LOCALIZATION OF FIELDS ON BRANE

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OUTLINE :

- Why study localization?
- The mechanism
- Localization of fermions in 5D models
- Gravity localization in 5D models
- Localization in 6D brane models
- Multiple warping : fermion localization
- Summary and Open Issues

WHY STUDY LOCALIZATION

Extra dimensions are hidden from us

$\Rightarrow$ Kaluza-Klein compactification

Assumption in warped braneworld models:

- Gravity can access the whole bulk spacetime
- But SM fields are localized on the brane

The idea is borrowed from string theory (D-branes)

Localization of fields $\Rightarrow$ alternative to compactification

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Localization of fields $\Rightarrow$ alternative to compactification

Are the fields really localized in different braneworld models ?

5D spacetime: $ds^2 = d\sigma^2 + e^{-2f(\sigma)}\eta_{\mu\nu}dx^\mu dx^\nu$

- Consider that all fields can access the whole bulk

  Consider fermion : $\Psi(x^\mu, \sigma) = \psi(x^\mu)\xi(\sigma)$

- Express the action of the field in canonical form

$$\int \sqrt{-g}\mathcal{L}_{Dirac}d^5x = \int \sqrt{-g}\,(i\bar{\Psi}\Gamma^a\mathcal{D}_a\Psi)d^5x$$

- Extract out the standard four dimensional part

$$i\gamma^\mu\partial_\mu\psi(x^\mu) = m\psi(x^\mu)$$

- Rest of the integral gives the localization condition

$$\int e^{-3f(\sigma)}\xi_m\xi_n d\sigma = \delta_{mn}$$

- Find the functional dependence on higher dimension $\rightarrow \xi(\sigma)$

THE MECHANISM : SM FIELDS

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- Find the functional dependence on higher dimension $\rightarrow \xi(\sigma)$

Whether this function, $\xi(\sigma)$, satisfies the localization condition ?

THE MECHANISM : GRAVITY

- metric for the 5D bulk spacetime

$$ds^2 = d\sigma^2 + e^{-2f(\sigma)}\eta_{\mu\nu}dx^\mu dx^\nu$$

- consider small fluctuation on the 4D part of the metric

$$ds^2 = d\sigma^2 + e^{-2f(\sigma)}(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu$$

$\longrightarrow$ variation of Einstein tensor $\longrightarrow$ equation of gravitons

$\longrightarrow$ consider $h_{\mu\nu}(x^\mu, \sigma) \sim \psi(\sigma)\hat{h}_{\mu\nu}(x^\mu)$, with $\Box_x \hat{h}_{\mu\nu} = m^2 \hat{h}_{\mu\nu}$

$\longrightarrow$ the equation correspoding to each graviton of mass m :

$$\frac{1}{\sqrt{-g}}\partial_\sigma \left(\sqrt{-g}\partial^\sigma \psi(\sigma)\right) = m^2 e^{2f(\sigma)}\psi(\sigma)$$

- localization condition:

