Phenomenology of warped extra dimensions at LHC

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- **TeV scale gravity models**
- **Phenomenology with them**
- **NLO QCD results for Drell-Yan at TeV scale**
- **Summary**

In collaboration with

Prakash Mathews, K. Sridhar, W. L. van Neerven & M.C. Kumar

Large Extra Dimensions

- Models of "Extra Dimensions" are now studied as serious contenders for "Physics Beyond SM"(BSM).
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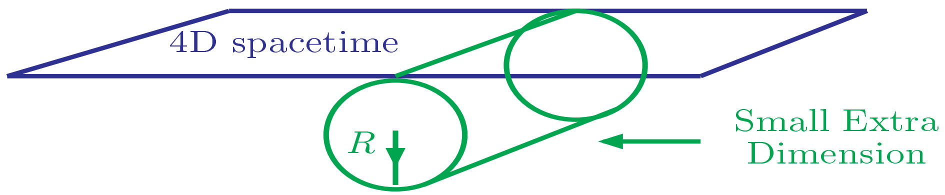

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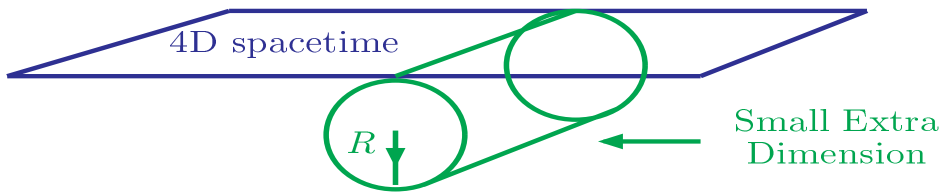

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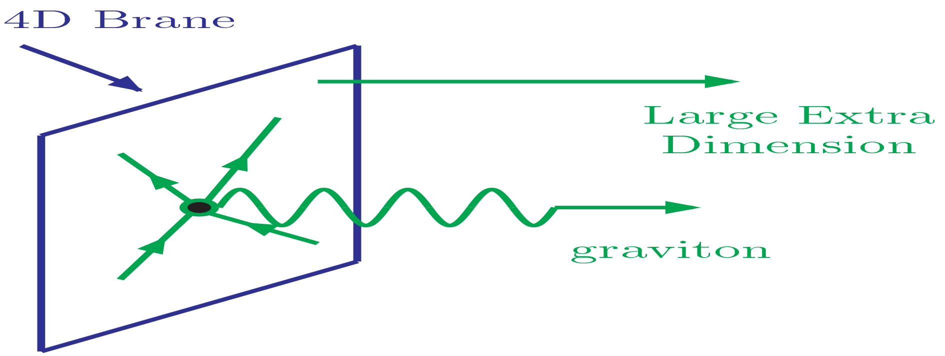

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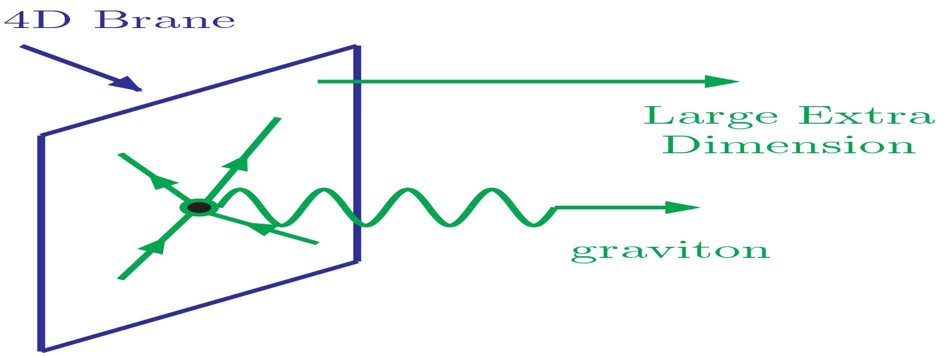

Brane-world Picture

- Gauss law in 4+d dim with d compact dim of radius R

$r \ll R$ $r \gg R$

$$V(r) \sim \frac{m_1 m_2}{M_S^{d+2}} \frac{1}{r^{d+1}} \qquad V(r) \sim \frac{m_1 m_2}{M_S^{d+2} R^d} \frac{1}{r}$$

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- r $\gg$ R gravitational flux lines in 4+d dim are constrained in the compact dim and hence the potential is effectively r^{-1} at large distances

$$\boxed{M_P^2 \sim M_S^{2+d}\, R^d}$$

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$$R \sim 10^{\frac{30}{d}-17}\ \text{cm} \qquad R^{-1} \sim 10^{\frac{-30}{d}+3}\ \text{GeV}$$

d	R (cm)	R^{-1}
1	10^{13}	10^{-27} eV
2	10^{-2}	10^{-3} eV
3	10^{-7}	100 eV
$\dots$	$\dots$	$\dots$
6	10^{-12}	10 MeV

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- Only Gravitational field can probe the full 4+n dim space, deviation from Newtonian gravity puts constraint on number of extra dim

$$d \geq 2 \ \ \textbf{Possible}$$

If Fundamental Planck Scale $M_S \sim 1$ TeV, "no Hierarchy problem".

Kaluza-Klein Modes

- Extra dimensions being compact, gravitational field will be periodic function in the extra dimension.
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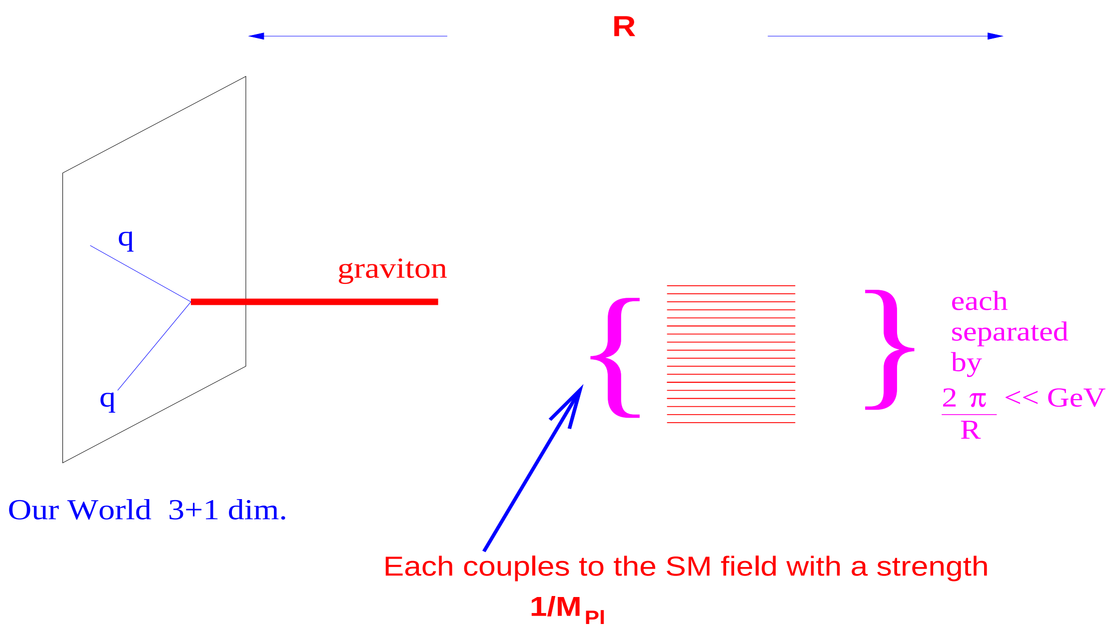
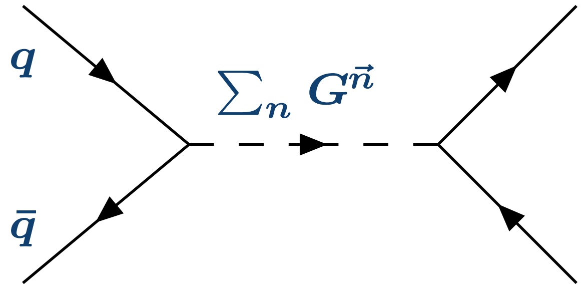


Massless graviton and KK modes couple with SM fields with coupling $M_P^{-1} \sim R^{\frac{d}{2}}$

Kaluza-Klein suppression



$$\sim \frac{1}{R^{\frac{d}{2}}} \quad \sum_n \frac{1}{Q^2 - m_{\vec{n}}^2 + i\epsilon} \quad \frac{1}{R^{\frac{d}{2}}}$$

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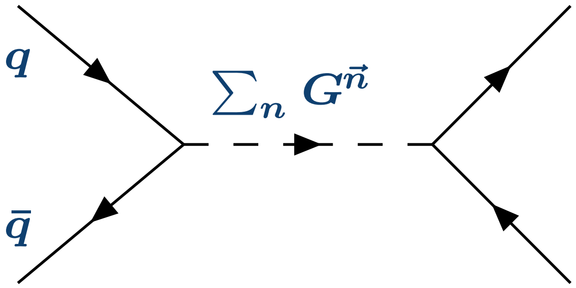

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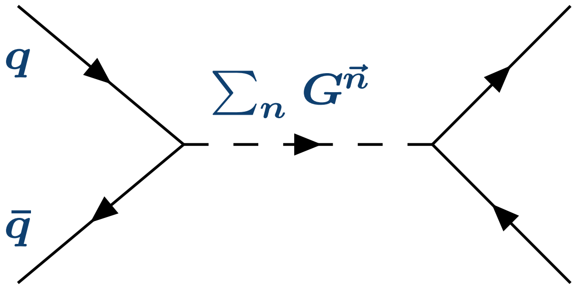

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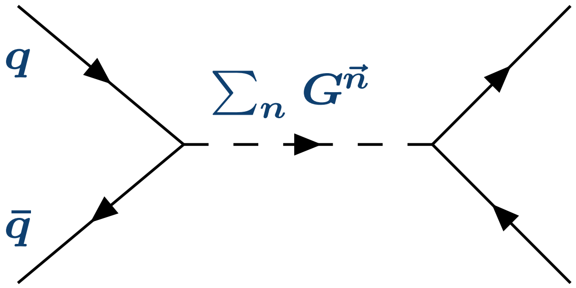

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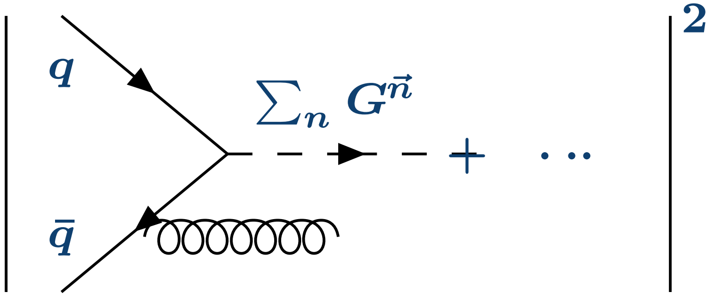

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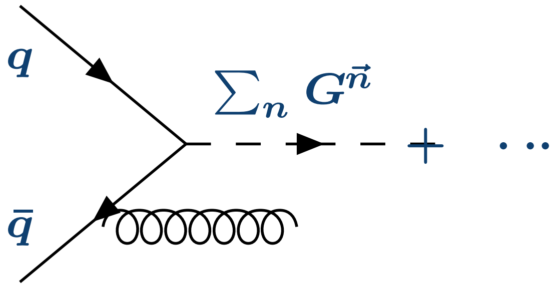

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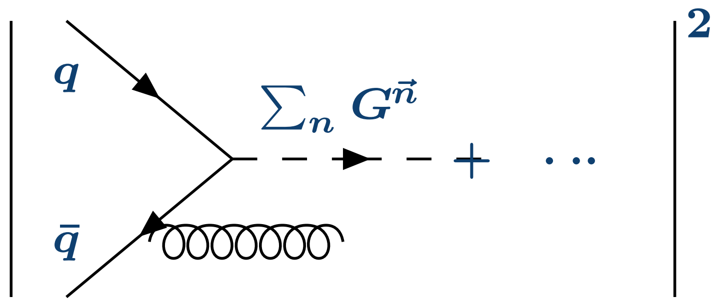

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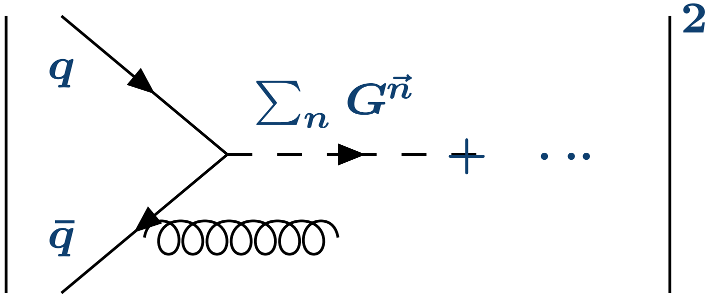

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- The couplings of the massive RS gravitons are enhanced by the exponential $e^{\pi \mathcal{K} R_c}$ leading to interactions of electroweak strength.
- Two basic parameters of the RS model are

$$m_0 = \mathcal{K} \, e^{-\mathcal{K} r_c \pi} \qquad\qquad c_0 = \frac{\mathcal{K}}{M_P}$$

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The propagator:

$$P_G(q) \quad = \quad \mathcal{D}(Q^2) B_{\mu\nu\lambda\rho}(q) \ ,$$

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M_n-the masses of the individual resonances, Γ_n are the widths.

