

$$[A'(y)]_i = \frac{\epsilon_i}{12M^3}\mathcal{V}_i \ ,$$

where $\epsilon_{hid} = -\epsilon_{vis} = 1$

The set of equations can be written with an arbitrary constant as

$$\begin{aligned} {}^4G_{\mu\nu} &= -\Omega g_{\mu\nu} \\ 6A'^2 &= -\frac{\Lambda}{2M^3} + 2\Omega e^{2A} \\ 3A'' &= \Omega e^{2A} \ . \end{aligned}$$

So, as is clear, Ω plays the role of physical cosmological constant on the brane. For example, for $\Omega > 0$ and $\Omega < 0$, correspond to dS-Schwarzschild and AdS-Schwarzschild brane respectively.

Negative Ω: ADS brane case

Defining a parameter $\omega^2 \equiv -\Omega/3k^2 \geq 0$

$$e^{-A} = \omega \cosh\left(\ln \frac{\omega}{c_1} + ky\right)$$

where c_1 is the integration constant.
corresponding brane tensions are

$$\mathcal{V}_{vis} = 12M^3k\left[\frac{\frac{\omega^2}{c_1^2}e^{2kr\pi} - 1}{\frac{\omega^2}{c_1^2}e^{2kr\pi} + 1}\right] \; ; \; \mathcal{V}_{hid} = 12M^3k\left[\frac{1 - \frac{\omega^2}{c_1^2}}{1 + \frac{\omega^2}{c_1^2}}\right]$$

Normalizing the warp factor to unity at the orbifold fixed point $y = 0$, we get:

$$c_1 = 1 + \sqrt{1 - \omega^2} \; .$$

ADS brane case...contd

- Which tells that ADS cosmological constant, $0 \leq \omega^2 \leq 1$ in order to have real solution.

To solve the hierarchy problem, we equate the warp factor at $y = r\pi$ to the ratio of the Higgs to the Planck mass:

$$e^{-A} = \omega \cosh\left(\ln \frac{\omega}{c_1} + kr\pi\right) = 10^{-n} \ .$$

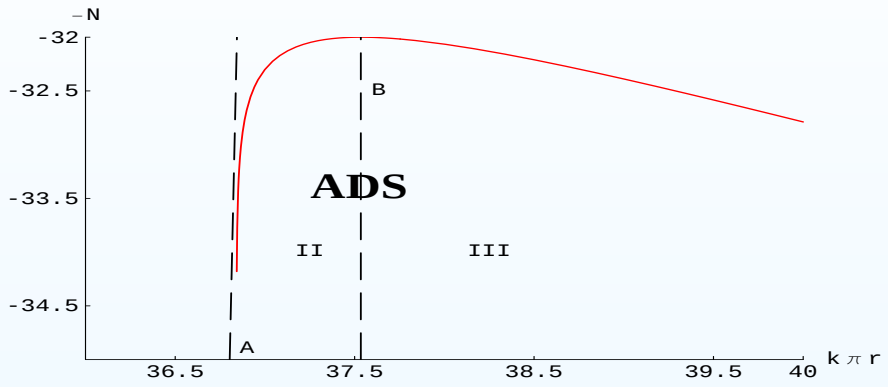
we call $n(\sim 16)$ is warp factor index

- we set the cosmological constant parameter $\omega^2 = 10^{-N}$

Then solution of the above equation comes out to be

$$e^{-kr\pi} = \frac{10^{-n}}{2}\left[1 \pm \sqrt{1 - 10^{-(N-2n)}}\right]$$

ADS brane case...contd



So, we have two allowed values for the brane separation for a fixed cosmological constant.

- For smaller value of kr, the visible brane tension appears to be negative. $\mathcal{V}_1 = -12M^3k$
- For higher value of kr, brane tension is positive.$\mathcal{V}_1 = 12M^3k$
- There exists a maximum value of the ADS cosmological constant $N_{max} = 2n$, which corresponds to zero visible brane tension.
- For the hidden brane, tension always happens to be positive.

Positive Ω: DS brane case

For $\Omega > 0$, solution of Einstein equation gives

$$e^{-A} = \omega \sinh\left(\ln \frac{c_2}{\omega} - ky\right) \ ,$$

- $\omega^2 \equiv \Omega/3k^2$ and again c_2 is constant of integration.
- corresponding brane tendons are

$$\mathcal{V}_{vis} = 12M^3k\left[\frac{\frac{\omega^2}{c_1^2}e^{2kr\pi}+1}{\frac{\omega^2}{c_1^2}e^{2kr\pi}-1}\right] \ ; \ \mathcal{V}_{hid} = 12M^3k\left[\frac{1+\frac{\omega^2}{c_1^2}}{1-\frac{\omega^2}{c_1^2}}\right]$$

- Normalization condition gives

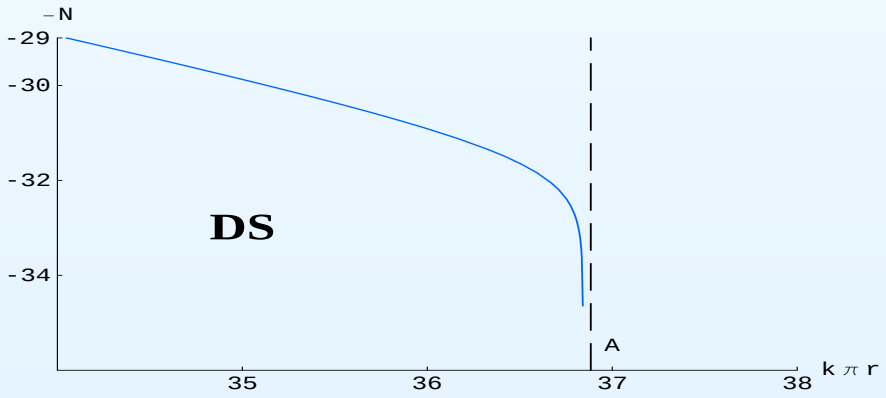
$$c_2 = 1 + \sqrt{1+\omega^2} \ .$$

DS brane case...contd

- In this case ω is not constrained as opposed to ADS case.
- Hierarchy constraints gives the solution

$$e^{-x} = \frac{10^{-n}}{c_1}\left[1+\sqrt{1+10^{2n-N}}\right].$$

- So, in this case we have only one solution



DS brane case...contd

- So, in DS case, we get single solution for a particular value of cosmological constant.
- Brane tension always happens to be negative in the visible brane along the curve in ($kr\pi - N$) plane for a particular value of hierarchy.
- For N = 124, which is the present estimated value of cosmological constant, $kr \sim 12.00$ which is very closed to the RS value.
- So, solution for hierarchy and smallness of cosmological constant simultaneously lead to the order of unity(Planck unit) separation between the two branes.

Conclusion

- Generalization of RS gives a posssiblity of having a intimate connection between the two fine tuning problems.
- In ADS visible brane case, we have two branches of solution, for a fixed value of cosmological constant, leading to hierarchy of scale.
- In this case, there axists a region of parameter space of kr, N, n, in which we can accomodate +ve tenson visible brane.
- There exists a zero brane tension when negative cosmologcial constant is maximum $\sim 10^{-32}$ in planck unit.
- But in DS brane case, the visible brane tension turns out to be necessarily negative, but we can have very tiny cosmological constant along with the hierarchy solution.
- Solution of hierarchy and smallness of cosmological constant simultaneously lead to the order of unity(Planck unit) separation between the two branes.

THANK YOU