$$\int \sqrt{-g}g^{00}(\sigma)\psi_m \psi_{m'} d\sigma = \delta_{mm'}$$

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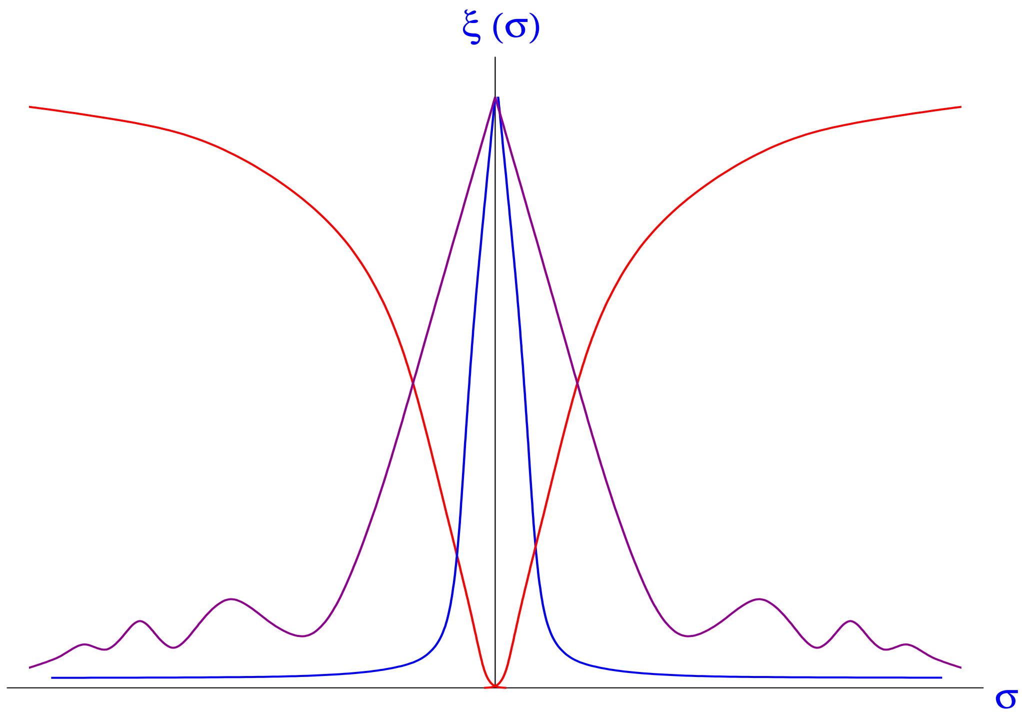
Finding $\psi(\sigma)$ check the localization condition

Functional dependence & localization scenario

- Function sharply peaked at the brane $\Rightarrow$ Localized
- Function not sharply peaked at the brane $\Rightarrow$ Quasi localized
- Function grows along the extra dimension $\Rightarrow$ Not localized



LOCALIZATION OF FERMIONS : SINGLE BRANE MODEL

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- Minimally coupled scalar with sine-Gordon potential :
  $e^{-2f(\sigma)} = \cosh^{2\nu}(b\sigma) \Rightarrow$ thick brane, decaying warpfactor
- Phantom scalar with sine-Gordon potential :
  $e^{-2f(\sigma)} = \cosh^{-2\nu}(b\sigma) \Rightarrow$ thick brane, growing warp factor
- Non standard (tachyon like) scalar:
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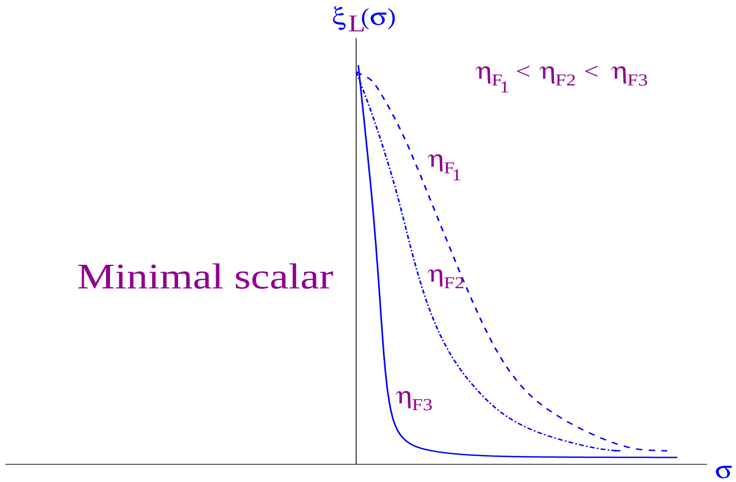
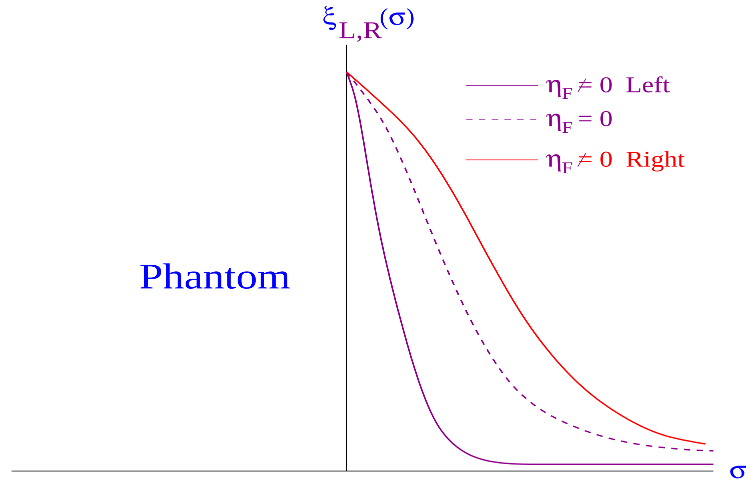
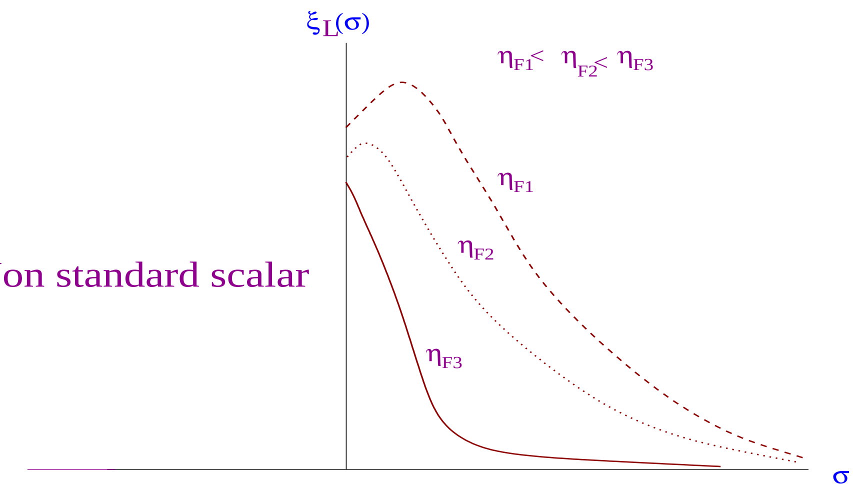
Switch on coupling between the bulk scalar and fermion

