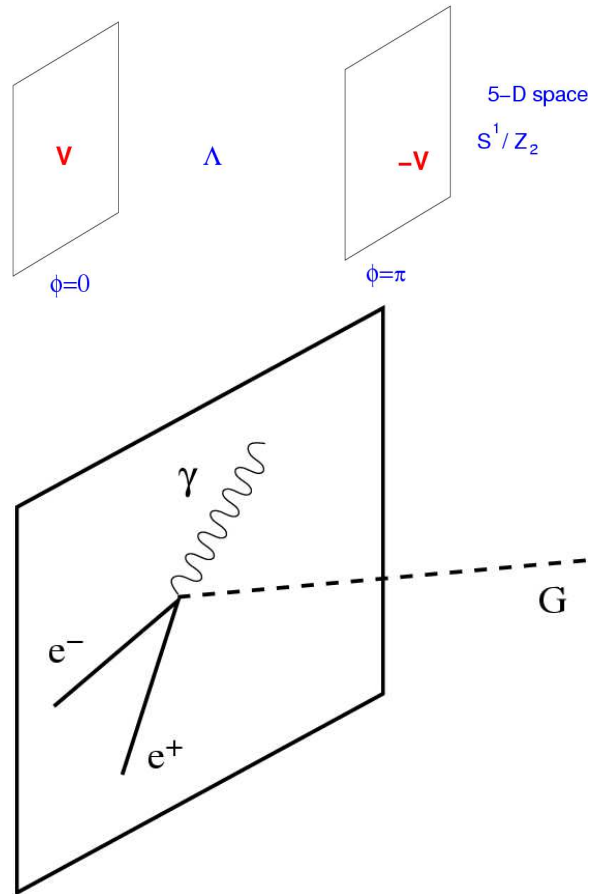


WORKSHOP ON THE PHYSICS OF WARPED EXTRA DIMENSIONS

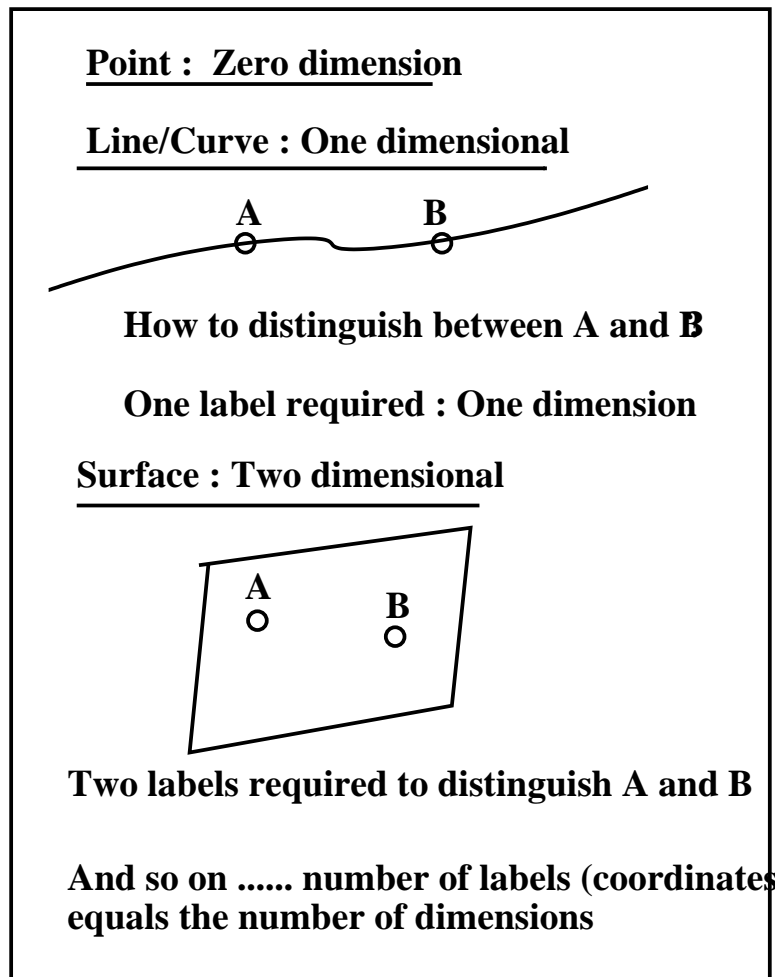
February 21-23, 2008, IIT Kharagpur



MEANING, MOTIVATION AND
OVERALL THEME

THE USUAL DIMENSIONS

What is a 'dimension' ?



