Unravelling Extra Dimensions at Colliders

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Space time is $4 + \delta$ *dimensional...*

- Two main classes of *Extra Dimensional Models*

- SM Fields are confined on $1 + 3$ dimensional subspace
  - Only *Gravity propagates in the bulk*
  - An effective *low energy* theory of gravity
  - Motivation : to solve the *Gauge Hierarchy Problem*

- All or some SM fields can access the full space time
  - e.g. Universal Extra Dimensional Models
  - Motivation : *Dark Matter, Low scale Unification*
  - Bonus : *2UED : Proton Decay, Number of Fermion Generations*

Anirban, Biplob, Kirtiman, Swarup's Talk

Agenda for this talk..

- Be familiar with Gravitons/ Gravisclars..
- Compactification a la AADD
  - Towers of gravitons/graviscalar..
  - Phenomenology with Gravitons
  - Higgs-Graviscalar mixing..
  - Possible impacts on Higgs Physics..
- Compactification a la RS
  - RS Gravitons..
  - Radion and radius stabilisation
  - Minimal model and its phenomenology
  - Radion -Higgs mixing and its phenomenology at the Colliders...
  - Unitarity and low enery effects

Note : Rubakov, Shaposnikov (RS) have considered extradimensional scenarios long back AADD or RS

Gravitons and Graviscalars...

Starting point : Einstein's equation in $D = 4 + \delta$ dimensions

- $M_D^{2+\delta}\ \left(\mathcal{R}_{MN} - \frac{1}{2}\, g_{MN}\mathcal{R}\right) = T_{MN};\ \ M, N = 1, 2.........., D$
- $T_{MN}(x, y) = \eta_{M\mu}\eta_{N\nu}T_{\mu\nu}(x)\delta(y)$
- $g_{MN} = \eta_{MN} + h_{MN}$
- $h_{MN}(x, y) = \sum_{n_1=-\infty}^{\infty} \cdots\cdots \sum_{n_\delta=-\infty}^{\infty} h^{\mathbf{n}}_{MN}(x) f_{\mathbf{n}}(y)$

f : depends on the compactification..

Propagating degrees of freedom after field redefinition :

$Radion:\ \ H^{\mathbf{n}} \sim h^{\mathbf{n}j}_{j},\ \ Scalar:\ \ S^{\mathbf{n}}_{ij} \sim h^{\mathbf{n}}_{ij} + cH^{\mathbf{n}},$

$Vectors:\ \ V^{\mathbf{n}}_{\mu j} \sim h^{\mathbf{n}}_{\mu j},\ \ Gravitons:\ \ G^{\mathbf{n}}_{\mu\nu} \sim h^{\mathbf{n}}_{\mu\nu} + (a\eta_{\mu\nu} + b\partial_\mu\partial_\nu)H^{\mathbf{n}}$

Equations of Motion:

$$(\partial_\mu\partial^\mu + m^2_{\mathbf{n}}) \begin{pmatrix} H^{\mathbf{n}} \\ S^{\mathbf{n}}_{ij} \\ V^{\mathbf{n}}_{\mu j} \\ G^{\mathbf{n}}_{\mu\nu} \end{pmatrix} = \begin{pmatrix} \frac{\kappa}{M_p} T^{\mu}_{\mu} \\ 0 \\ 0 \\ \frac{1}{M_p}(-T_{\mu\nu} + (\eta_{\mu\nu} + \partial_\mu\partial_\nu)T^{\rho}_{\rho}) \end{pmatrix}$$

Tensors and Scalars in AADD

Compactification on circle, $f^{\mathbf{n}}(y) = f^{\mathbf{n}}(y + 2\pi\mathbf{R})$

- $f^{\mathbf{n}}(y) \sim e^{\frac{2\pi i \mathbf{n}.\mathbf{y}}{\mathbf{R}}}$
- $m_{\mathbf{n}}^2 = \frac{|\mathbf{n}|^2}{R^2}$

Single fundamental scale M_D in the full theory

$$S_{\rm bulk} = -\frac{1}{2} M_D^{\delta+2} (2\pi R)^{\delta} \int d^4x \sqrt{-g^{(4)}} R^{(4)}$$

$$M_{Pl}^2 = M_D^{\delta+2} (2\pi R)^{\delta}$$

$$\Delta m \sim \frac{1}{R} \sim \left(\frac{M_D}{1TeV}\right)^{\frac{\delta+2}{2}} \; 10^{\frac{12\delta-31}{\delta}} \text{ eV}$$

$M_D = 1TeV$

δ	R	Δm
1	$10^{13} cm$ (Excluded)	-
2	$10^{-1} mm$ (Allowed)	0.0003 eV
3	$10^{-6} cm$ (Allowed)	0.02 eV
4	$10^{-8} cm$ (Allowed)	18 KeV

Tower is quasi continuous for small δ!!

