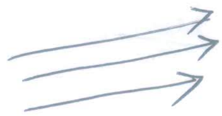
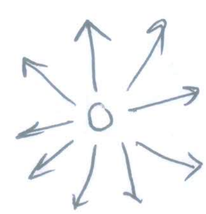
Radiation.

~~If~~ It is ~~exp~~ justified to treat E-M radiation as rays that propagate along straight lines, if the length-scales are much larger than the wavelength.



Flux

A source radiates isotropically if it sends out same amount of radiation in all directions.



e.g. a spherical star.

L - luminosity Energy / sec ~~J/s~~ watts.

The energy, at a distance r, is equally spread out over a sphere $4\pi r^2$.

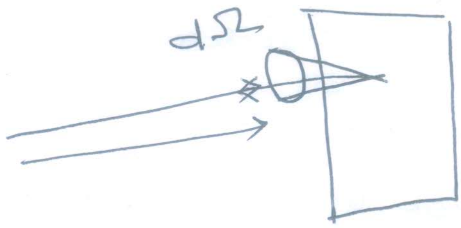
$$\text{Flux} = \frac{\text{Energy}}{\text{Area} \cdot s} = \frac{L}{4\pi r^2} = F.$$

This is the energy carried by all the rays.

Specific Intensity or Brightness

quantifies the energy carried by "individual rays."

Not meaningful to talk of a single direction, rather a small spread in directions.



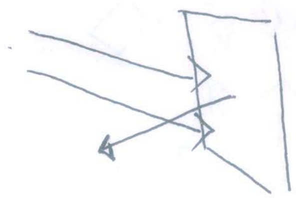
place dA normal to ray

$$dE_\nu = I_\nu \, dA \, dt \, d\nu \, d\Omega$$

I_ν - specific intensity or brightness.

$J \,/\, m^2 \,/\, s \,/\, Hz \,/\,$ steradian.

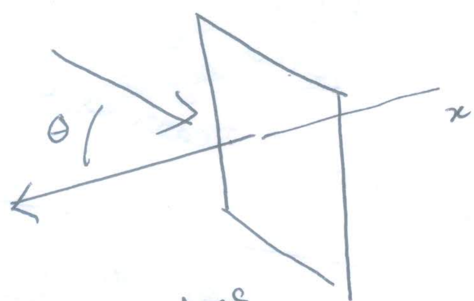
Flux and Momentum.



radiation at an angle will see a projected surface

$$F_\nu = \int I_\nu(\hat{n}) \cos\theta \, d\Omega$$ - Energy Flux

~~Momen~~ **Momentum**



$$p = \frac{E}{c} \quad \text{for photons}$$

$$P_\nu = \int \frac{I_\nu}{c} \cos^2\theta \, d\Omega$$

Angular moments

We can also integrate over frequency to obtain

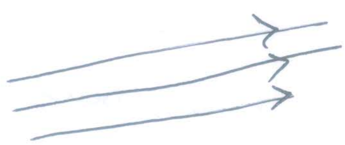
$$F = \int F_\nu \, d\nu \quad \text{etc.}$$

Energy Density

~~Spec~~

u_ν - specific energy density

$$dE_\nu = u_\nu(\cancel{\ \ }) \, dV \, d\nu \, d\Omega$$



Consider a ray.
What is energy density.

$$dE_\nu = I_\nu \, dA \, dt \, d\nu \, d\Omega$$

$$dE_\nu = u_\nu(\Omega) \, dA \, c \, dt \, d\nu \, d\Omega$$

$$\Rightarrow \quad u_\nu(\Omega) = \frac{I_\nu}{c}$$

$$u_\nu = \int \frac{I_\nu}{c} \, d\Omega = \frac{4\pi J_\nu}{c}$$

$$J_\nu \equiv \frac{1}{4\pi} \int I_\nu \, d\Omega \quad \text{mean specific intensity.}$$

$$u = \int u_\nu \, d\nu.$$

Radiation Pressure - Isotropic Radiation.