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M_n-the masses of the individual resonances, Γ_n are the widths. λ is defined as

$$\lambda(x_s) \quad = \quad \sum_{n=1}^{\infty} \frac{x_s^2 - x_n^2 - i\frac{\Gamma_n}{m_0}x_n}{x_s^2 - x_n^2 + \frac{\Gamma_n}{m_0}x_n} \ , \qquad x_s = Q/m_0 .$$

We have to sum over all the resonances to get the value of $\lambda(x_s)$.

Phenomenology with Extra-Dimension

In the SM, the partonic cross sections decreases with the energy scale (Q or p_T involved):

$$\frac{d}{dQ^2}\hat{\sigma}_{ab}^{SM}\left(\hat{s}, Q^2\right) \sim \frac{1}{\hat{s}Q^2}$$

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- The processes where the virtual/real KK gravitons contribute significantly:

  (1) Di-lepton or Drell-Yan production at large invariant mass Q

  (2) Di-photon or Di-boson production at large Q, P_T

  (3)Observables with missing energy (...) $\cdots$

Drell-Yan Process

$$P_1(p_1) + P_2(p_2) \longrightarrow [\gamma, Z, G] + \text{hadronic states}(X)$$
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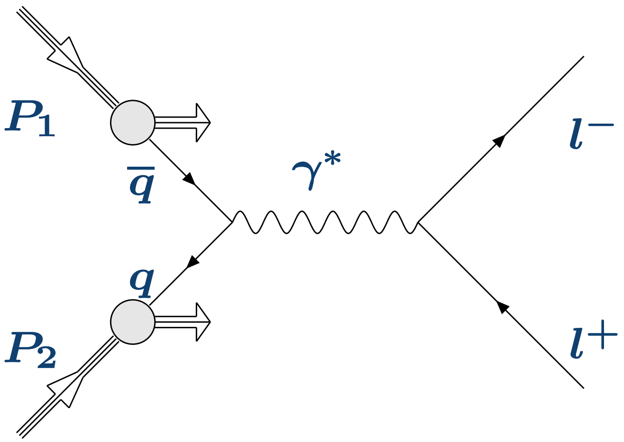

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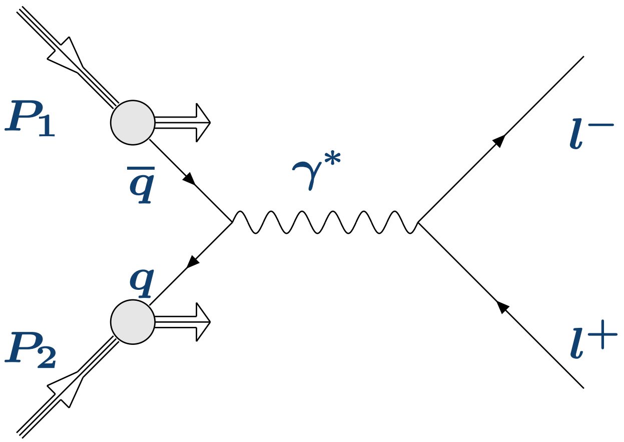

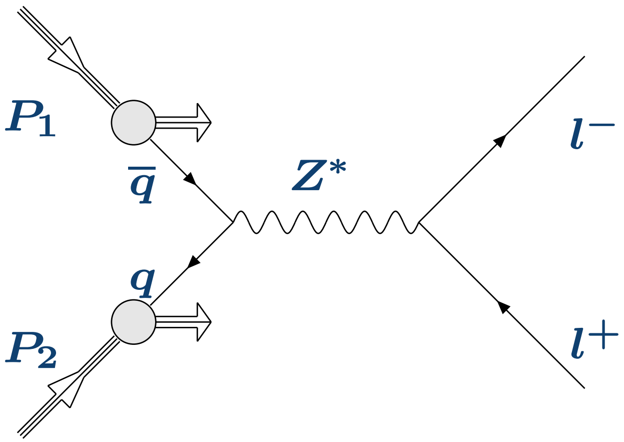

SM

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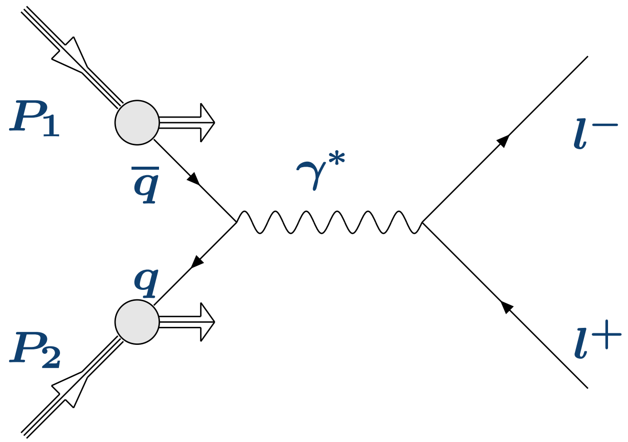

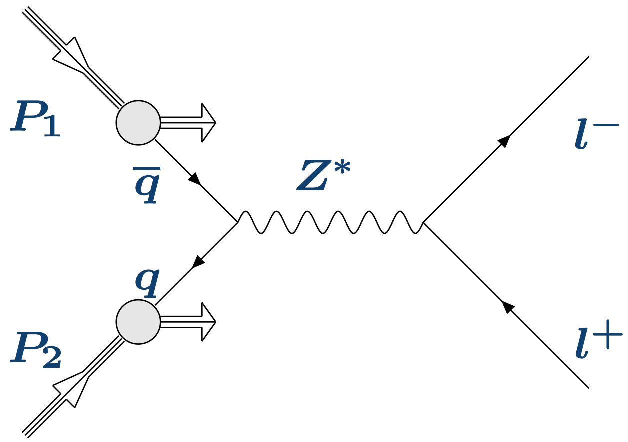

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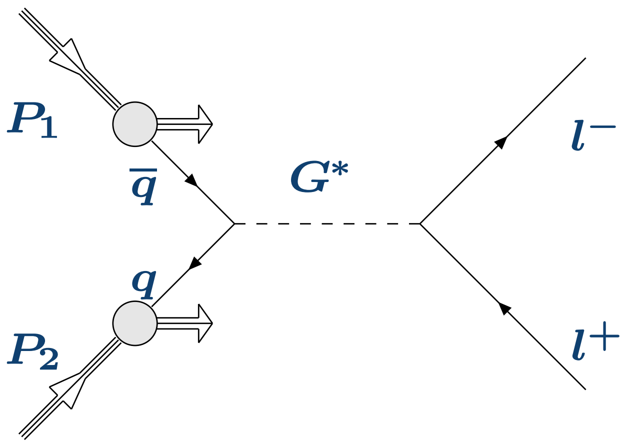

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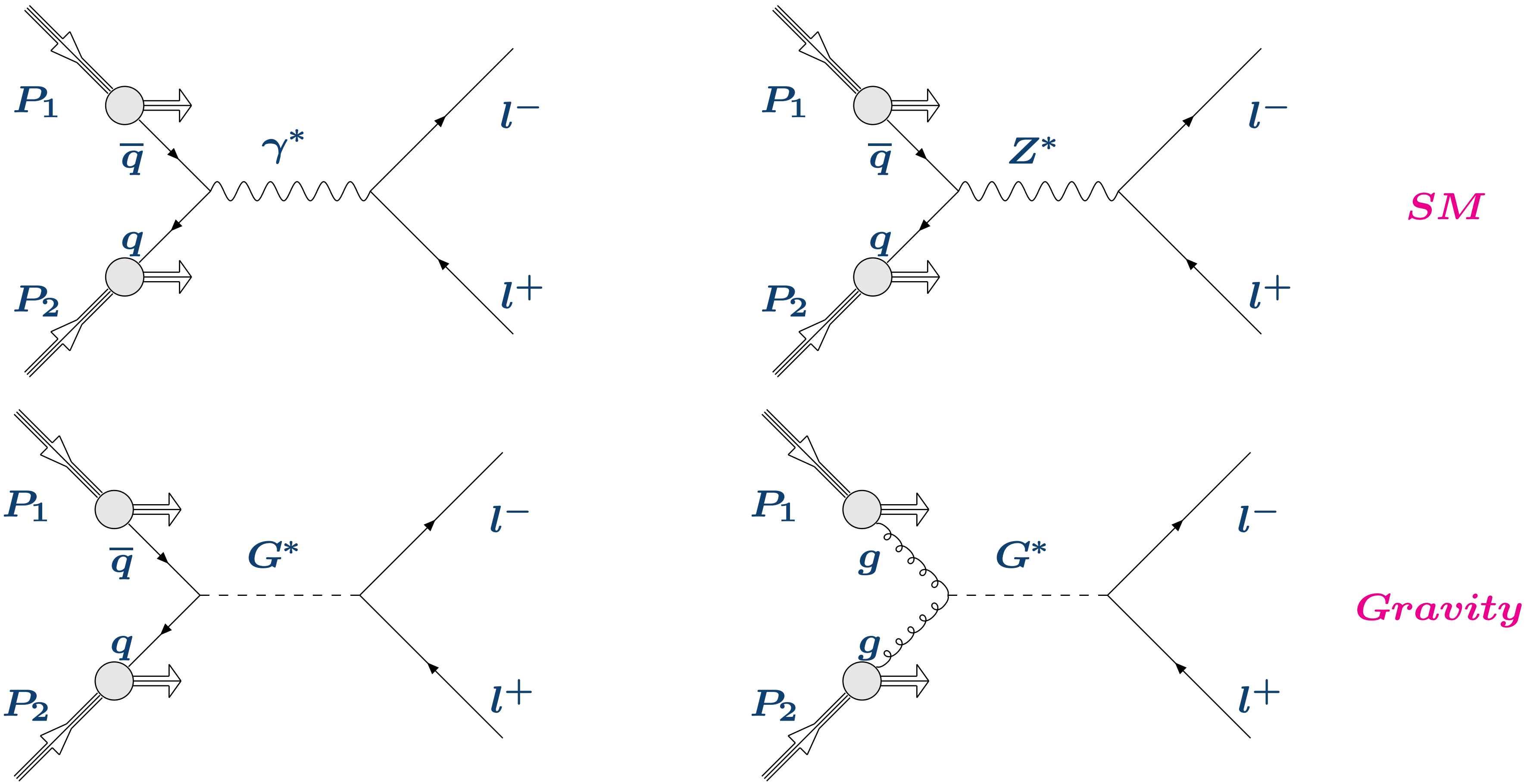

Parton Model

Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

$$2S\, d\sigma^{P_1 P_2}\left(\tau, Q^2\right) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}\left(x, \mu_F\right) 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, Q^2, \mu_F\right)$$