Density of states :

$$dN = S_{\delta-1} \frac{M_{pl}^2}{M_D^{2+\delta}} m^{\delta-1} dm$$

AADD Phenomenology

$$\mathcal{L}_{int} = \sum \left[-\frac{1}{M_{Pl}} G^{n}_{\mu\nu} T^{\mu\nu} + \frac{\kappa}{M_{Pl}} H^{\mathbf{n}} T^{\mu}_{\mu} \right]$$

Coupling to matter is *model independent and universal*

- Real Graviton Production

$e^+e^-, pp, p\bar{p} \to G^{n}_{\mu\nu} + X$ (X is SM particle (one or more)

$\sigma_{Total} = \sum_n \sigma_n$

Sum can be approximated by an integral !!

Upper limit of the integral : Collider Energy

Signature : Large missing energy and momentum !!

- *Effects of virtual Gravitons on SM processes*

$e^+e^-, pp, p\bar{p} \to X$ *(X is SM particle): Graviton Propagator.*

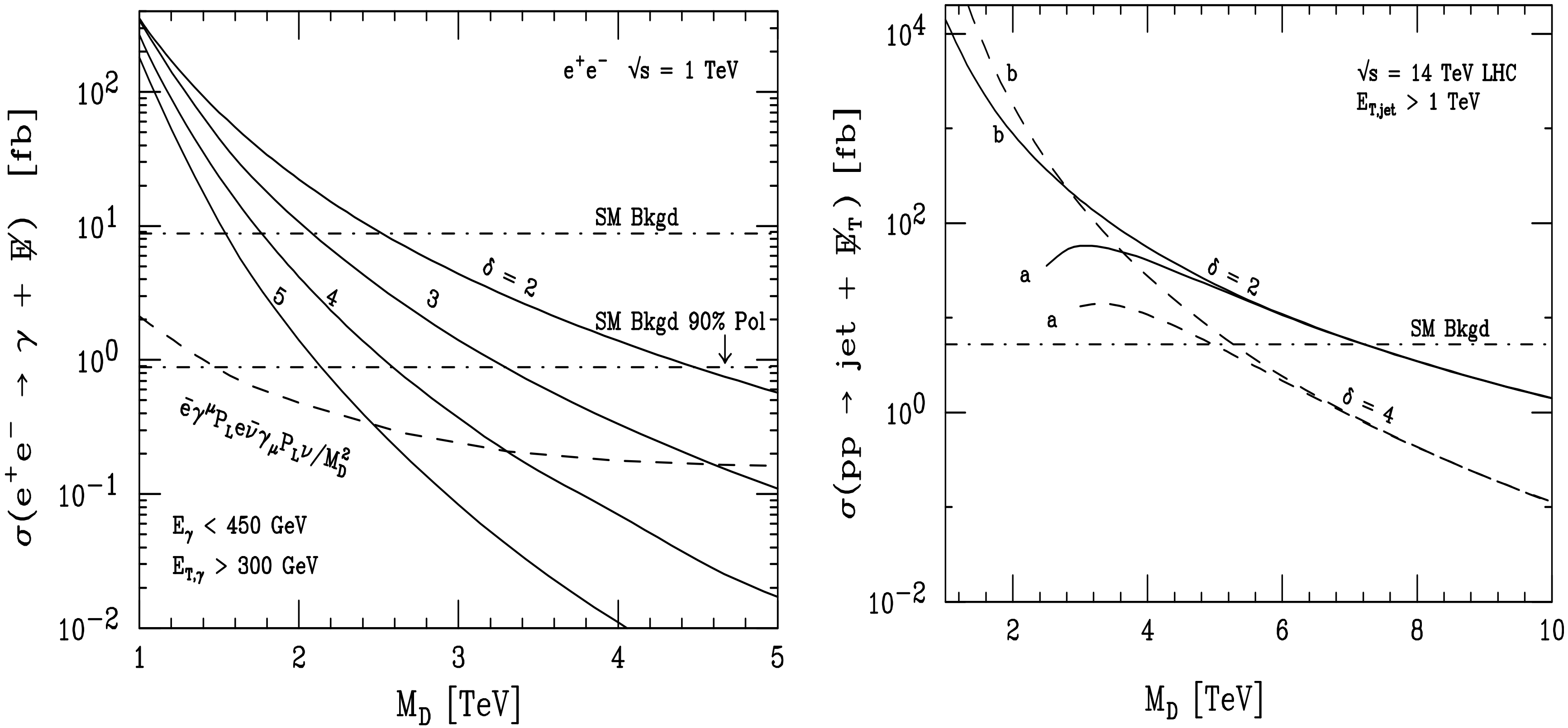
$\sigma_{Total} = \sum_n \sigma_n$

Upper limit of the integral : Unknown; Results are cut-off dependent

Signature : Change in the SM cross-section, Unexpected (from SM) final states..