$$\sqrt{-g}\mathcal{L}_{Dirac} = \sqrt{-g}\left(i\bar{\Psi}\Gamma^a\mathcal{D}_a\Psi - \eta_F\bar{\Psi}\mathsf{F}(\Phi)\Psi\right)$$

Fermion localization scenario: left chiral zero modes







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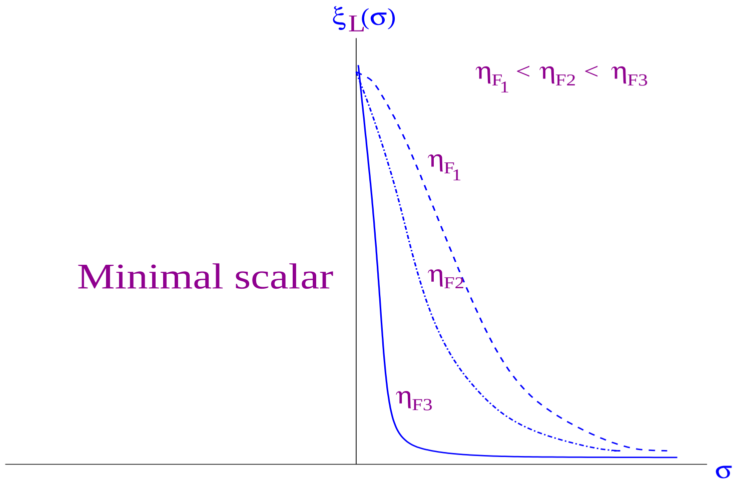