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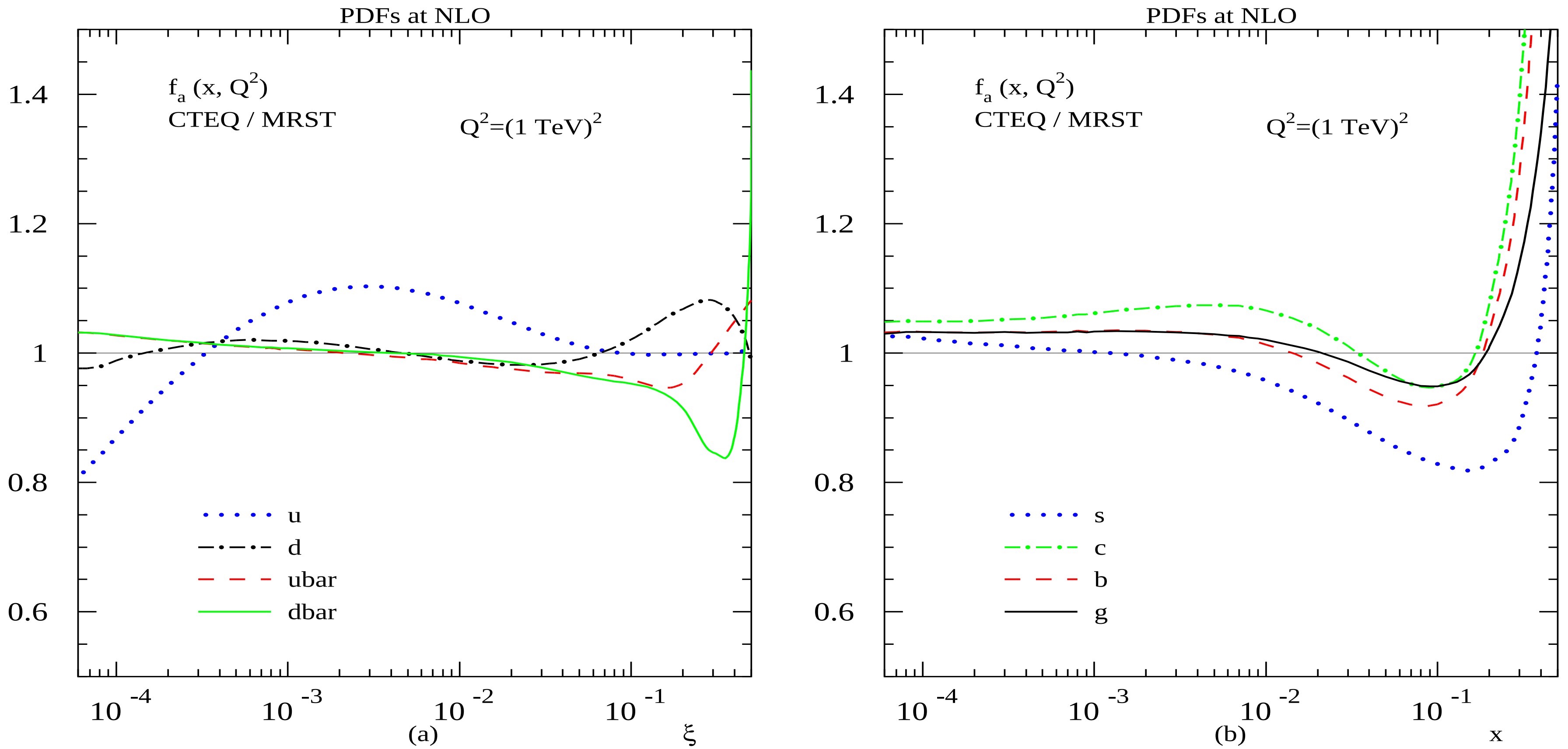
It is hence important for extra dimension searches to have better control over the theoretical uncertainties

CTEQ/MRST

Various groups perform global analysis of wide rage of DIS and other hadron scattering data to get best fits to a particular order in QCD



○ PDFs satisfies the general constraint, but could differ from each other as experimental and theoretical uncertainties are involved, various assumptions and initial conditions are used

Contributing Subprocess

Leading Order:

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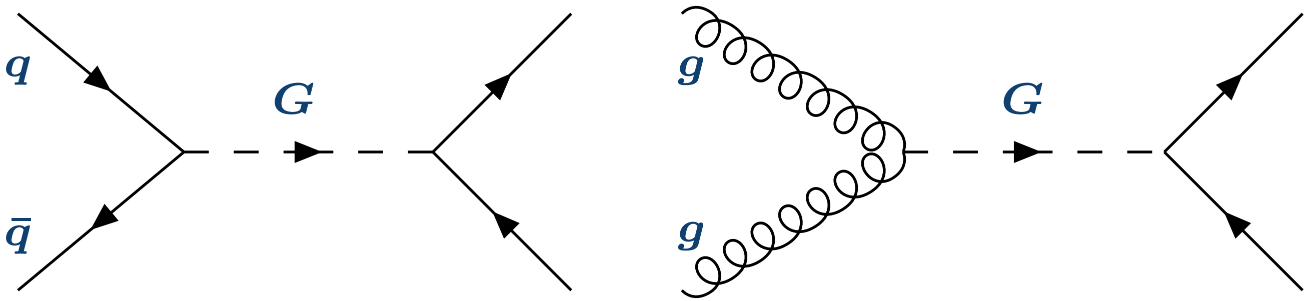

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- How serious is it?

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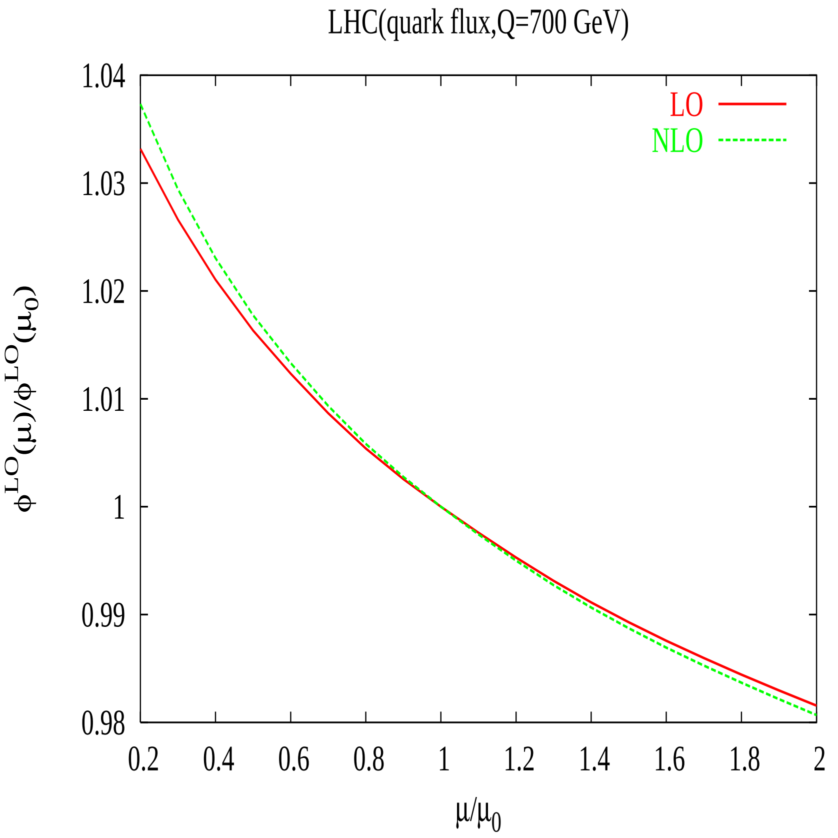

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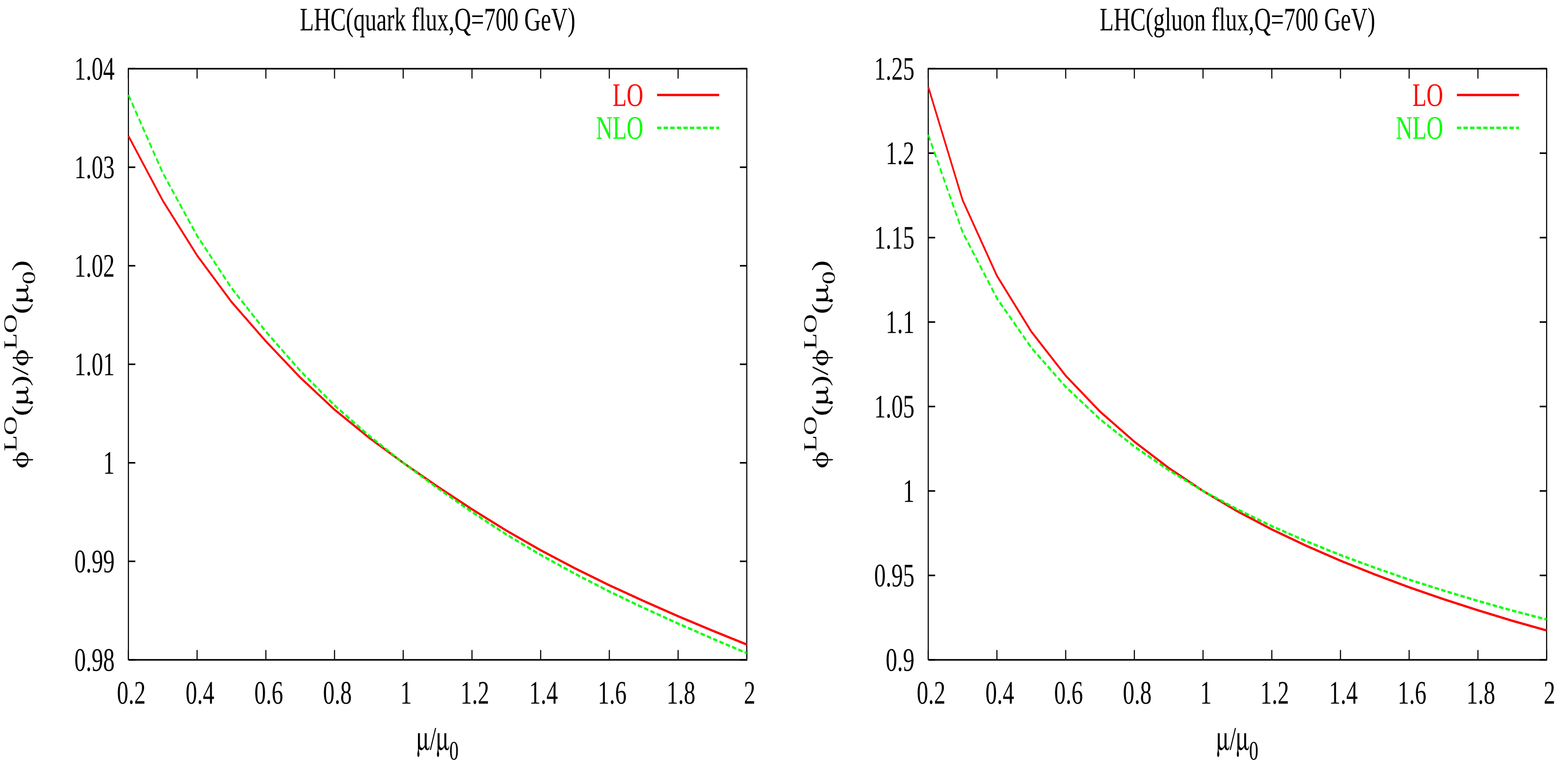

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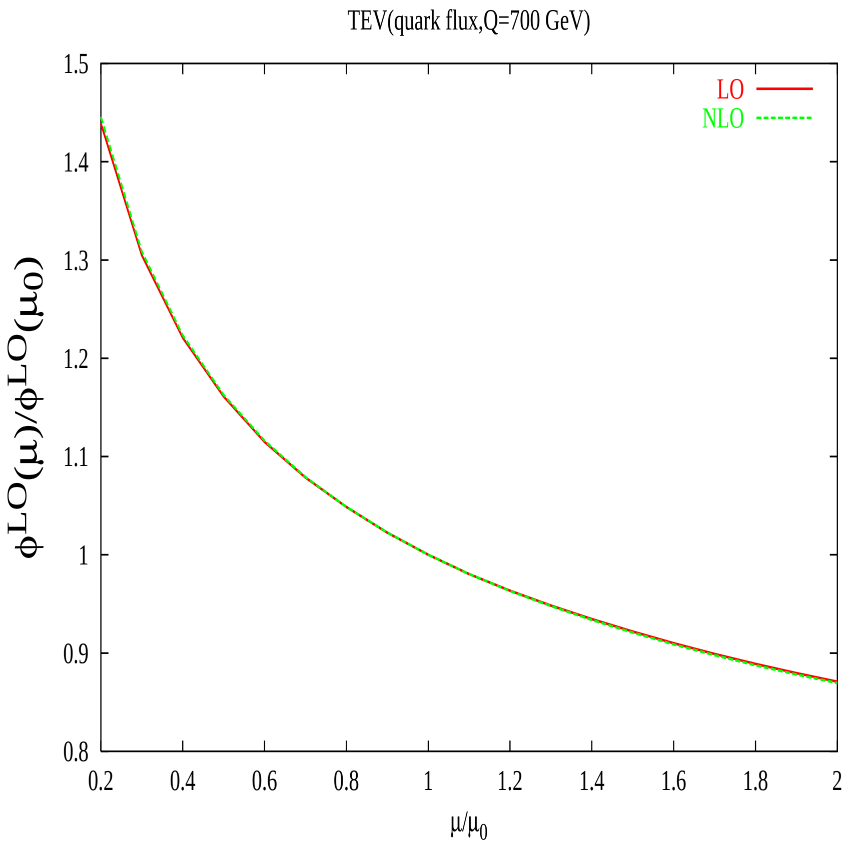