- Normal human perception is of one, two and three dimensions
- Special Relativity brings in a fourth, time

THE EXTRA DIMENSIONS

- Dimensions beyond the usual four (x,y,z,t) are **EXTRA**
- Infinitesimal distance in Special Relativity

$$(\Delta s)_1^2 = -(c\Delta t)^2 + (\Delta \vec{x})^2$$

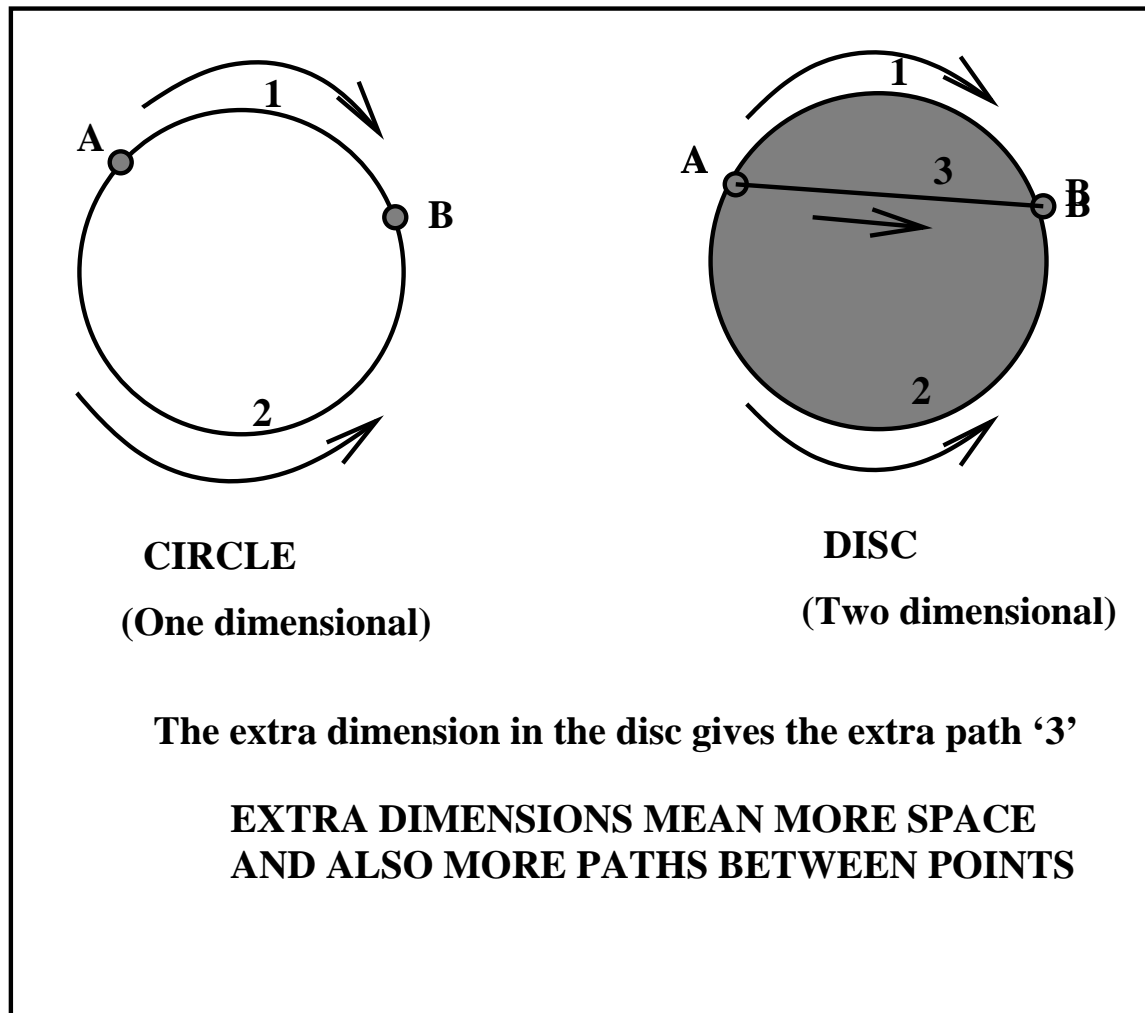
- Note the crucial **MINUS** sign
- Introduces the notion of timelike, spacelike and null intervals