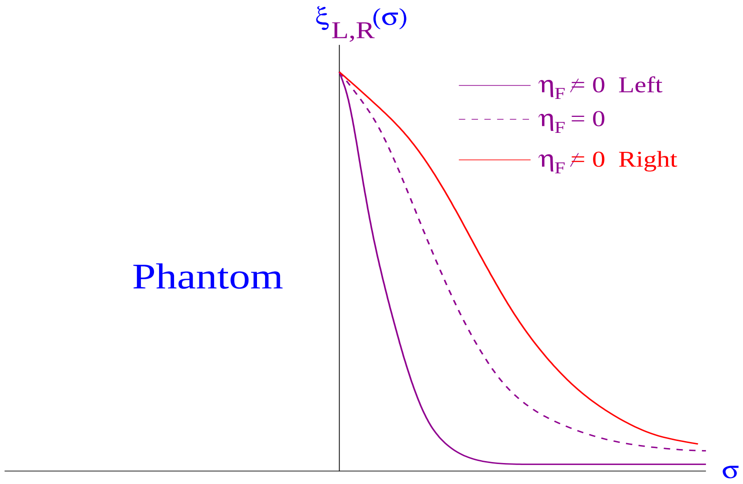

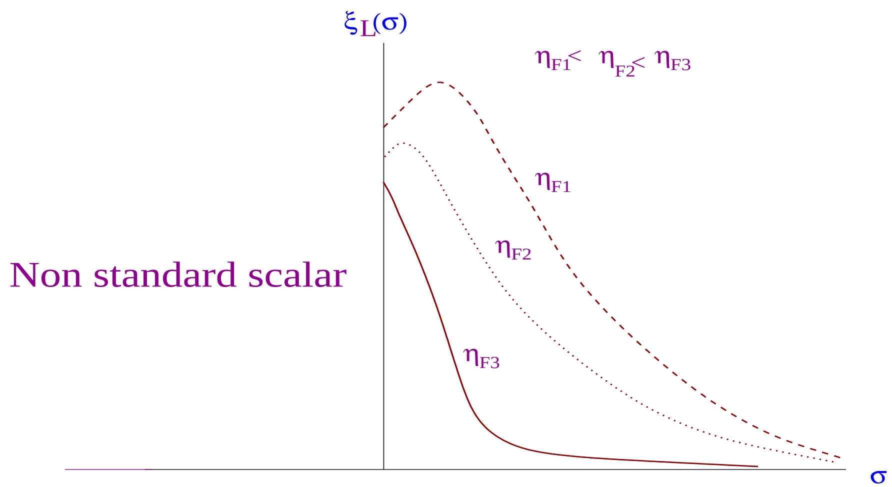

Stronger coupling enhances localization

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$\Rightarrow$ Zero modes are localized on the brane

$\Rightarrow$ Gravitational potential between two test particles m_1 and m_2

$$V(r) = G_N \frac{m_1 m_2}{r} \left(1 + \frac{1}{r^2 k^2}\right)$$

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Localization of gravity zero modes in models with bulk scalars :

Minimal scalar : localized

Tachyon like scalar : localized

Phantom scalar : not localized

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The models with decaying warp factor localize gravity

The models with growing warp factor do not localize gravity

LOCALIZATION : IN MODELS WITH TWO EXTRA DIMENSIONS

6D spacetime: $ds^2 = e^{2f(r)}\eta_{\mu\nu}dx^\mu dx^\nu + dr^2 + e^{2g(r)}L^2 d\theta^2$

Bulk phantom scalar $\Rightarrow ds^2 = e^{\frac{k}{2r}}\eta_{\mu\nu}dx^\mu dx^\nu + dr^2 + e^{-2kr}L^2 d\theta^2$

Localization conditions for zero modes :

- Gravity & Scalar : $\int_0^\infty e^{2f(r)+g(r)}\psi_m\psi_{m'}dr = \delta_{mm'}$
- Fermion : $\int_0^\infty e^{-f(r)}\xi_m\xi_n dr = \delta_{mn}$
- Gauge field : $\int_0^\infty e^{g(r)}\phi_m\phi_n dr = \delta_{mn}$