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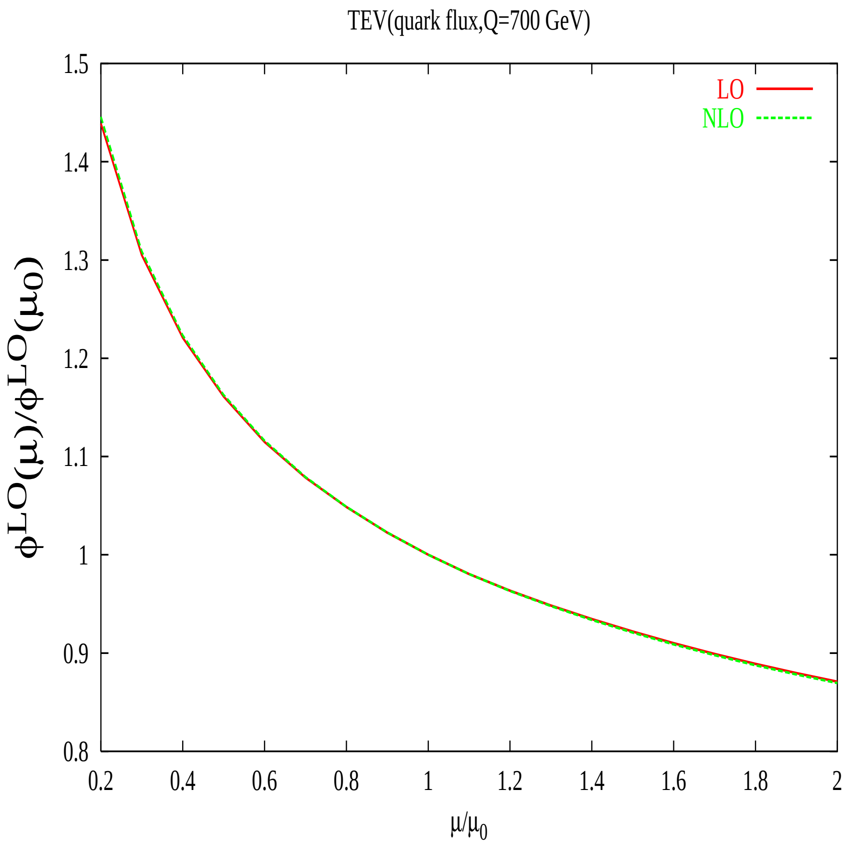

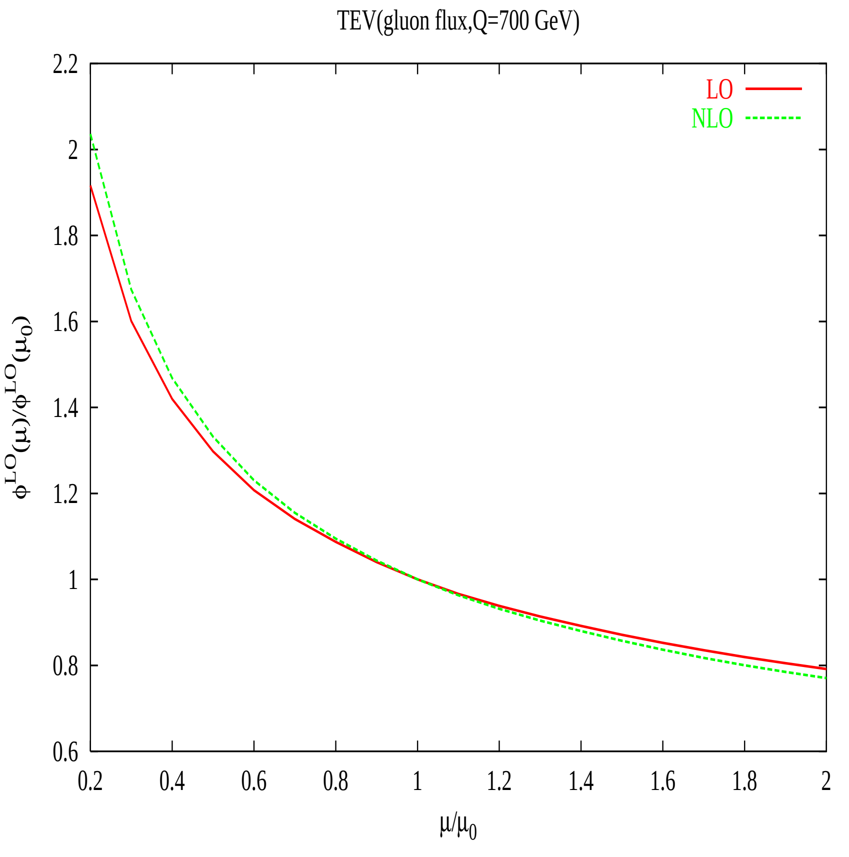

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QCD corrections are larger than other EW and gravity corrections.

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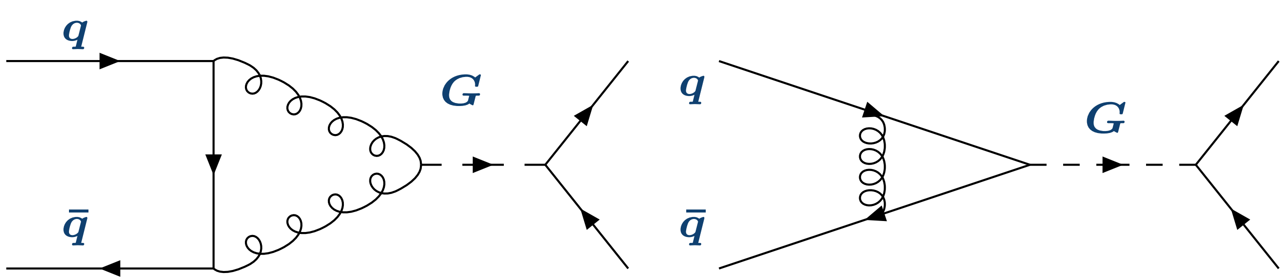
- Collinear mass factorisation is done in $\overline{MS}$ scheme.

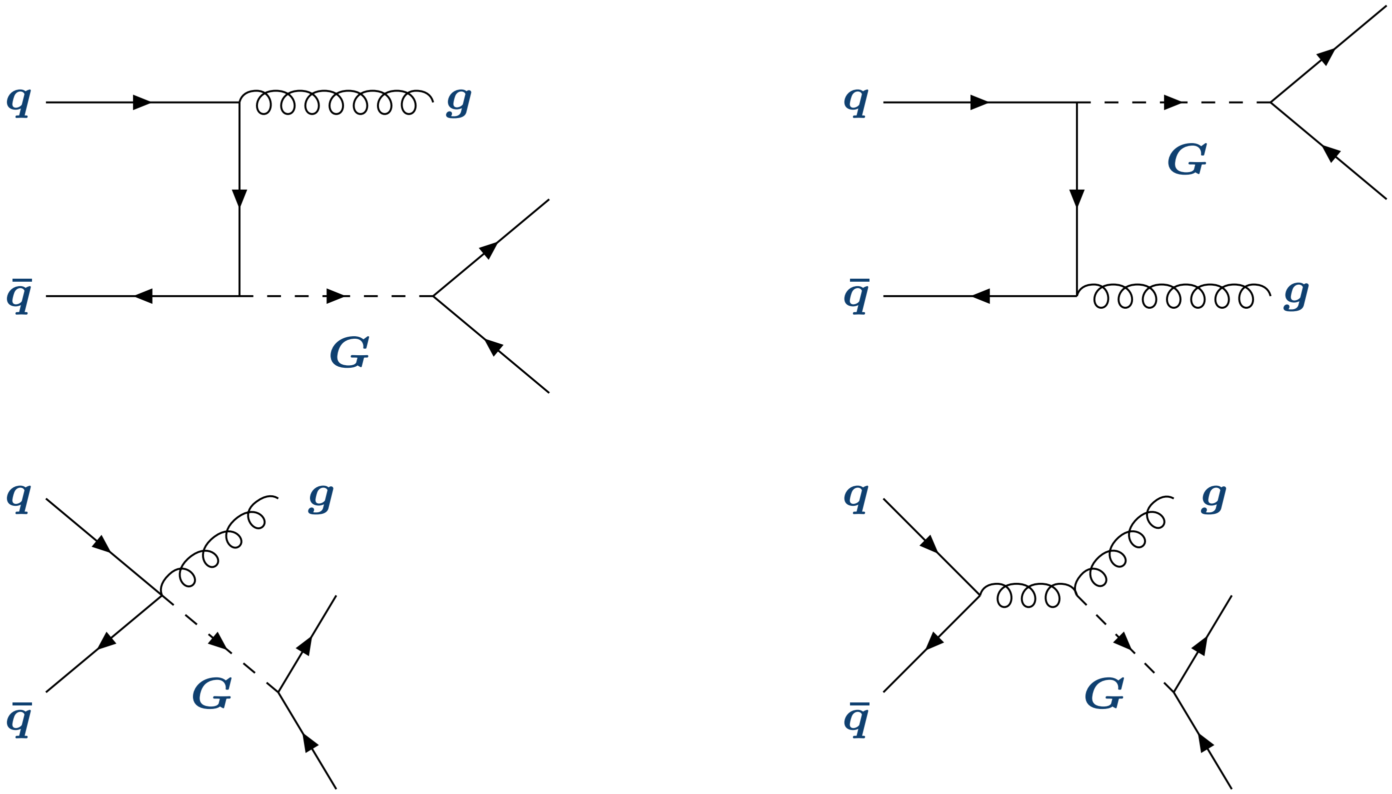
Virtual Corrections, $q\,\bar{q} \longrightarrow\ G$

$$\bar{\boldsymbol{\Delta}}^{G}_{q\bar{q}} = \boldsymbol{\Delta}^{(0)G}_{q\bar{q}} + \boldsymbol{a_s}\,\frac{\boldsymbol{2}}{\boldsymbol{\varepsilon}}\boldsymbol{\Gamma}^{(1)}_{qq} \otimes \boldsymbol{\Delta}^{(0)G}_{q\bar{q}} + \boldsymbol{a_s}\boldsymbol{\Delta}^{(1)G}_{q\bar{q}}$$

$q + \bar{q} \longrightarrow G$ (1 loop):



Real emission, $q\,\bar{q} \longrightarrow g\,G$



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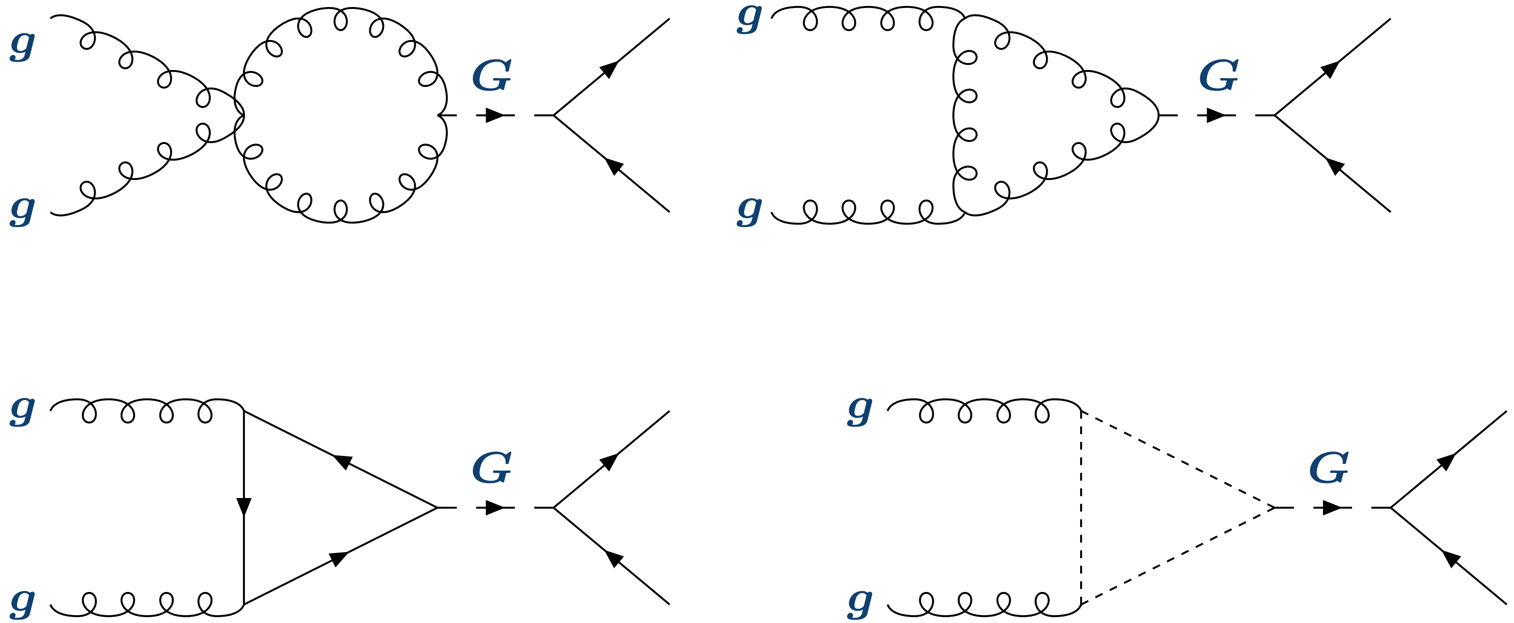
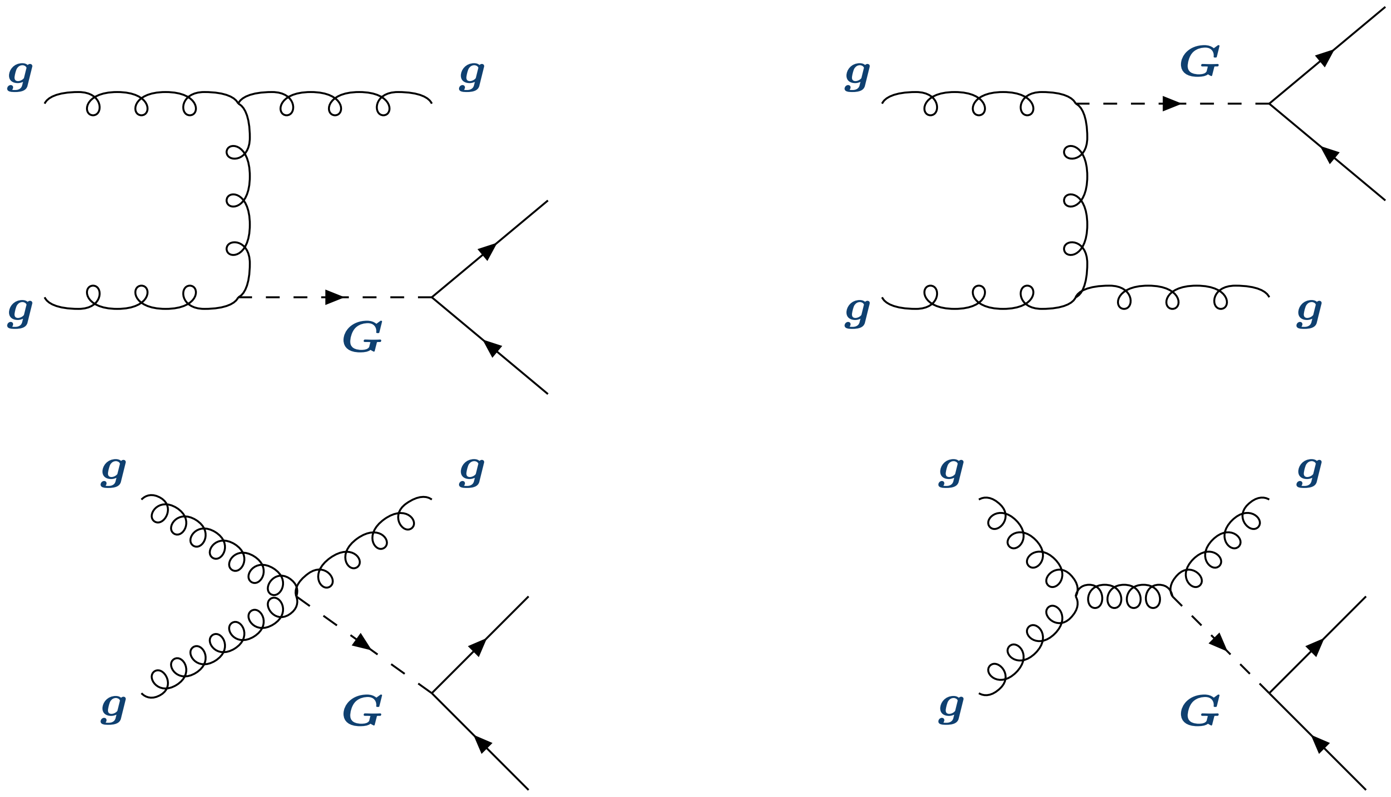