- What sign should we put in the distance if we have an extra dimension (σ)?

$$(\Delta s)^2 = (\Delta s)_1^2 \pm (\Delta \sigma)^2$$

- A minus sign would signal two times → unrealistic (Kleinian signature)
- So, we choose a positive sign

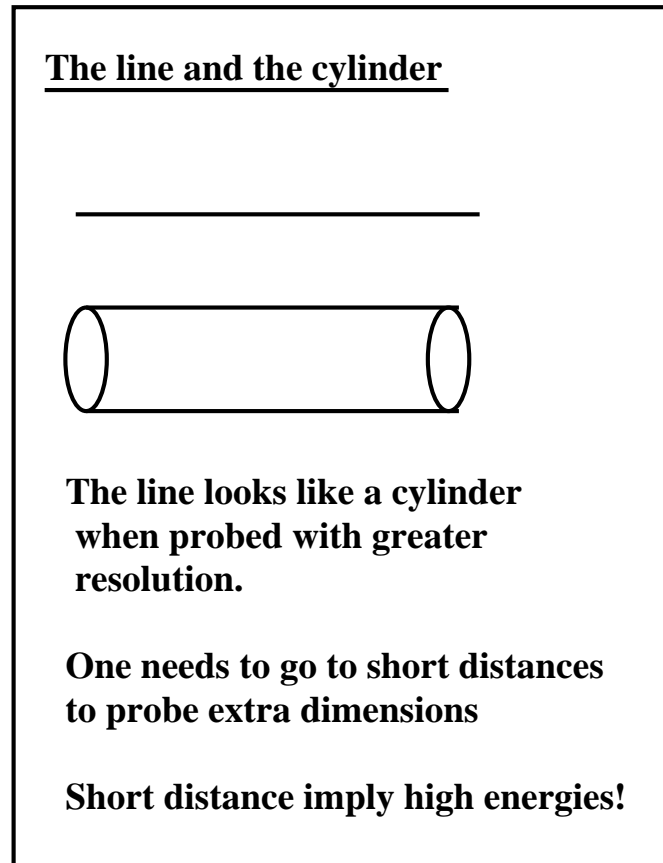
NOVELTIES WITH EXTRA DIMENSIONS



- Besides '3' there are actually infinitely many more in the disc but there are only '1' and '2' for the circle!
- Similar extensions for 4D spacetime with one or more extra dimensions

THE PHYSICISTS MOTIVATION

- If there are extra dimensions are they large/small? Original ideas speak of small extra dimensions, not visible at large scales