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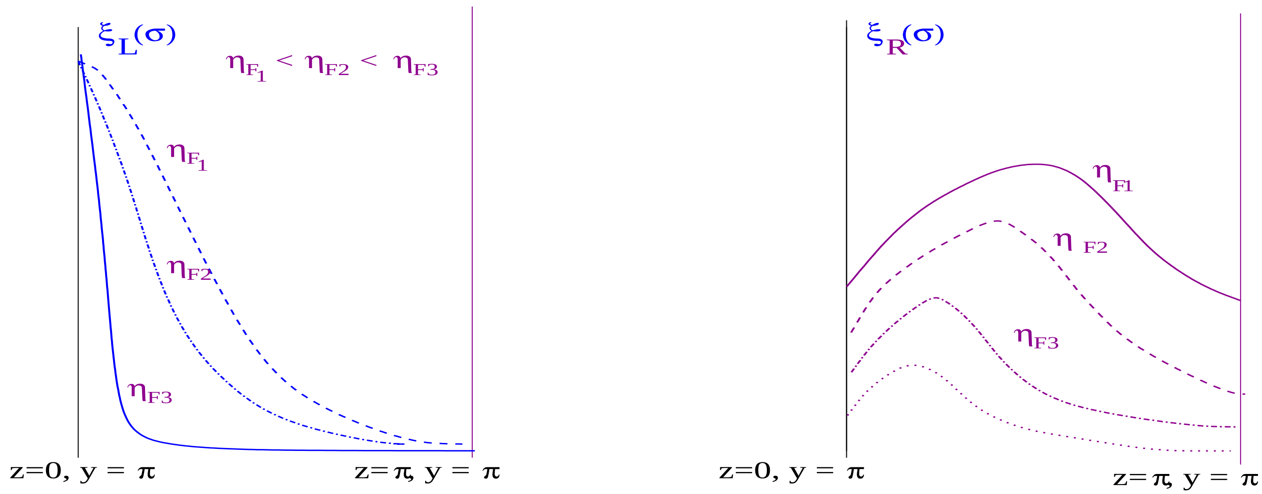
All zero modes may be localized on a single brane

$$ds_6^2 = \frac{\cosh^2(kz)}{\cosh^2(k\pi)}\left[\exp(-2c|y|)\eta_{\mu\nu}dx^\mu dx^\nu + R_y^2 dy^2\right] + r_z^2 dz^2$$

4-branes at $y = 0, \pi$ have coordinate dependent brane tension

$$V_{y=0} = -V_{y=\pi} = Asech\ (kz)$$

Chiral zero modes on 3-branes for coupling with brane tension $\Rightarrow$



Multiple warping : fermion localization

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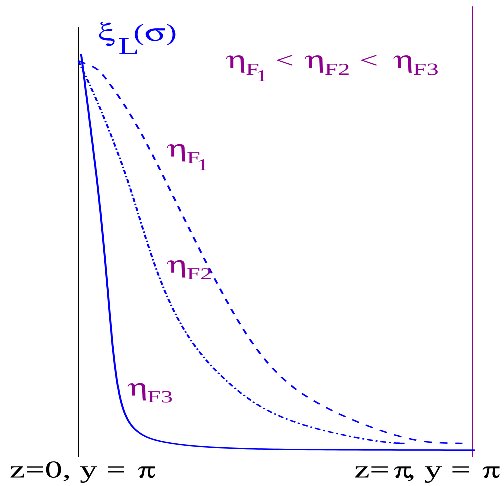

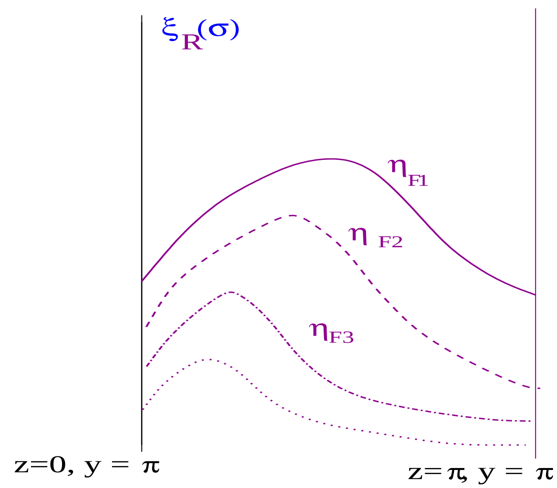

No need to introduce bulk scalars by hand
Coordinate dependent brane tension naturally gives localization

SUMMARY AND OPEN ISSUES

- Coupling with bulk scalars enhances fermion localization
- Multiple warping of spacetime gives natural localization
- All fields may be localized at a single brane in 6D
- A model which localizes all fields plus solves hierarchy problem
  : still not found