Real emission, $g\, g \to g\, G$



Real emissions, $q\,\bar{g} \longrightarrow q\,G$

$$\boxed{\bar{\boldsymbol{\Delta}}^{G}_{qg} = a_s \frac{1}{\varepsilon}\left(\boldsymbol{\Gamma}^{(1)}_{qg} \otimes \boldsymbol{\Delta}^{(0)G}_{q\bar{q}} + \boldsymbol{\Gamma}^{(1)}_{gq} \otimes \boldsymbol{\Delta}^{(0)G}_{gg}\right) + a_s \boldsymbol{\Delta}^{(1)G}_{qg}}$$

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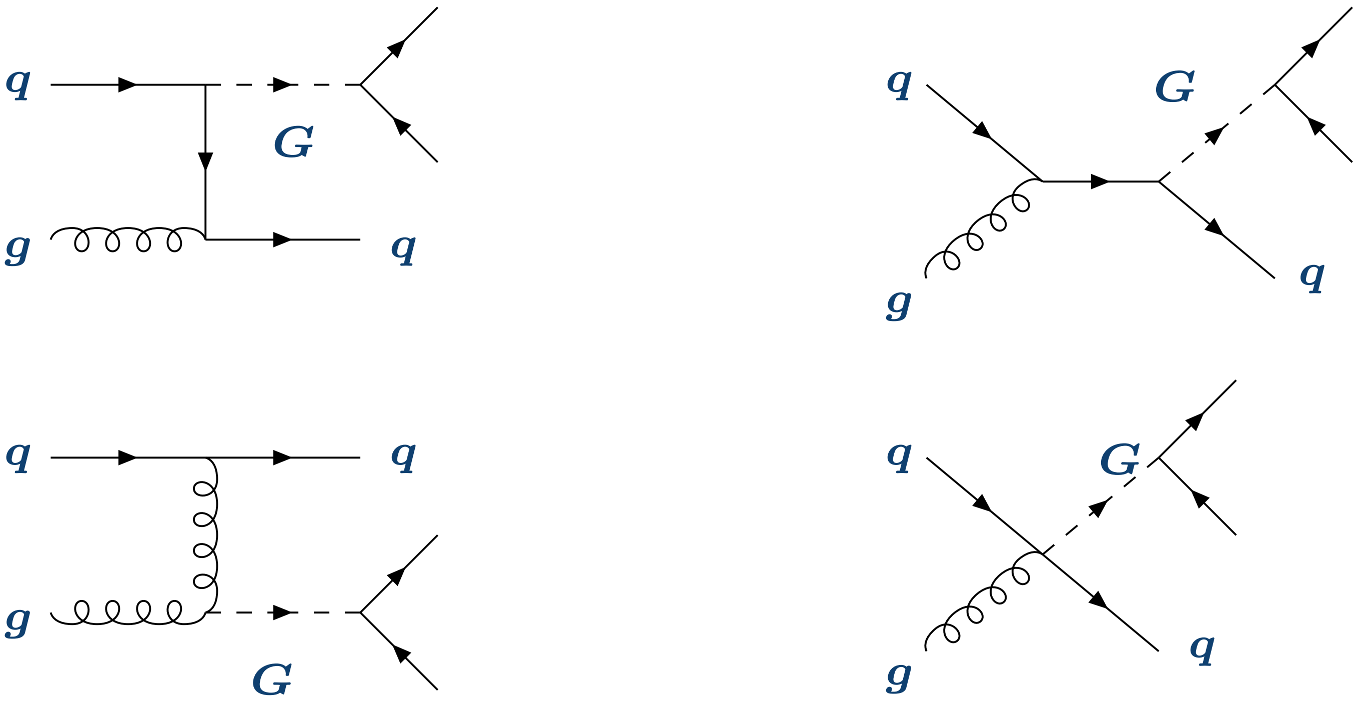

Invariant mass distribution of lepton pair

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$$2S\,\frac{d\sigma^{P_1P_2}}{dQ^2}(\tau,Q^2) =$$

$$\sum_q \mathcal{F}_{SM,q}\Bigg[H_{q\bar{q}}(\tau,Q^2)\otimes\left(\Delta^{(0)\gamma Z}_{q\bar{q}}(\tau,Q^2)+a_s\,\Delta^{(1)\gamma Z}_{q\bar{q}}(\tau,Q^2)\right)$$

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Coefficient Functions Independent of ADD or RS Model

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	LHC	TEVATRON
$\sqrt{S}$	14 TeV	1.96 TeV

PDF	MRST 2001 LO & NLO
Choice of scale	$\mu_F = \mu_R$ & $\mu_F = Q$

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$$\frac{d\sigma^I(Q)}{dQ} \qquad \left.\frac{d\sigma^I(Q,Y)}{dQ\,dY}\right|_{Q_0}$$

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Partonic Cross Section versus its Flux

- Parton level cross section increases monotonically with invariant lepton pair mass Q upto m_o.

$$\frac{d}{dQ^2}\hat{\sigma}^{RS}_{ab}\left(\hat{s}, Q^2, M_S\right) \quad \sim \quad c_o^4 \lambda \left(\frac{Q^6}{\hat{s}\ \ m_o^8}\right), \qquad\qquad Q < m_o$$

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$$\Phi_{ab}\left(x = \frac{Q^2}{S}, \mu_F\right)$$

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- At large Q the gravity "cross section" is comparable to SM.

- At large Q the parton "cross section" dominates over "Flux" leaving "observable effect".

Flux at LHC and Tevatron

$$\Phi_{ab}(x,\mu_F) \quad = \quad \int_x^1 \frac{dz}{z} f_a\left(z,\mu_F\right) f_b\left(\frac{x}{z},\mu_F\right) \qquad\qquad x = \frac{Q^2}{S}$$

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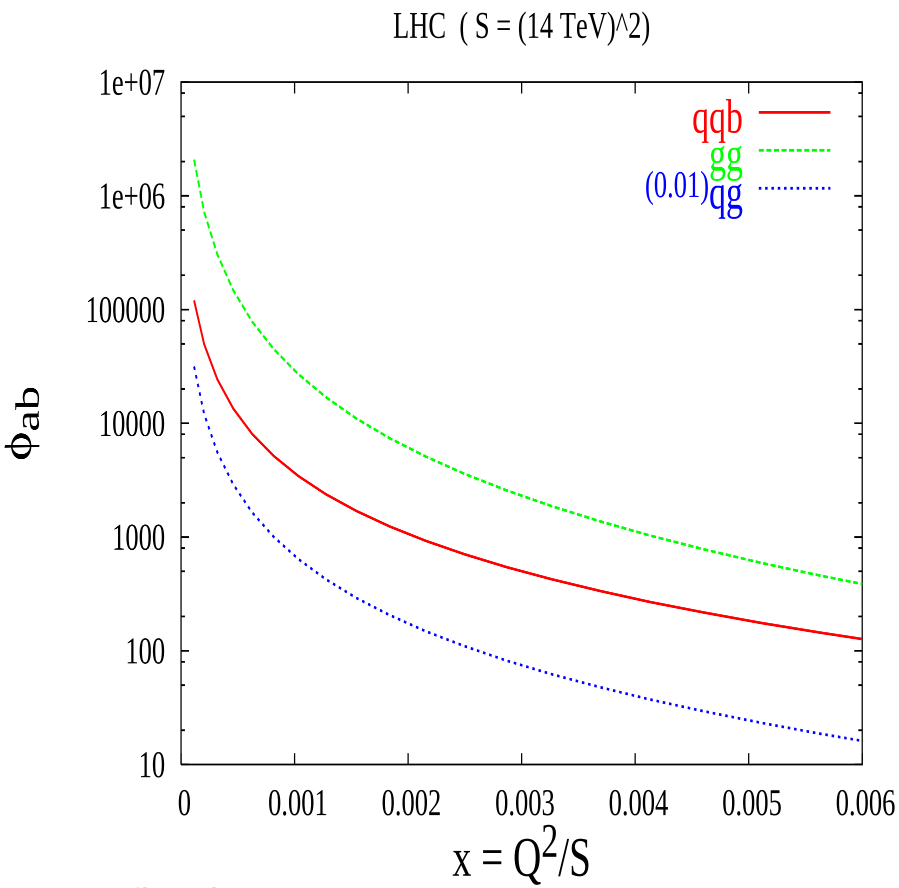

Gluon flux is largest at LHC

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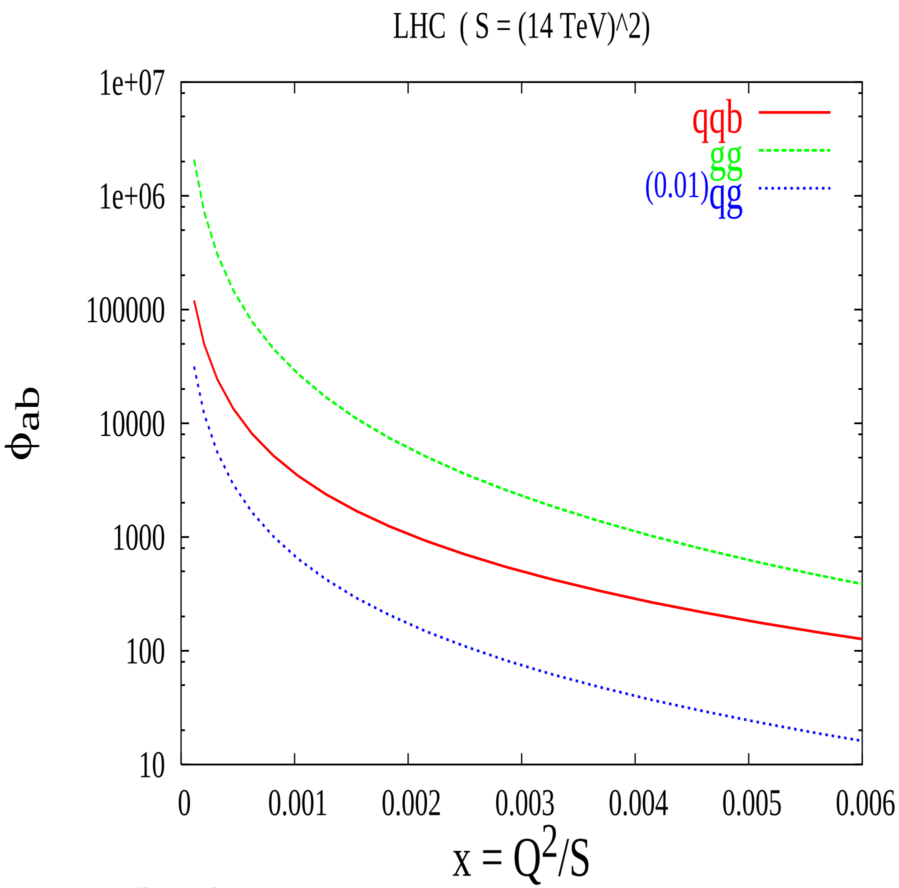