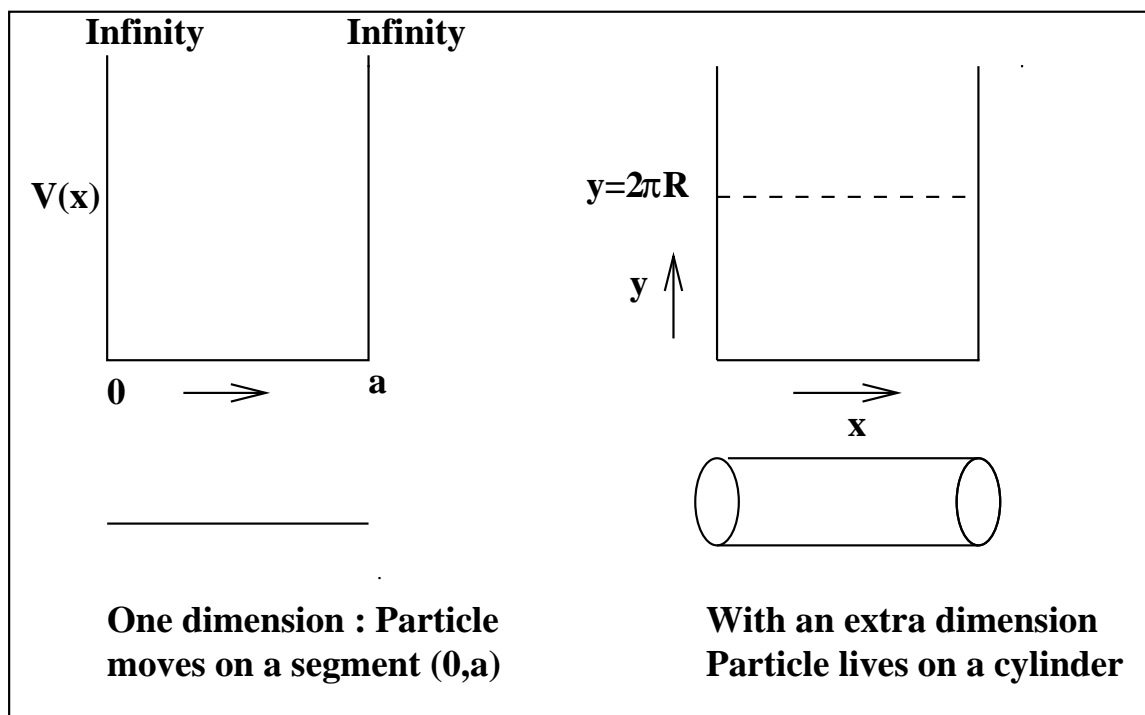


- So, extra dimensions are important at high energies ,BUT they may leave some SIGNATURES in today's world

A QUANTUM PERSPECTIVE

- Since extra dimensions are important at small length scales it is obvious that we need quantum mechanics to study them!

Particle in a box with an extra dimension!



→ **Note** $y \sim y + 2\pi R$

- What are the energies in the presence of the extra dimension?

Wavefunction: $\Psi_{k,l} = \psi_k(x)\phi_l(y)$

Solutions :

$$\begin{aligned}\psi_k(x) &= c_k \sin \frac{k\pi x}{a} \\ \phi_l(y) &= a_l \sin \frac{ly}{R} + b_l \cos \frac{ly}{R}\end{aligned}$$

- **Physics along x direction unchanged**
- **Along y** $\phi_l(y) = \phi_l(y + 2\pi R)$
- **Nonvanishing constant solution for $l = 0$**
($\phi_0 = b_0$)

Energy levels :

$$E_{k,l} = \frac{\hbar^2}{2m} \left[\left(\frac{k\pi}{a} \right)^2 + \left(\frac{l}{R} \right)^2 \right]$$

→ **For $l = 0$ we have the one dimensional spectrum $E_{k,0}$**

→ **Doubly degenerate states for $l \neq 0$**

- What is the lowest new level? $\rightarrow E_{1,1}$

$$E_{1,1} = \frac{\hbar^2}{2m} \left[\left(\frac{\pi}{a} \right)^2 + \left(\frac{1}{R} \right)^2 \right]$$

\rightarrow For $R \gg a$, second term is small and first term dominates

\rightarrow For $R \ll a$, the energy of this new level is comparable to the energy of the k th eigenstate in the old problem where $k \sim \frac{1}{\pi} \frac{a}{R}$. Note k is very very large.

- Hence to see these new states we need very high energies

- These states are the Kaluza–Klein infinite tower of states, characteristic of extra dimensions

- Note that if we consider only $l = 0$ the 2D system would look effectively one dimensional!!

KALUZA–KLEIN MODELS

- Idea of extra dimension : Nordstrom (1914)

Kaluza (1921) : Gravity in five dimensions

$$\text{5D Gravity} \equiv \text{4D Gravity} + \text{Electromagnetism} + \text{a scalar field}$$

- Scalar field chosen to have fixed value
- The first example of unification of forces

Klein (1926) : Curled up extra dimension

- Klein gave the idea of a small, circular extra dimension and showed how extra dimensions can still be around even though we cannot see them. We need very high energies to observe them