Real Graviton Production

Examples for Real Graviton Producion



Giudice, Rattazzi, Wells, NPB 544 (1999)

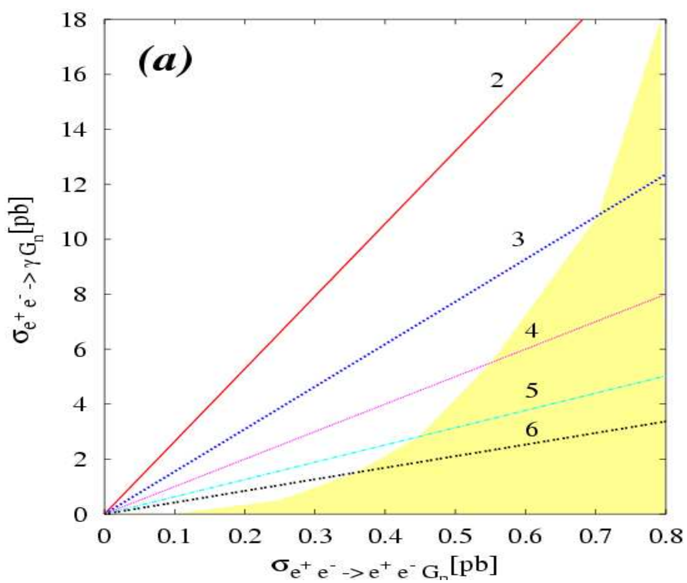
Real Graviton Production

Another interesting example..

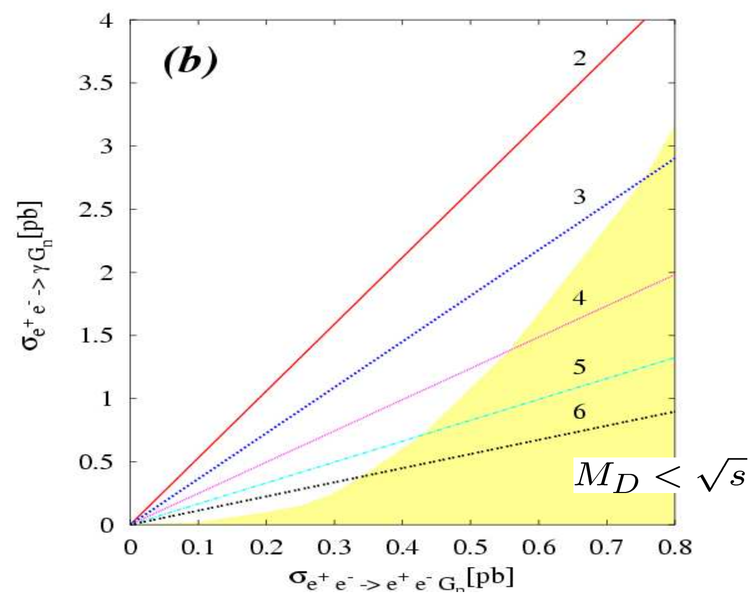
Determination of number of extra-dimensions

Interplay of $\begin{cases} e^+e^- \to \gamma + G_{KK} \\ e^+e^- \to e^+ + e^- + G_{KK} \end{cases}$

Sreerup Raycahudhuri et al.,
PRD 50 6872, 1994



$\sqrt{s} = 500\text{GeV}$



$\sqrt{s} = 1\text{TeV}$

Virtual Graviton effects

Examples..

- Limits om M_D: Di-jet production at Tevatron (Mathews et al.JHEP 0007:008,2000).
- Top pair production at LHC (Lola et al.hep-ph/0010010).
- Di-jet production at $\gamma\gamma$ collider (Ghosh et al. HEP 9911:004,1999).
- And many others....

Virtual effects are not always cut-off dependent..

- Effects of virtual graviton $e^+e^- \to f\bar{f}$ on the Z-pole at LEP.

Datta, Gabrielli, Mele, PLB 552 2003

What about the Graviscalars ?

- Tower of graviscalars couples strongly to massive field
- coupling suppressed by M_p
- signature similar like graviton (practically invisible)

For all practical purposes coupling has no effect on phenomenology but on 4-dim sub-space one can add curvature -Higgs mixing term

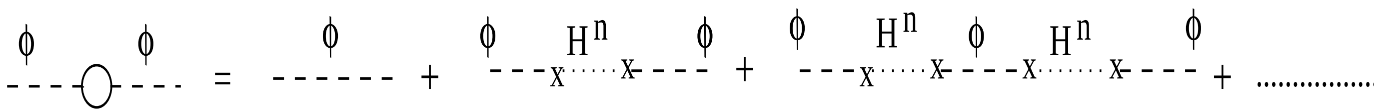
- $\mathcal{S} = -\xi \int d^4x \sqrt{-|g|}\, \mathcal{R}(y=0)\, \Phi^\dagger \Phi$
- $\mathcal{L} = \frac{2\kappa\xi v m_\phi^2}{m_P}\, \phi \sum H^{\mathbf{n}}$
- Effectively bilinear mixing of Higgs and graviscalar

An infinite dimensional scalar mass-matrix !!