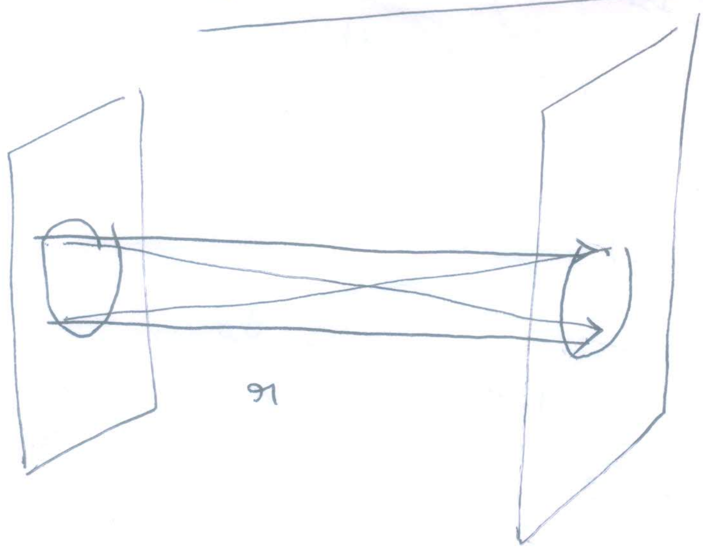
$$P_\nu = \frac{2}{c}\int I_\nu \cos^2\theta \, d\Omega$$ - reflecting cavity. integral over 2π only

$$u_\nu = \frac{1}{c}\int I_\nu \, d\Omega$$

Isotropy $\Rightarrow$ $P = \frac{2}{3c} I_\nu \, 2\pi = \frac{1}{3}u$

$$\boxed{P_\nu = \frac{1}{3}u_\nu}$$

Conservation of I_ν in Free space



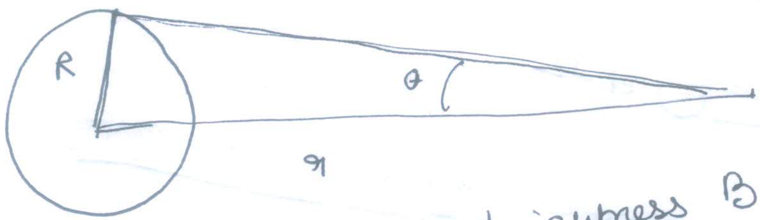
consider dA_1 and dA_2 on 2 surfaces, both normal to a ray such that same set of rays pass through dA_1 and dA_2

$$dE_\nu = I_\nu(1)\, dA_1 \, d\Omega_1 \, dt \, d\nu = I_\nu(2)\, dA_2 \, d\Omega_2 \, dt \, d\nu$$

$$d\Omega_1 = \frac{dA_2}{r^2} \qquad d\Omega_2 = \frac{dA_1}{r^2}$$

$$\Rightarrow I_\nu(1) = I_\nu(2) \Rightarrow \frac{dI_\nu}{ds} = 0.$$

$1/r^2$ Law



Sphere of uniform brightness B.

At a distance r, the flux from this source is

$$F = \int I_\nu \cos\theta \, d\Omega = B \int d\phi \int_0^{\theta'} \cos\theta \, \sin\theta \, d\theta$$

where $\theta' = \sin^{-1}\left(\frac{R}{r}\right)$

$$\int_0^{\theta'} \cos\theta \, d\cos\theta = -\frac{\cos^2\theta}{2}\Big|_0^{\theta'} = \frac{1}{2}\left[1 - \cos^2\theta'\right] = \frac{1}{2}\frac{R^2}{r^2}$$

$$= \frac{\pi B R^2}{r^2} = \frac{\pi R^2 B}{r^2}$$

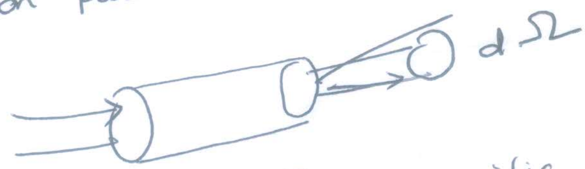
At surface

$F = \pi B$.

Specific Intensity – constant, solid angle comes down.

Radiative Transfer

There is ~~radiation~~ emission, absorption & scattering when radiation passes through a medium.