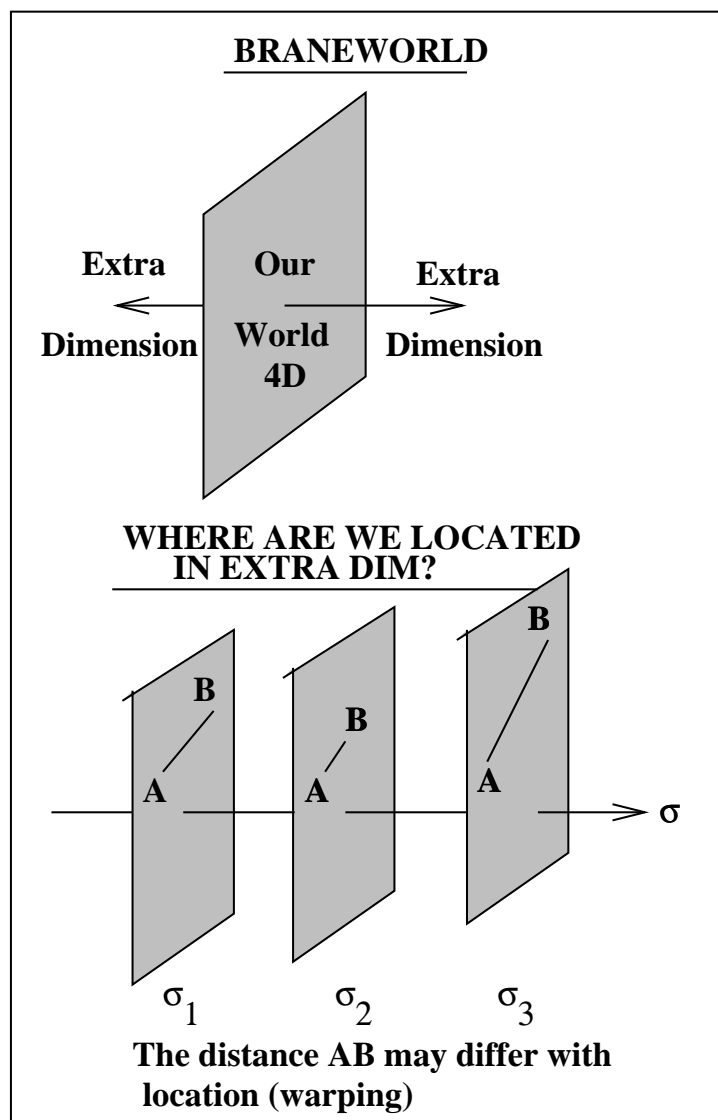
- Einstein was instrumental in publishing Kaluza's work and spend his later life in the quest of unified field theories
- Major problem with KK theories : how to incorporate chiral fermions ?

STRINGS AND EXTRA DIMENSIONS

- String theory starts out with the assumption that the basic entities are one dimensional objects : closed loop or open string
- Length scales where point particles are seen as resonant modes of the vibrating string $\rightarrow 10^{-33}m$, Energies $\rightarrow 10^{19}GeV$.
- String theories predict their own space-time dimension through consistency conditions : 10 (supersymmetric), 26 (bosonic)
- One must learn how to 'compactify' (Klein) \rightarrow no unique compactification procedure exists

BRANEWORLD MODELS

- ‘Brane’ derived from ‘Mem(brane)’
 - A surface in a background, like a sheet of paper (2D) in 3D
 - Generalise to 4D ‘surface’ in say, 5D



LARGE EXTRA DIMENSIONS (1999)

Antoniadis, Arkani-Hamed, Dimopoulos, Dvali

- Gap between Electroweak (100GeV) and the Planck (10^{19}) GeV scales: Hierarchy problem
- Assume 4D spacetime to have 'n' extra dimensions
- Relation between '4' and 'n' dimensional Planck scales

$$M_{Planck(4D)}^2 \sim r_n^n M_{Planck(4+n)}^{n+2}$$

→ $M_{Planck(4+n)} \sim 1 \text{ TeV}$ and $M_{Planck(4D)} \sim 10^{19} \text{ GeV}$ one gets:

$$r_n \sim 10^{\frac{30}{n}-17}$$

→ $n = 1$ $r_1 \sim 10^{13}$ (ruled out)
→ $n = 2$ $r_2 \sim 1$ mm (possible!)
→ $n > 2$ one can reach $(TeV)^{-1}$ sized extra dimensions!

- Since we don't see them it may be assumed that all Standard Model fields are stuck on a brane, while gravity propagates in the full $4 + n$ dimensions

- At TeV energies one should see the effects of this $4 + n$ dimensional gravity!

- Modifications to Newtonian gravity should also be seen at sub-millimetre distances!!

- Newtonian gravity has not been tested at such short distances (inverse square law may change!)

- However, this brings in a new hierarchy : hierarchy of length scales of the extra dimensions 'r'

THE TEVATRON 'ANT'

(Taken from the Fermilab website)

Question: The TeV ANT moves very slowly across a desk. Suppose it take 30 seconds to travel 5.5 feet (165 cm) and suppose its mass is 0.1 grams. What is its energy in TeV?

The ant's Kinetic energy is $E = \frac{1}{2}mv^2$

If the ant moves across 5.5 feet (165 cm) in 30 seconds its velocity is 5.5 cm/sec.