Higgs invisible decay

A clever way to investigate Higgs graviscalar mixing

- Effect of this bilinear term on the Higgs propagator

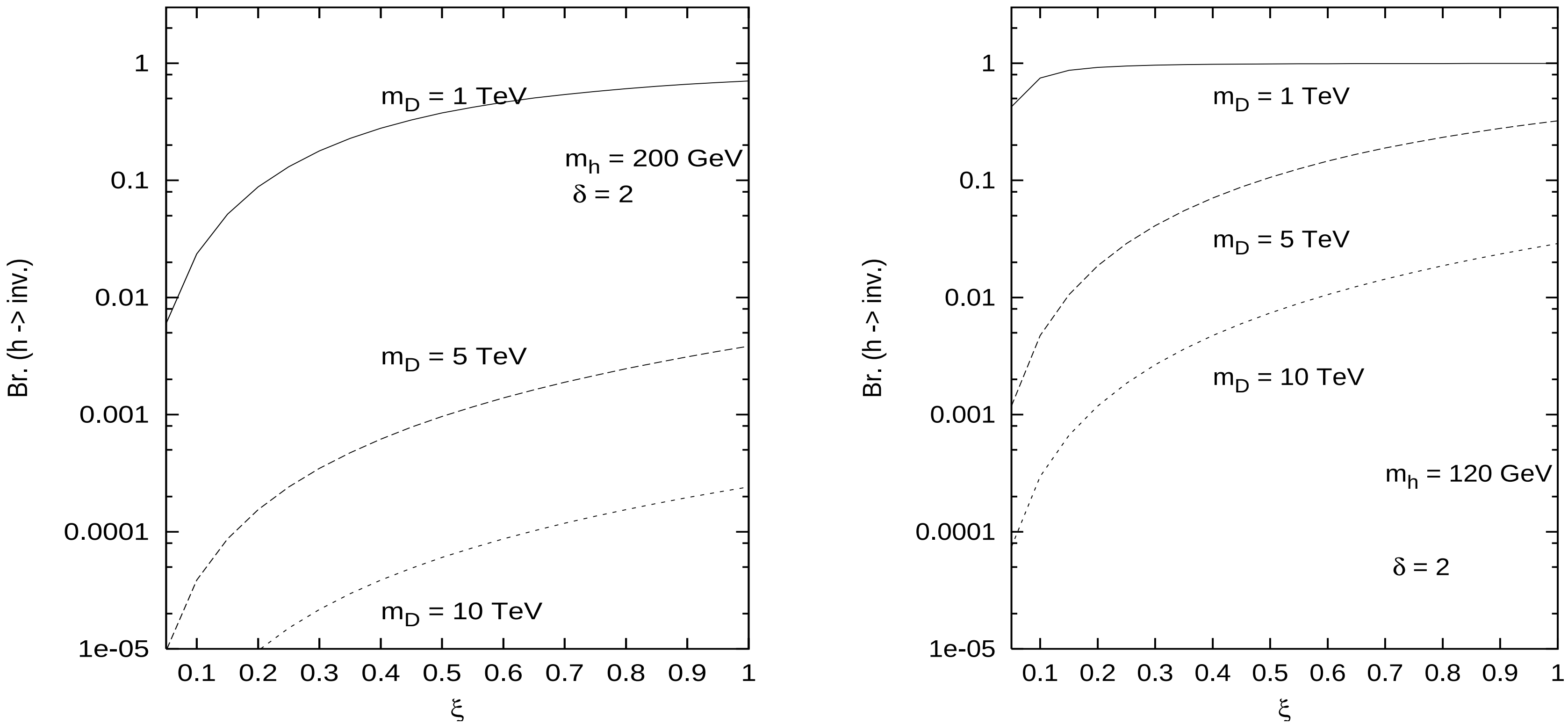


Effectivve Higgs propagator

- $G_h(q^2) = \frac{i}{q^2 - m_h^2 + i\epsilon} \longrightarrow \frac{i}{q^2 - m_h^2 + \Sigma(q^2) + i\epsilon}$
- $\Sigma(q^2) = -(\frac{2\kappa\xi v m_\phi^2}{m_P})^2 \sum \frac{1}{q^2 - |\mathbf{n}/r|^2 + i\epsilon}$
- Quasi-continuos tower of graviscalars: $\sum_{\mathbf{n}} \longrightarrow \int \rho(m) dm$
- integration is not well defined..but imaginary part of $\Sigma(q^2)$ is easily calculable..
- $m_\phi \Gamma = Im.\left[\Sigma(q^2 = m_\phi^2)\right] = \pi\kappa^2\xi^2 v^2 \frac{m_\phi^{1+\delta}}{M_D^{2+\delta}}$
- Important : even for $\delta = 2$ $m_\phi < 2m_W$; $\Gamma \sim m_\phi^3$

Invisible branching ratio

$$Br_{inv.} = \frac{\Gamma_{inv}}{\Gamma_{SM}+\Gamma_{inv}}$$



Invisible branching ratio decreases with number of extra dimensions, δ

Effects of inivisble decay on higgs physics : Datta, Huitu, Mukhopadhyaya, Laamanen, PRD 70 2004.

Some more on mixing?

Whether we can reformulate the Higgs-graviscalar mixing in another way ?

- Diagonalise inf. dim. mass matrix
- Higgs-graviscalar oscillation ?

Can we translate the model independent bounds on inv. Higgs ?