$dA \, dS$

$\dot{j}_\nu$ - specific <u>emission coefficient</u>.

$$\boxed{dE_\nu = \dot{j}_\nu \, dV \, dt \, d\nu \, d\Omega}$$

$$\cancel{dE_\nu} = \;=\; \dot{j}_\nu \, dA \, dS$$

$$dt \, d\nu \, d\Omega$$

$$I_\nu \, dA \, dt \, d\nu \, d\Omega$$

$$+ \; \dot{j}_\nu \, dA \, dS \, dt \, d\nu \, d\Omega$$

$$= I_\nu(s+\Delta s) \, dA \, dt \, d\nu \, d\Omega$$

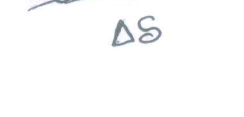
$$\Rightarrow I(s+\Delta s) = I(s) + \dot{j}_\nu \, \Delta S$$

$$\boxed{\Delta I_\nu = \dot{j}_\nu \, \Delta S}$$

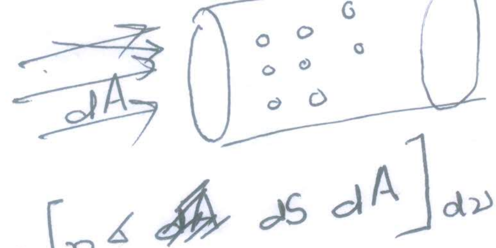
For isotropic $\dot{j}_\nu = \frac{P_\nu}{4\pi}$

$$\cancel{\frac{dI}{dS}} =$$

Absorption coefficient



α_ν - $\quad \Delta I_\nu = - I_\nu \, \alpha_\nu \, \Delta S$



$$- I_\nu \, d\Omega \left[n \sigma \, dS \, dA \right] d\nu \, dt$$

I_ν

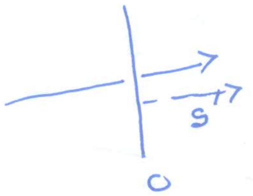
$$\alpha_\nu = n\,\sigma_\nu$$

$$\boxed{\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu} \rightarrow \text{radiative transfer through a medium}$$

s is the distance along the ray.

Solution.

Only emission.



$$I_\nu(s) = I_\nu(0) + \int_0^s j_\nu \, ds'$$

Only Absorption.

$$I_\nu(s) = I_0 \, \cancel{\int_0^s \alpha_\nu(s')} \, e^{-\int_0^s \alpha_\nu(s')\,ds'}$$

Optical Depth and Source Function.

The radiative transfer equation takes a particular simple form if we define

$$\tau(s) = \int_0^s \alpha_\nu(s')\,ds' \quad \Big| \quad d\tau_\nu = \alpha_\nu \, ds.$$

~~The radiative~~

A medium is said to be optically thick or opaque if $\cancel{\tau > 1}$. $\tau > 1$. It is optically thin or transparent if $\tau < 1$.

$\tau < 1$ indicates that a typical photon of frequency ν ~~travers~~ leaves the medium without getting absorbed.

$\tau > 1$ - typical photon of frequency ν cannot traverse the entire medium without getting absorbed.

The radiative transfer equation now becomes

$$\frac{dI_\nu}{d\tau} = \cancel{I_\nu}\ S_\nu - I_\nu$$

$S_\nu = \dfrac{j_\nu}{\alpha_\nu}$ is called the source function.

The source function is often a simpler physical quantity than the emission coefficient.

Also, τ_ν ~~is the relevant~~ reveals more clearly the important intervals along a ray as far as interaction with matter is concerned.

Hence it is often more convenient to use this form of the radiative transfer equation.

Formal solution.- regard everything as a function of τ.