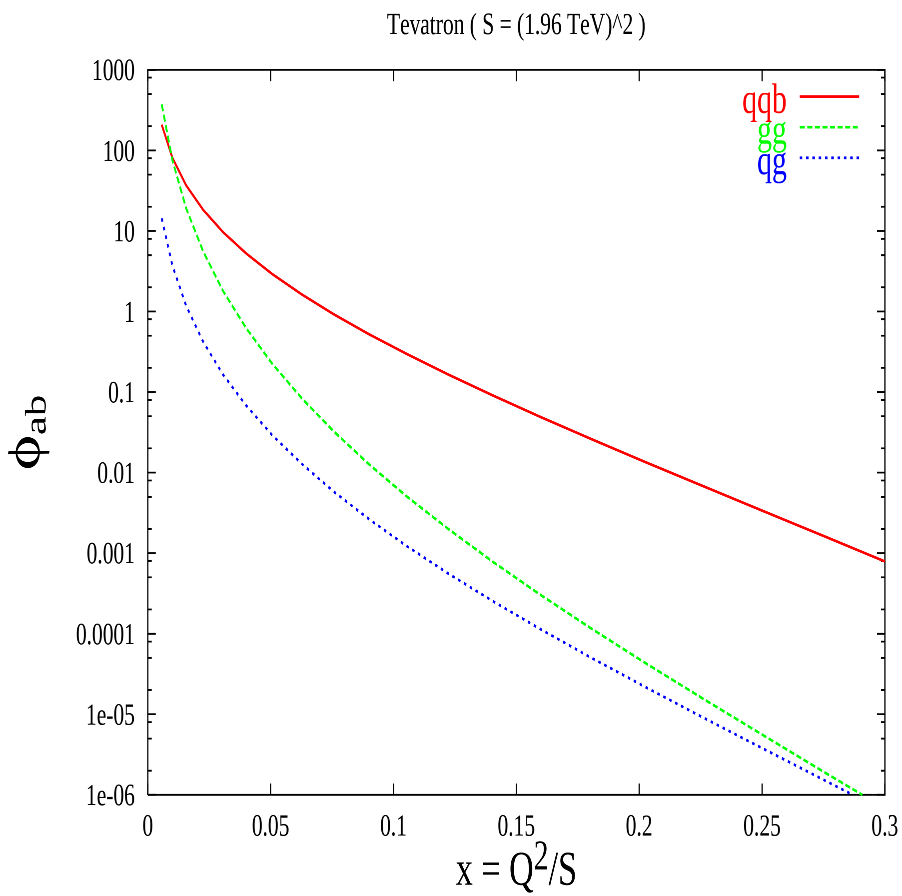

Gluon flux is largest at LHC

Quark-anti quark flux is largest at Tevatron

RS Scenario Results

$$\mathcal{D}(Q^2) = \sum_{n=1}^{\infty} \frac{1}{s - M_n^2 + iM_n\Gamma_n} \equiv \frac{\lambda}{m_0^2}$$

$$\frac{c_0^2}{m_0^2}\mathcal{D}(Q^2) = \frac{c_0^2}{m_0^4}\,\lambda$$

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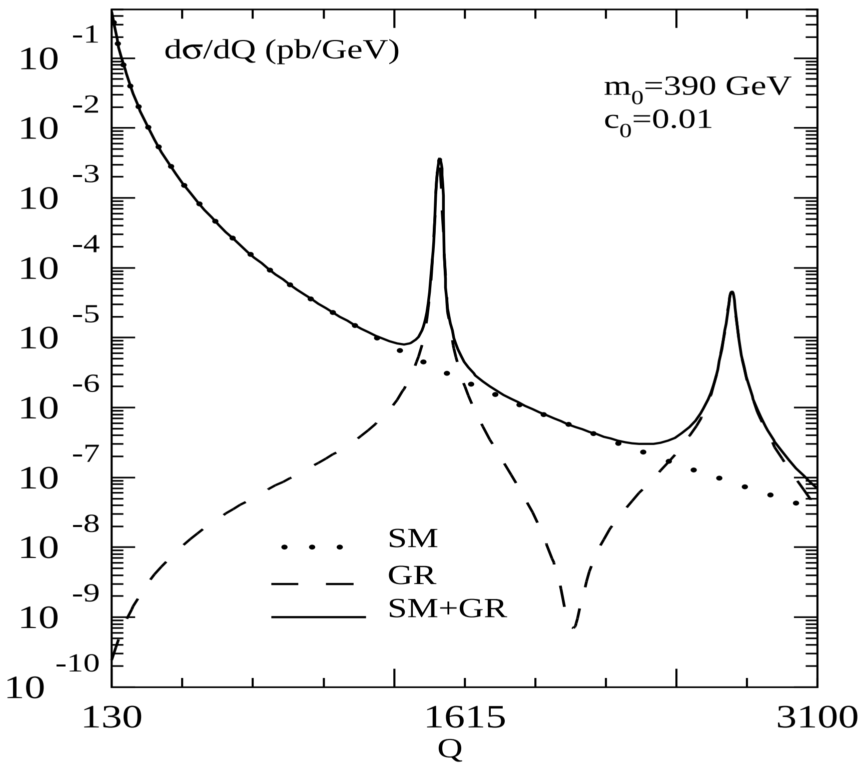
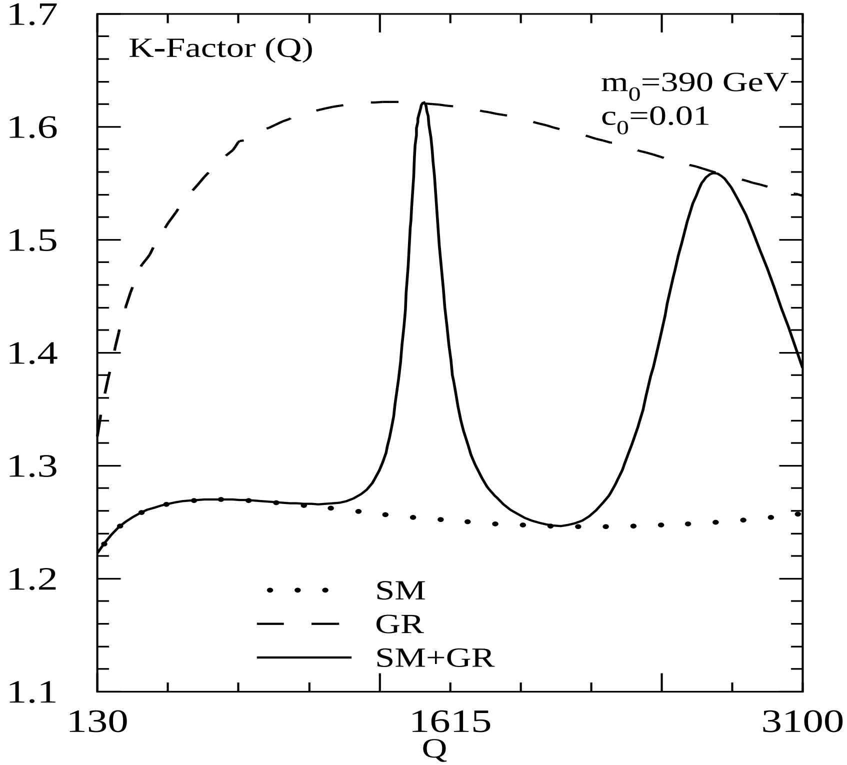
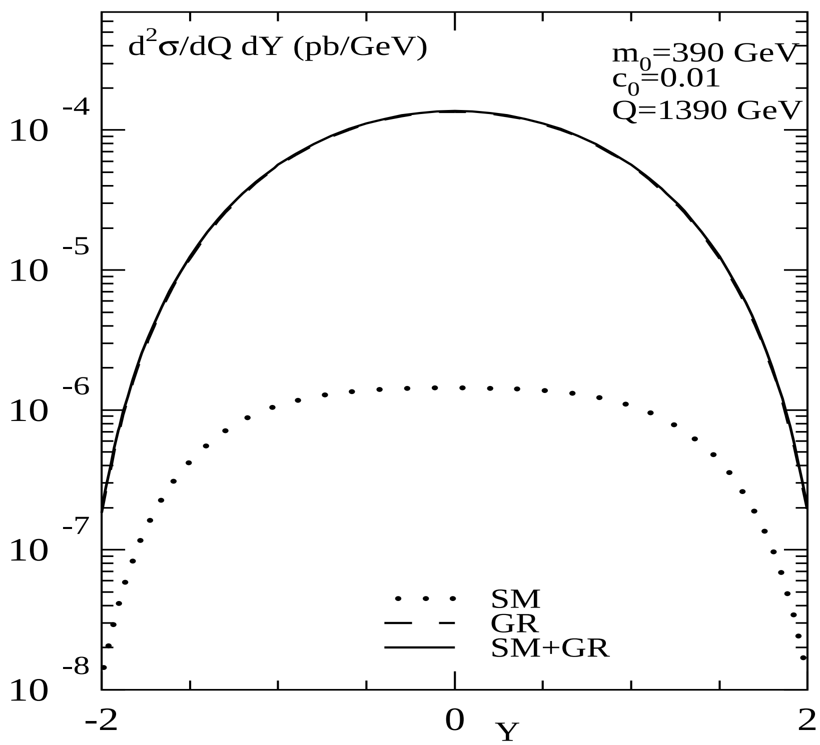
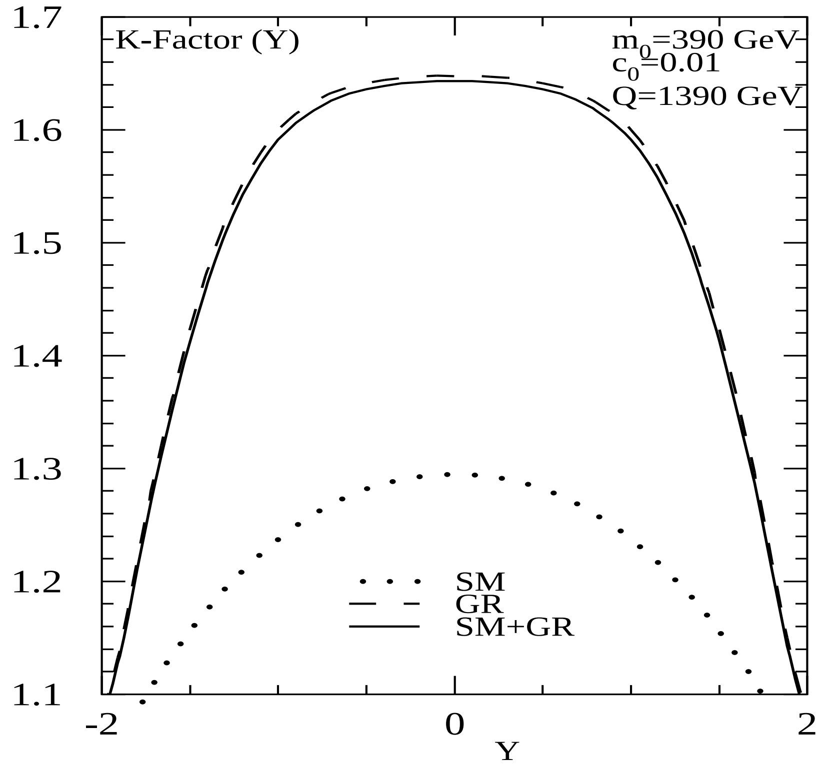




- Away from the resonance region gravity contribution is negligible
- K-Factor behavior can be understood from the $K^{(0)}$ behavior for the RS model.