- Another source of invisibility : spin 2 tower !
- common unknown 'input parameters : δ, M_D

Example : $e^+e^- \to Z\phi(\to inv.)$ & $e^+e^- \to ZG^{\mathbf{n}}$

Any signal of invisible higgs in AADD : to be compared with graviton production

Higgs production in VBF channel : translate the model ind. bound

hepph/0402262; without consdering graviton production

Further studies : $pp \to W\phi;\ pp \to t\bar{t}\phi$.....

Which is the best channel for exploring $\xi - M_D$ plane ?

Gravitons/Graviscalars in RS

Compactification on S_1/Z_2 with one extra dimension

- Two 4-dim subspaces (branes) at the orbifold fixed points.
- SM lives in $y = \pi$ brane
- Ansatz for the metric : $ds^2 = e^{\sigma(y)}\ \eta^{\mu\nu} dx_\mu dx_\nu + R_c^2 dy^2$
- metric is a solution of Einstein's eqns. with a fine tuning between bulk and brane cosmological constant..$\sigma(y) = 2kR_c y$
- all dimensionfull parameters are scaled by a factor $e^{-kR_c\pi}$ at $y = \pi$ sub-space)
- $M_{Pl}^2 = \frac{M^3}{k}(1 - e^{-2\pi k R_c})$
- Towers of Graviton : $m_n = x_n k e^{-\pi k R_c}$, x_n : zeros of J_1
- Members of the tower are well seperated..

Interaction with matter : $-\frac{1}{M_{Pl} e^{-\pi k R_c}} G_{\mu\nu}^{(1)} T^{\mu\nu}$

$M_{Pl} e^{-\pi k R_c} \simeq \frac{m^{(1)} M_{Pl}}{4k}$

Free parameters : $m^{(1)}$ and $\frac{M_{Pl}}{k}$

Graviton Production in RS

An Example..

KK graviton resonance production @ LC

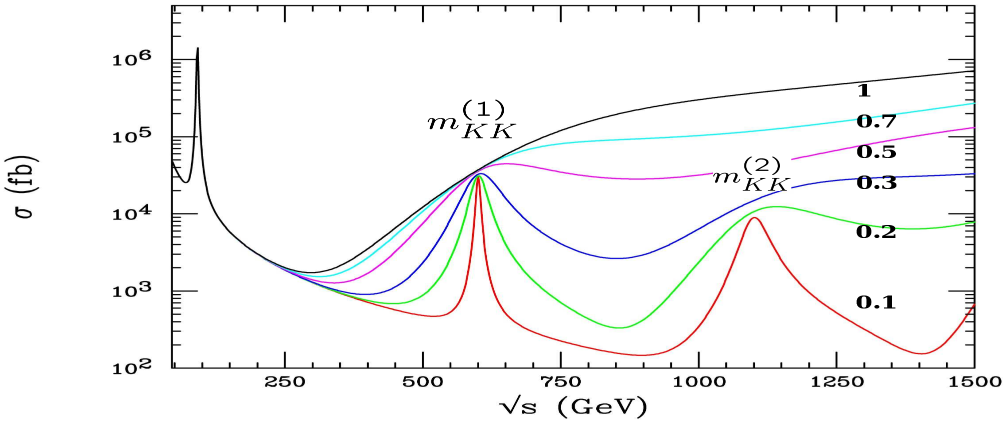
Davoudiasl, Hewett & Rizzo,
PRL 84 (2000) 2080