~~Multiply the equation with e~~

Multiply the equation with e^{τ_ν}.

$$e^{\tau}\frac{dI}{d\tau} = -e^{\tau} I + S e^{\tau}$$

$$\Rightarrow \quad \frac{d}{d\tau}\left(e^{\tau} I\right) = S e^{\tau}$$

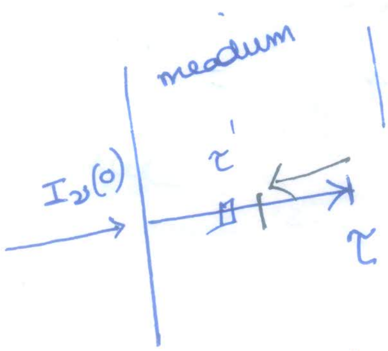
$$\Rightarrow \quad \frac{d\mathcal{J}}{d\tau} = \mathcal{S} \qquad \text{where} \quad \mathcal{J} = e^{\tau} I, \quad \mathcal{S} = e^{\tau} S$$

$$\Rightarrow \quad \mathcal{J}(\tau) = \mathcal{J}(0) + \int_0^{\tau} \mathcal{S}(\tau')\, d\tau'$$

$$\Rightarrow \quad I_\nu(\tau) = \cancel{I_\nu(0)}\, e^{-\tau} I_\nu(0) + e^{-\tau}\int_0^{\tau} S(\tau')\, e^{\tau'}\, d\tau'$$

Formal Solution of the R.T. equation.

$$I_\nu(\tau) = I_{0\nu}(0)\, e^{-\tau} + \int_0^{\tau} S(\tau')\, e^{-(\tau-\tau')}\, d\tau'$$



(1) Incident light is absorbed. → attenuated

(2) Source is attenuated

Suppose $S_\nu(\tau)$ is a constant.

$$I_\nu(\tau) = I_\nu(0)\, e^{-\tau} + \cancel{S e^{-\tau} + S\left[1-\right.} S e^{-(\tau-\tau')}\Big|_0^{\tau}$$

$$= I_\nu(0)\, e^{-\tau} + S\left(1 - e^{-\tau}\right)$$

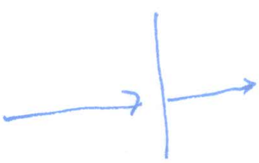
$$= S + \left(I_\nu(0) - S\right) e^{-\tau}$$

Points.

1. $\tau \to \infty \qquad I_\nu(\tau) \to S$
2. if ~~$S > I$~~ $S_\nu > I_\nu \Rightarrow \frac{dI_\nu}{d\tau} > 0$ I_ν tends to increase along the way
3. $S_\nu < I_\nu \quad \frac{dI}{d\tau} < 0$, I_ν tends to decrease along the way
4. $I_\nu \to S_\nu$ given sufficient optical depth. — relaxation problem.
5. Scattering problem. S_ν depends on I_ν. S_ν cannot be specified apriori.

Mean Free Path

What is the average distance that a photon travels in a medium before it gets absorbed?



Probability of a photon getting absorbed is $e^{-\tau}$. Mean optical depth at which the photon gets absorbed is

$$\langle \tau_\nu \rangle = \int_0^\infty \tau e^{-\tau} d\tau = 1.$$

Assuming a homogeneous medium

~~$\langle \tau \rangle =$~~

$$\langle \tau_\nu \rangle = \alpha_\nu \langle l_\nu \rangle.$$

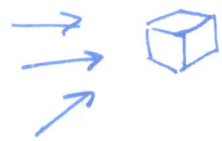
$$\Rightarrow \langle l_\nu \rangle = \frac{1}{\alpha_\nu} = \frac{1}{n \sigma_\nu}$$

In an inhomogeneous medium, we can think of an local mean free path.

Radiation Force.

When radiation is absorbed by a medium it also transfers momentum ≡ Force.

$$\vec{F}_\nu = \int \frac{I_\nu}{c} \alpha_\nu \hat{n} \, d\Omega$$

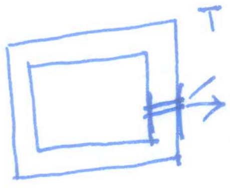


6

Thermal Radiation.

Radiation ~~produced~~ emitted by matter which is in thermal equilibrium.

Black body radiation. Consider a cavity at temperature T.



such that the radiation itself is in equilibrium with the walls of the cavity.

We let the radiation come to equilibrium, and then let a small part of it out without disturbing the equilibrium. ~~Th~~ The radiation in such a cavity is called black body radiation.

We use thermodynamic principles and the fact that photon number is not conserved to determine some properties.

We expect the number of photons to adjust to the temperature T.

An important property is that I_ν ~~is~~ does not depend on the properties of the cavity, only on the temperature T. Further, it is an universal function of T.