Rapidity Y distribution:



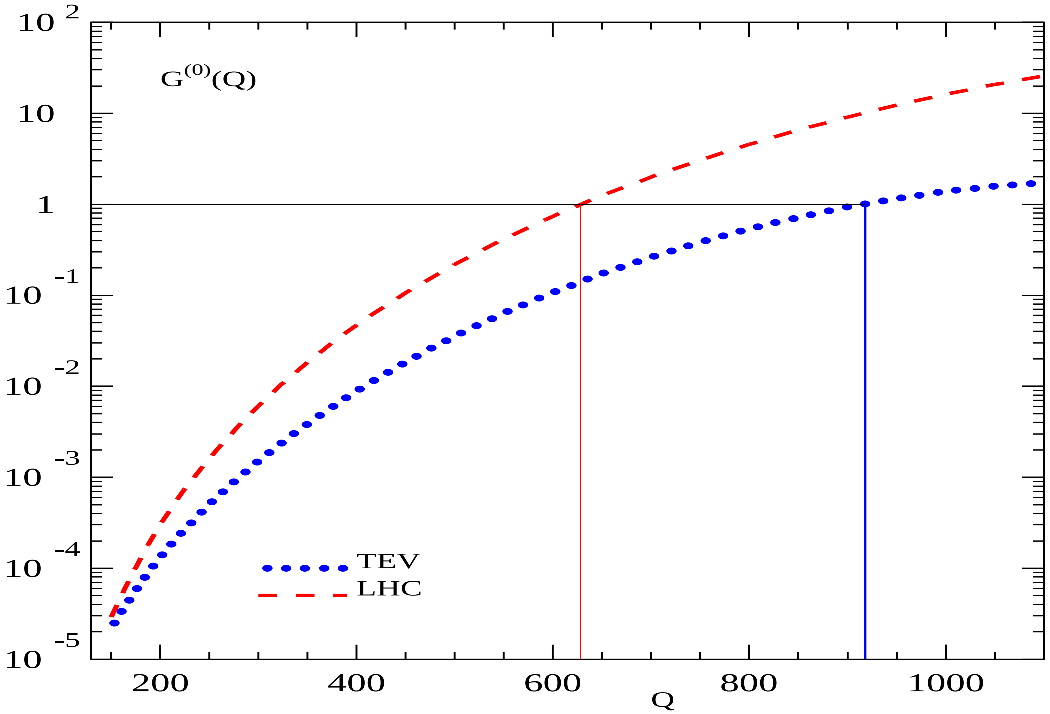


- K-Factor for rapidity distribution, close to to the first KK resonance $m_1 \sim 1.5$ TeV
- Gravity dominates the resonance region as can be understood from the $K^{(0)}$ behavior of RS model.

K-Factor

$$K^{(SM+GR)}(Q) = \frac{K^{SM} + K^{GR}\, G^{(0)}}{1 + G^{(0)}}$$

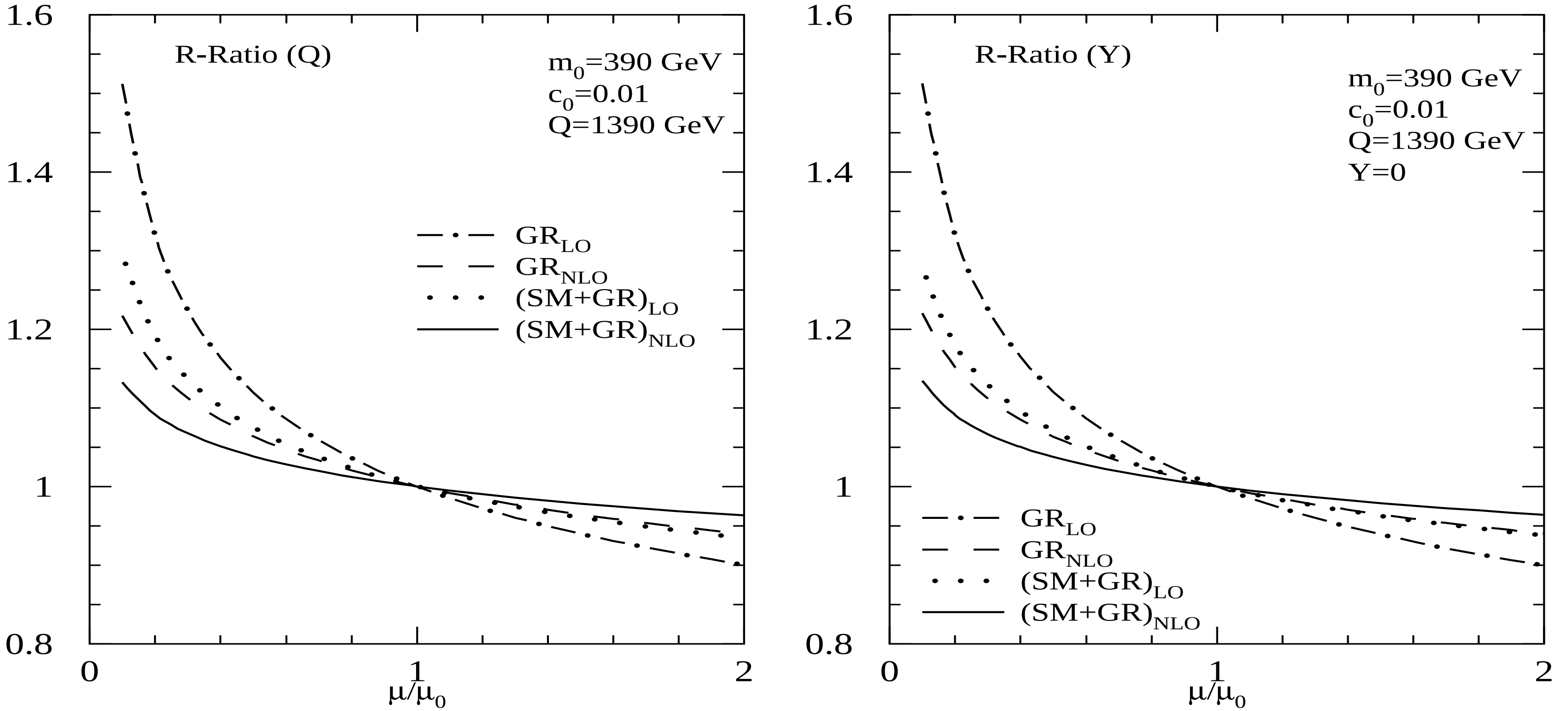
$$G^{(0)}(Q) = \left[\frac{d\sigma^{SM}_{LO}(Q)}{dQ}\right]^{-1} \left[\frac{d\sigma^{GR}_{LO}(Q)}{dQ}\right]$$



- $G^{(0)}(Q)$ behavior is governed by a competing 'couplings' and PDF flux at LHC and TEV
- At high Q when Gravity contribution becomes comparable to SM, the PDF flux dictates the proceedings

R-Factor:

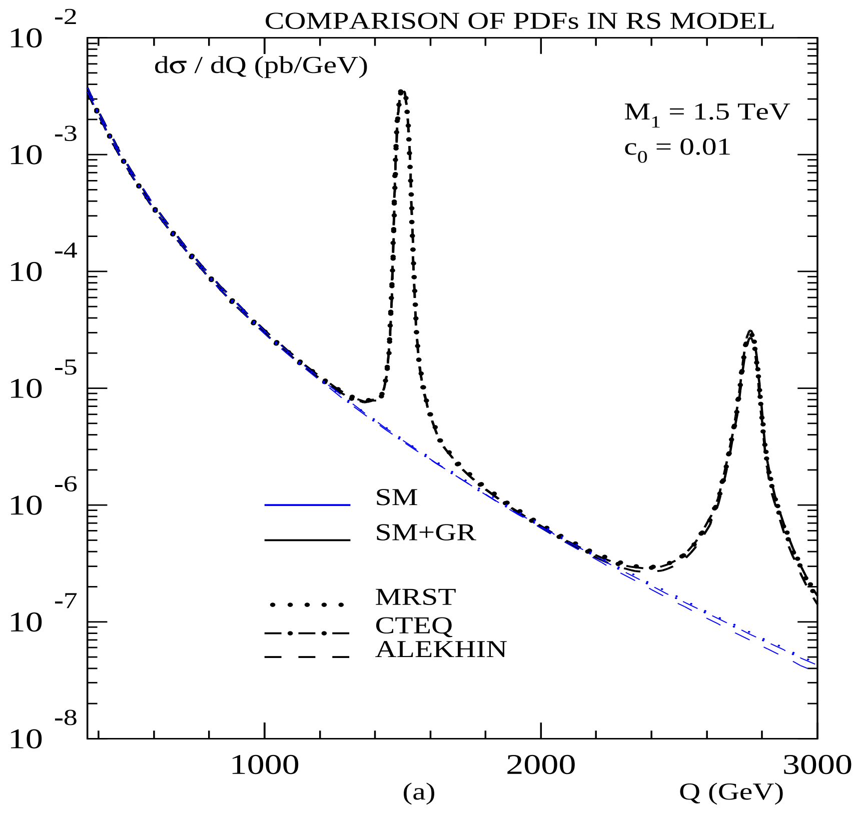
$$R^I_{LO,NLO} = \left[\frac{d\sigma^I_{LO,NLO}(Q,\mu=\mu_0)}{dQ}\right]^{-1}\left[\frac{d\sigma^I_{LO,NLO}(Q,\mu)}{dQ}\right]\Bigg|_{Q=Q_0}$$

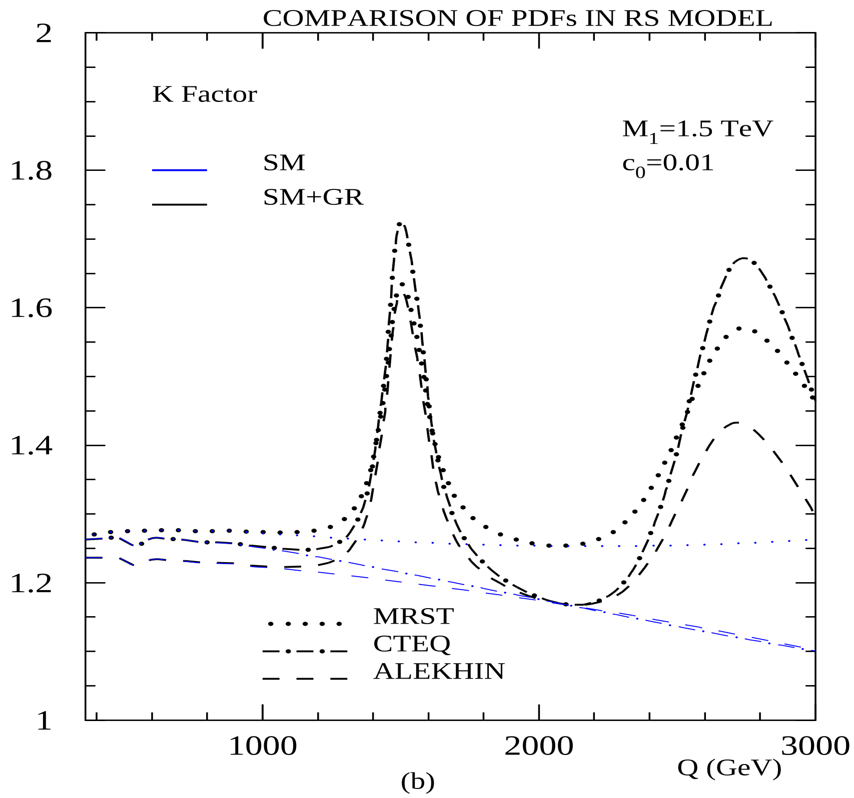


- Scale variation reduced considerably in going from LO $\longrightarrow$ NLO
- Inclusion of SM to GR also reduces scale variation

RS Scenario Results

$$\frac{c_0^2}{m_0^2}\mathcal{D}(Q^2) \quad = \quad \frac{c_0^2}{m_0^2}\sum_{n=1}^{\infty}\frac{1}{s - M_n^2 + iM_n\Gamma_n} \quad \equiv \frac{c_0^2}{m_0^4}\lambda\Big(\frac{Q}{m_0}\Big)$$