$$e^+e^- \to g_{KK}^{(1)} \to \mu^+\mu^-$$

$$\Gamma_1 = c \times m_{KK}^{(1)} \frac{k}{\bar{M}_4}$$

c: calculable numerical factor

$$m_{KK}^{(1)} = 600\text{GeV}$$



$\frac{k}{M_4}$

Width becomes large

as k/M4 becomes large

A better option $e^+e^- \to G^{(1)}\gamma$. Rai and Raychaudhuri JHEP 0310 2003

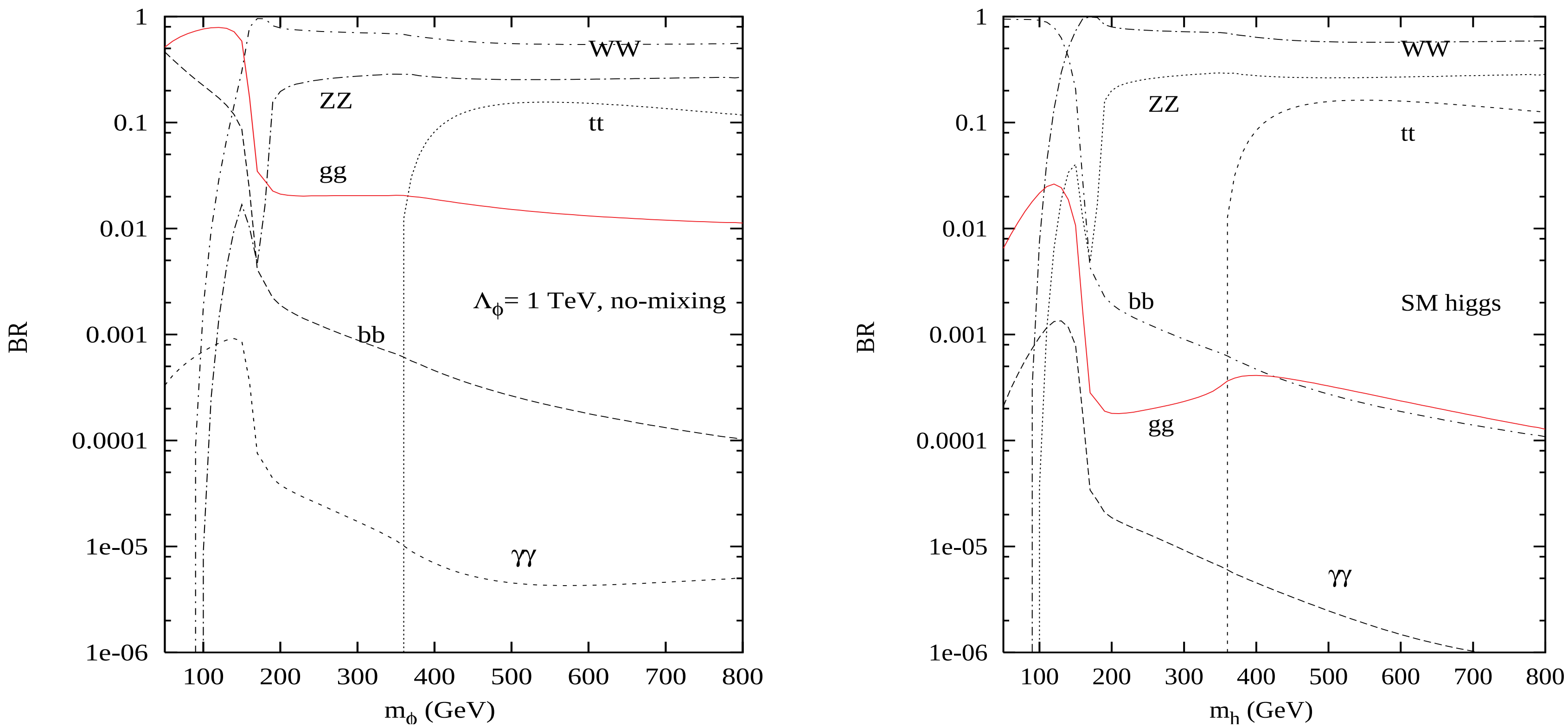
Graviscalar in RS

- The only scalar mode is radion : zero mode of the G_{55} (massless).
- Radion is size of the extra dimension : distance between two branes.
- Need of a mechanism to stabilise this two brane system.
- Same mechanism generates an effective potential and mass for the radion .
- Goldberger and Wise : with a scalar field in the bulk..
  - Does not consider bacrkeaction on the brane.
- Generates an effective potential on sub-spaces at $y = \pi$
- implies a mass and dynamically generated vev (Λ_ϕ) for the $radion$ ...
- Phenomenologicaly interesting: : $\Lambda_\phi \sim 1$ TeV and $m_\phi < 1$ TeV !!
- Radion couples to matter/SM : $\mathcal{L} = \frac{1}{\Lambda_\phi}\, \phi\, T^\mu_\mu$

$$
\begin{aligned}
T^\mu_\mu \quad &= \quad \sum m_f \bar{f} f + m_V^2 V_\mu V^\mu + (2m_h^2 h^2 - \partial_\mu h \partial^\mu h) \\
&\quad + \frac{\beta(g_s)}{2g_s}\, G^{\mu\nu a} G_{\mu\nu a} + \frac{\beta(e)}{2e}\, F^{\mu\nu} F_{\mu\nu}
\end{aligned}
$$

- Couplings are very similar to the SM Higgs

Phenomenology: branching ratio



- gg width/decay branching ratio is drastically enhanced w.r.t the SM Higgs
- At LHC, enhanced coupling to gg was exploited: $pp(gg) \to \phi \to \gamma\gamma, ZZ, WW$

Datta, Mahanta PLB 483 2000; Giudice, Rattazzi, Wells NPB 544 1999

Higgs Radion mixing...

Almost similar to the previous case..

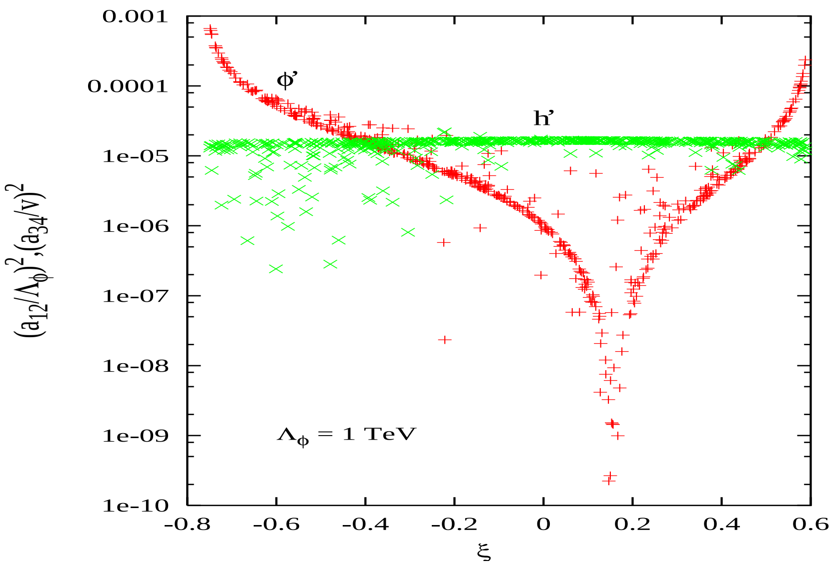
- Remember now the non-trivial metric...