So $E = \frac{1}{2}(0.1gm)(5.5cm/sec)^2 = 1.51ergs$

convert ergs to eV (divide by 1.602×10^{-12})

$E = 0.944 \times 10^{12}eV$ or $0.944TeV$

Question again: How can you explain that it takes a multi-billion dollar complex chain of accelerators to accelerate one proton to a TeV when that's the same energy as a slow-moving ant?

Remember Avogadro's number from high school chemistry!

$$N_A = 6.022 \times 10^{23} \text{ molecules/mole}$$

Make some simple assumptions: the ant is made of Carbon (its really much more complicated than that), so one mole is 12 grams.

So, the TeV ANT has $(0.1/12) \times 6.022 \times 10^{23} = 5 \times 10^{21}$ molecules of carbon in it. Each carbon nucleus has 6 protons and 6 neutrons so that means the TeV ANT has 6×10^{22} nucleons in it to share the TeV of energy.

A single proton in the Tevatron has one TeV of energy. But is the proton a single, indivisible particle? NO. It is made of even smaller particles called partons: quarks and gluons. An approximate number of partons in the proton is 10. So each parton carries on the average about 1/10 of the TeV of energy.

WARPED EXTRA DIMENSIONS

Rubakov and Shaposhnikov, Visser, Gogberashvili,
Randall and Sundrum

- **Location-specific distances. Quantify.**

$$(\Delta s)^2 = e^{2f(\sigma)} \left[-(c\Delta t)^2 + (\Delta \vec{x})^2 \right] + (\Delta \sigma)^2$$

- **At $\sigma = \sigma_0$ (4D world) the interval is:**

$$(\Delta s)^2 = e^{2f(\sigma_0)} \left[-(c\Delta t)^2 + (\Delta \vec{x})^2 \right]$$

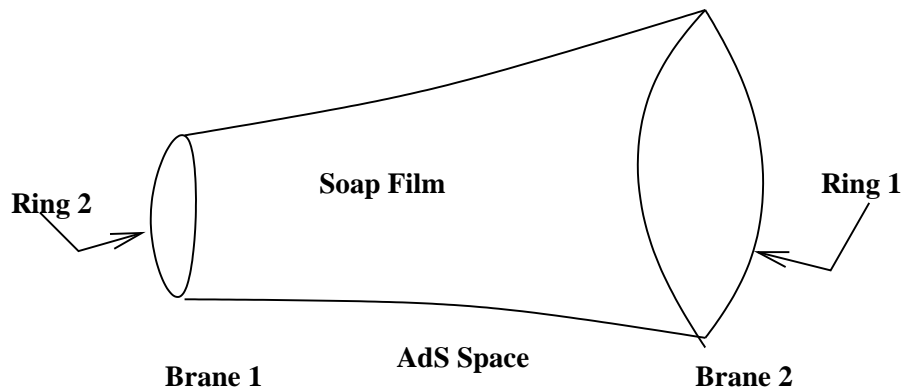
- **Note this is the distance used in Special Relativity BUT with a scale factor $e^{2f(\sigma_0)}$**

• SCALING CHANGES EVERYTHING!! •

AN ANALOGY

2-brane model vs soap films between rings!

ANALOGY



Soap Film : Action= S (2d area) + S (boundary1) + S (boundary 2)

Brane-bulk : Action= S (Bulk) + S (bdy1) + S (bdy(2))

With appropriate tensions and coupling constants!

- The analogy works to a great extent
- One can study fluctuations too and there are similarities!

See, L. Durand, Stability and oscillations of a soap film: an analytic treatment, Am. Jr. Phys, 49, 334 (1981)

ARE THERE EXTRA DIMENSIONS ?

Answer

- NOT YET SEEN •

Latest experiments :

- (1) Fermilab Tevatron D0 Run II
PRL, 95, 091801 (2005)
PRL, 95, 161602 (2005)
 - (2) Fermilab Tevatron CDF II Detector
PRL, 99, 171802 (2007)
PRL 99, 171801 (2007)
- give lower bounds on quantities
characterising extra dimensions

Hope

- More experiments forthcoming •
(LHC, Stanford, Eot-Wash)

- Concrete models + calculations exist •

Existence and non-existence

- Non-existence will, in a way, prove why the world is four dimensional
- Existence will be a triumph for unification enthusiasts (eg. string theorists)

THE WORKSHOP IS ABOUT SPECULATIONS ON THE POSSIBILITIES THAT EMERGE IF EXTRA DIMENSIONS, OR MORE SPECIFICALLY, WARPED EXTRA DIMENSIONS, ARE THERE