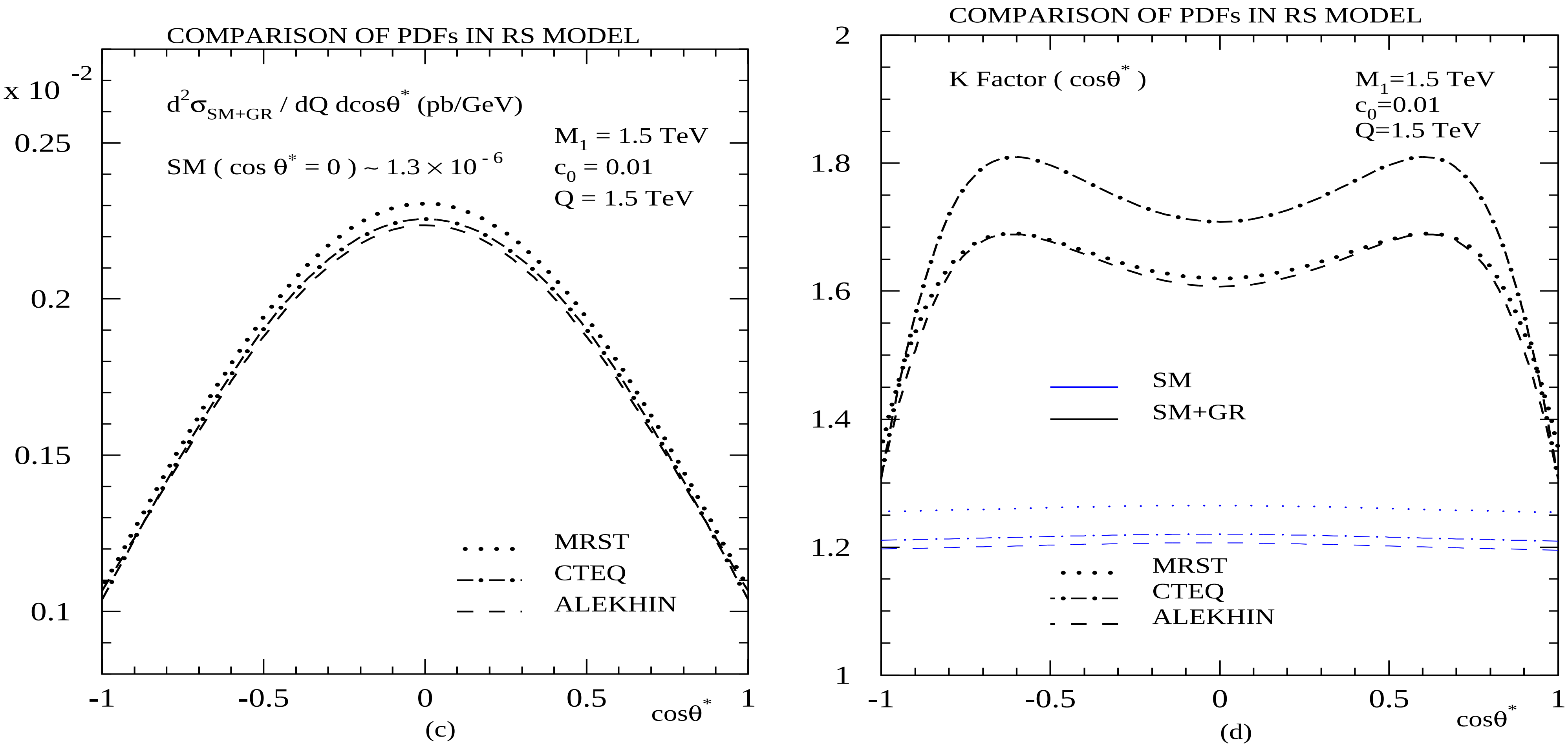




- **Away from the resonance region gravity contribution is negligible**
- **K-Factor behavior can be understood from the $K^{(0)}$ behavior for the RS model**

Angular distribution:

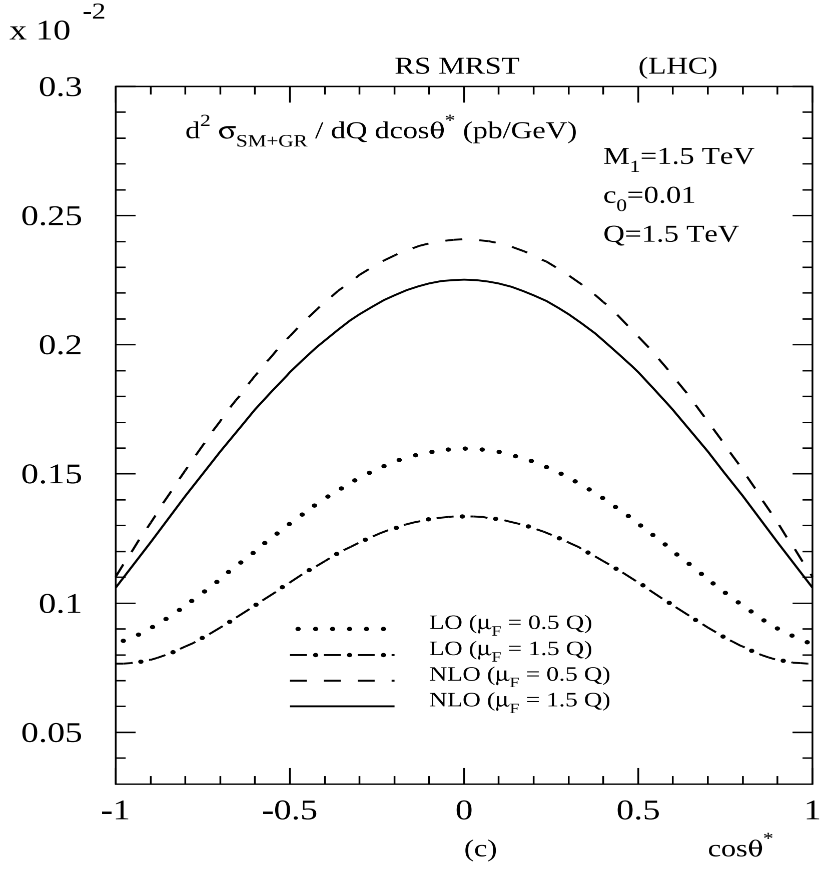
$$\left.\frac{d\sigma(Q,\cos\theta)}{dQ\,d\cos\theta}\right|_{Q_0}$$

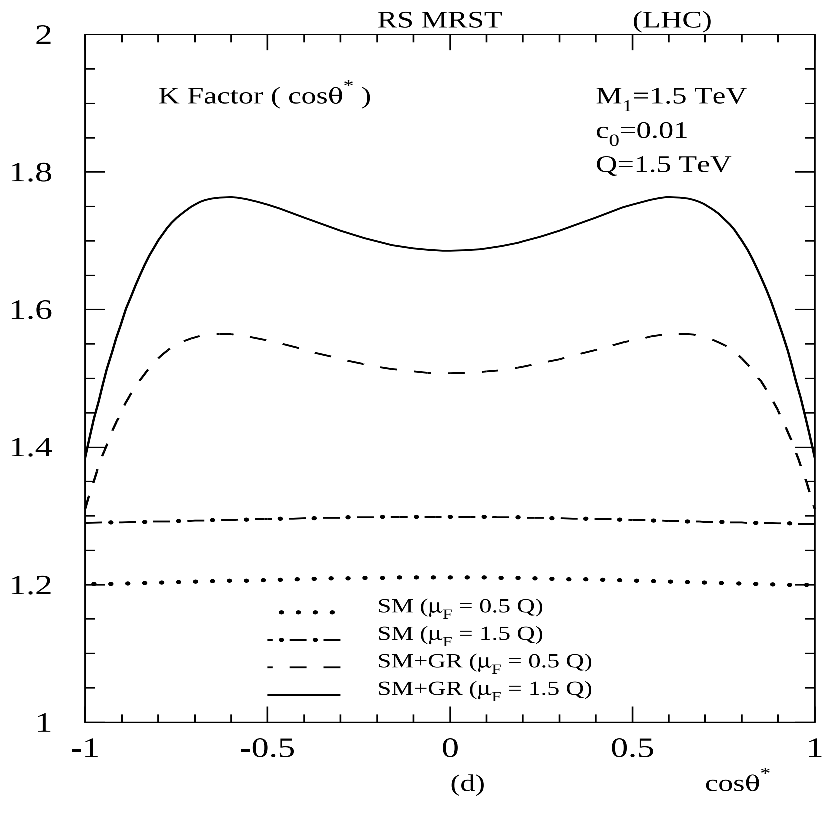


Factorisation scale dependence of angular distribution:

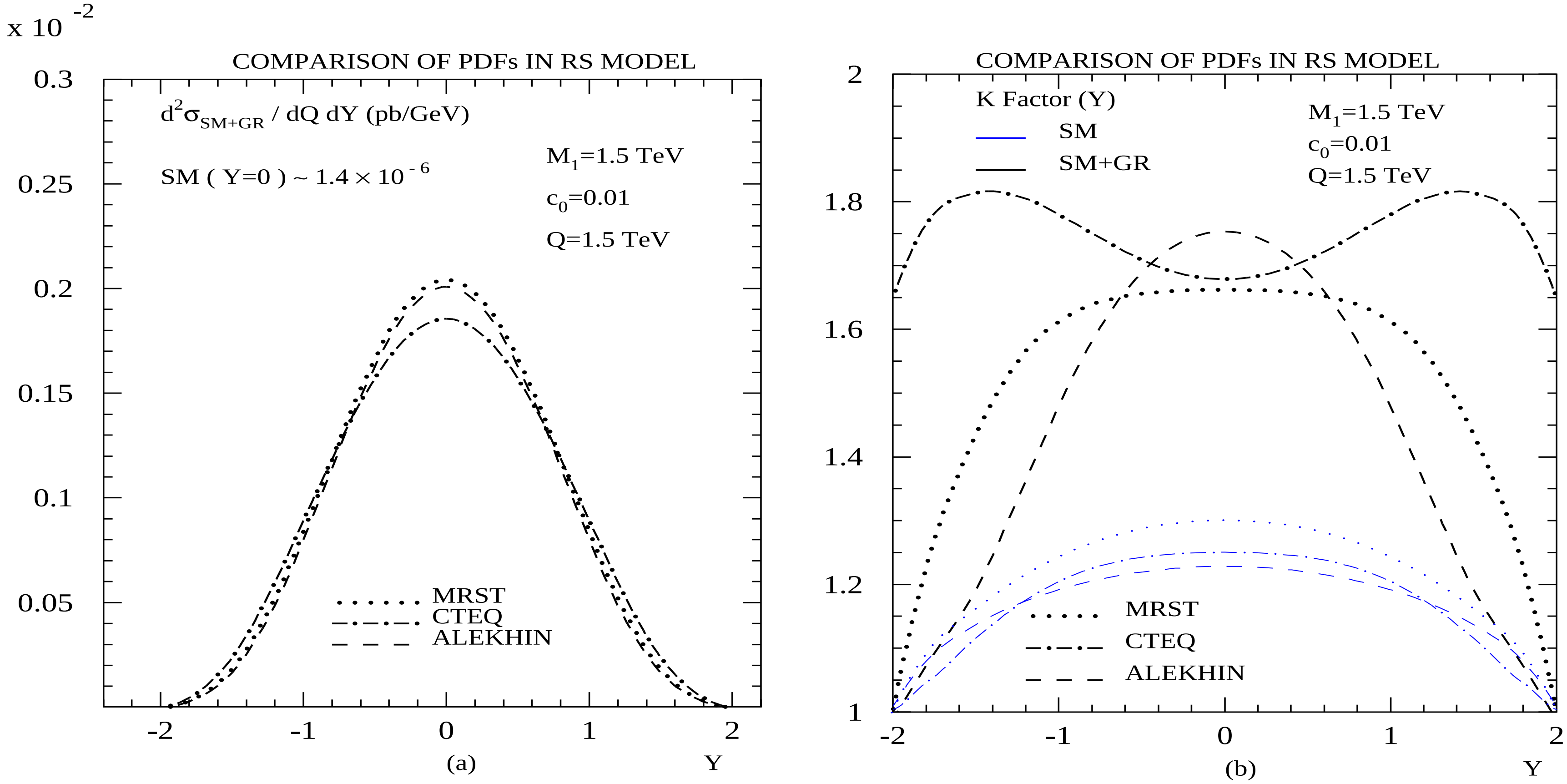
$$\frac{d\sigma(Q,\cos\theta)}{dQ\,d\cos\theta}\Big|_{Q_0}$$

Distributions	Tevatron		LHC	
	LO	NLO	LO	NLO
$d^2\sigma/dQdY$	23.2	7.7	18.7	6.9
$d^2\sigma/dQdcos\theta$	24.2	8.0	18.4	6.8





RS Rapidity



x 10 -2
COMPARISON OF PDFs IN RS MODEL
$d^2\sigma_{SM+GR}$ / dQ dY (pb/GeV)
SM (Y=0) ~ 1.4×10^{-6}
M_1=1.5 TeV
c_0=0.01
Q=1.5 TeV
MRST
CTEQ
ALEKHIN
0.3
0.25
0.2
0.15
0.1
0.05
-2
-1
0
1
2
(a)
Y
COMPARISON OF PDFs IN RS MODEL
K Factor (Y)
SM
SM+GR
M_1=1.5 TeV
c_0=0.01
Q=1.5 TeV
MRST
CTEQ
ALEKHIN
2
1.8
1.6
1.4
1.2
1
-2
-1
0
1
2
(b)
Y

Summary

- Next to Leading Order coefficient functions for DY process in models of TeV-scale gravity are available now.
- Various distributions *viz.* $\boldsymbol{Q}$, $\boldsymbol{x_F}$, $\boldsymbol{Y}$ distributions and $\boldsymbol{A_{FB}}$ asymmetry at NLO are studied for ADD & RS models.
- Theoretical uncertainties get significantly reduced at NLO level
- Quantitative impact of the QCD corrections for searches of extra dimension at hadron colliders investigated