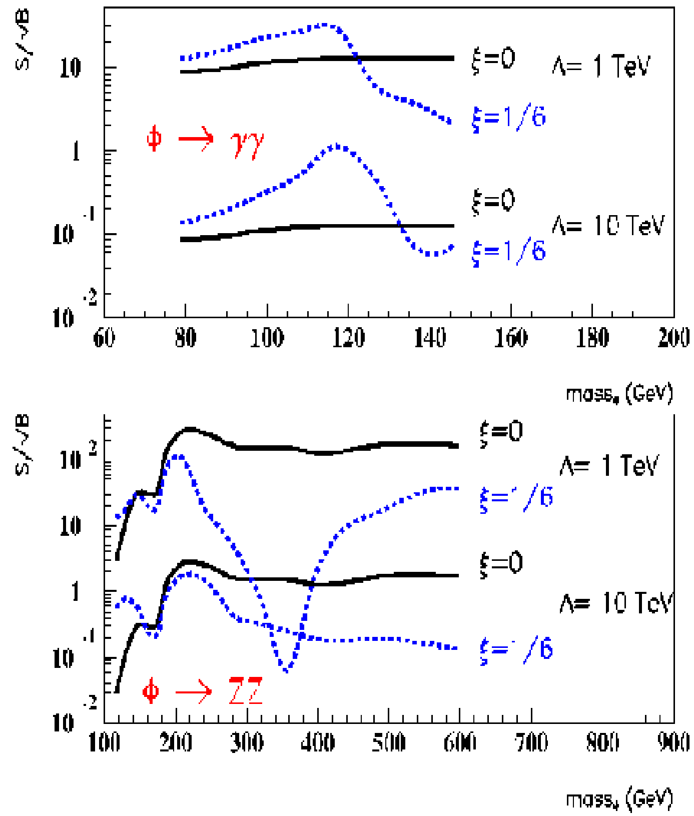
$$\mathcal{L} = -\frac{1}{2}\,\phi\left[(1-6\xi\gamma^2)\partial_\mu\partial^\mu + m_\phi^2\right]\phi - \frac{1}{2}\,h\left[\partial_\mu\partial^\mu + m_h^2\right]h - 6\xi\gamma\;\phi\partial_\mu\partial^\mu h$$

- Life is simpler, 2×2 mixing matrix involving the kinetic terms
- Two step diagonalisation
  - get rid of the off-diagonal $\phi\partial_\mu\partial^\mu h$ term
  - Renormalisation of the rotated fields.
- Mixing matrix is not Unitary !!
- Kinetic terms of the physical fields must have the correct signs
  - $-\left(1+\sqrt{1+4\gamma^2}\right) \le 12\xi \le -\left(1-\sqrt{1+4\gamma^2}\right)$
- 4 input parameters : $\xi,\ \Lambda_\phi, m_h, m_\phi$
- Radion couples more strongly than the AADD graviscalars
- No more invisibility !

Higgs Radion mixing...

ATLAS study : Decay to $\gamma\gamma$ and $ZZ^*(4l)$ channel

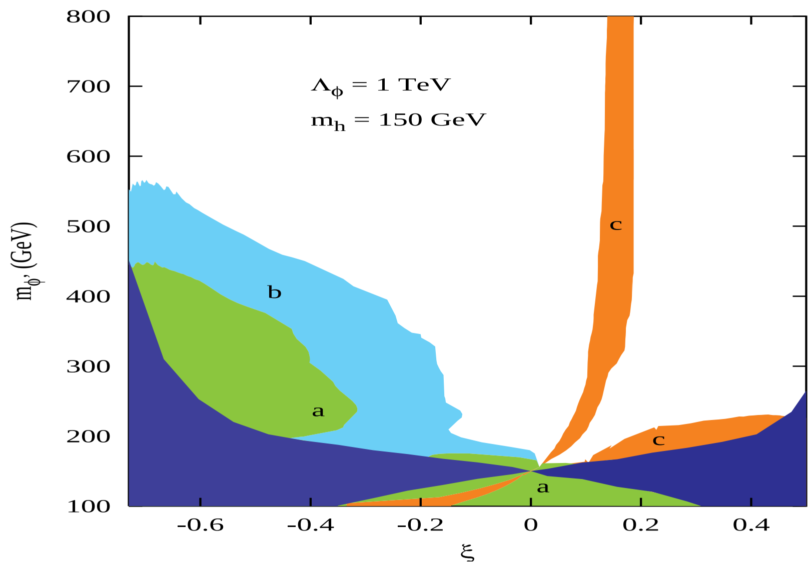




signals so far studied are similar in nature with no-mixing case.., $gg \to \phi, h \to \gamma\gamma, ZZ$

Prospect at e^+e^- ?

from $e^+e^- \to h, \phi\ \nu\bar{\nu} \to gg \quad ; \sqrt{s} = 0.8, 1, 3$ TeV



- a similar exclusion plot at the LHC ?
- Which channel to look for ?

possible ways to discriminate Higgs/radion ?

- gg mode...
- at LHC , production by gg is good, but this decay channel is of no use

possible ways to discriminate mixing/no-mixing case ?

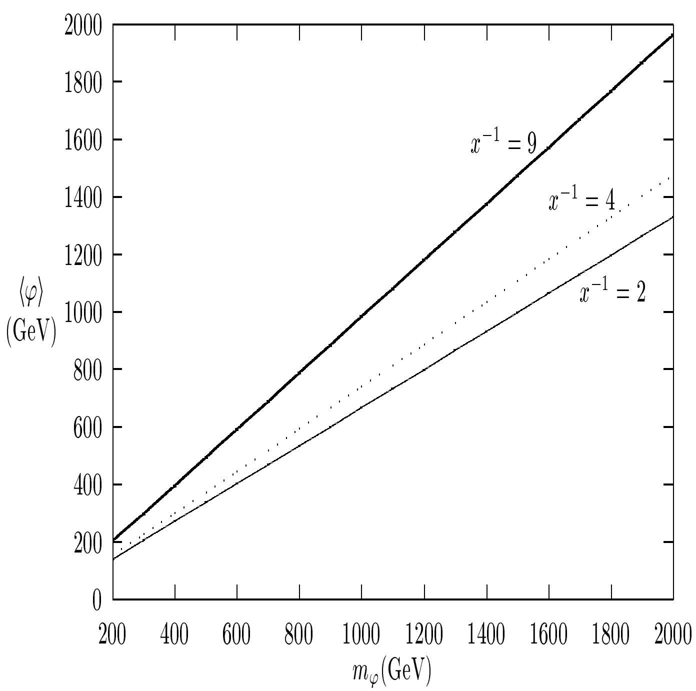
determine the parameters $\Lambda_\phi, \xi, m_h, m_\phi$

Unitarity Consideration

Afterall this is an effective theory...!!

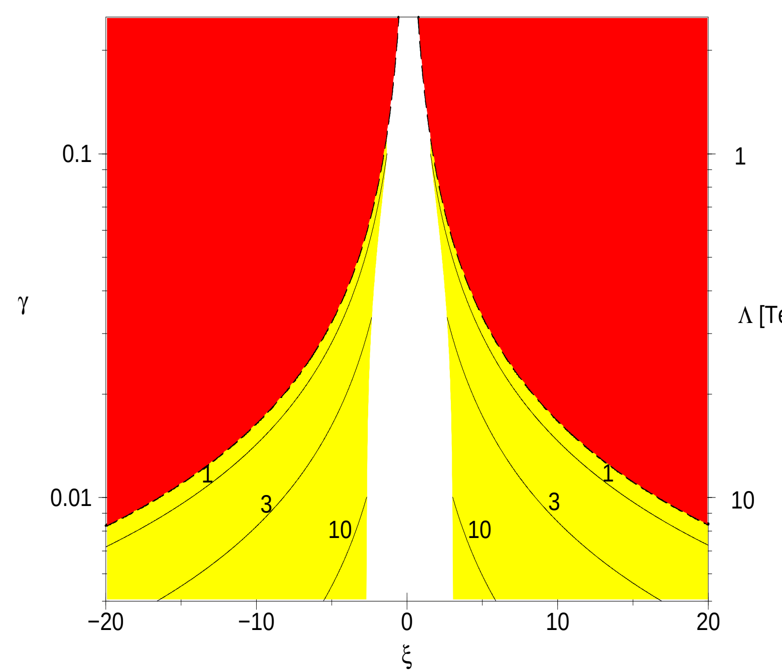
- It has important effects on the scalar sector
- Important to check UNITARITY constraints from gauge bososn scattering..

Choudhury, Mahajan, Raichoudhury



Coupled channel involving W, Z, h ($a_0 < \frac{1}{2}$)

Han, Kribbs, McElerth



Only W_L scattering ($a_0 < \frac{1}{2}$)

To summarise....

- We are at the doorsteps of a new era that will begin with the first run of LHC !!
- A beautiful testing ground to check whether a new dynamics exists at TeV scale or not
- Extra dimensional scenarios offers an alternative to SUSY and are testable at LHC and future leptonic colliders..
- Let us hope for some excitements in the coming years...

Apologies : Many other people have contributed to this very interesting field..all the references are not shown in this talk...

Thanks to my collaborators

Masud Chaichian

Emidio Gabrielli

Katri Huitu

Jari Laamanen

Uma Mahanta

Masud Chaichian

Barbara Mele

Biswarup Mukhopadhyaya

Zeng-H. Yu