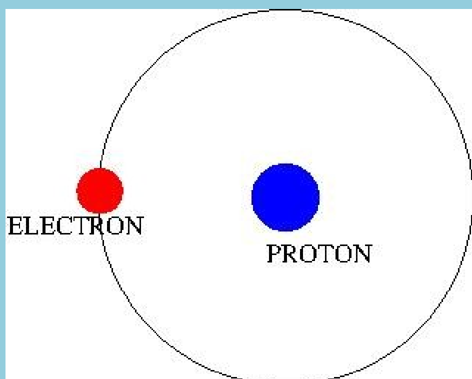
21-cm Physics and Cosmology

Somnath Bharadwaj
IIT Kharagpur

Hydrogen atom



Spin of proton and electron
Is ignored

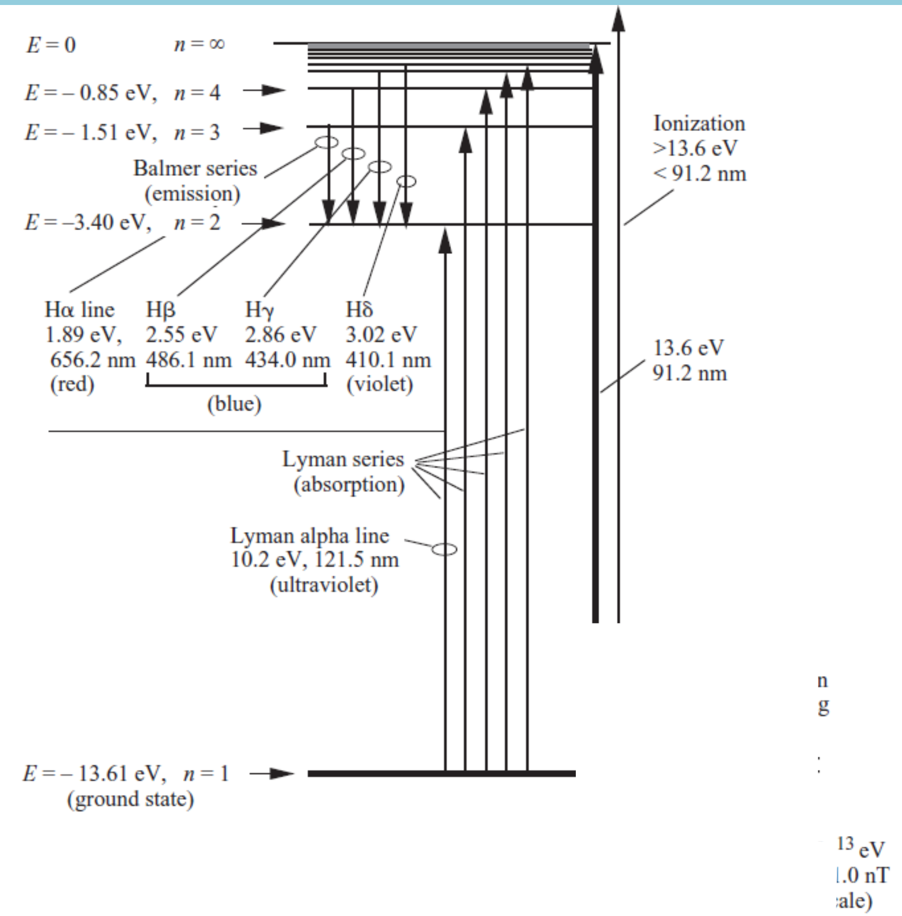
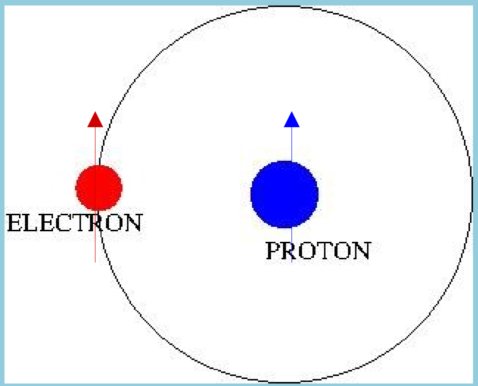


Fig. 10.3: Energy levels of the hydrogen atom. Downward Balmer and upward Lyman transitions are shown. The minuscule hyperfine splitting of the ground state is shown with greatly exaggerated spacing together with the even finer Zeeman splitting of the upper hyperfine level. The energy difference between the upper and lower Zeeman states is given for a 1-nT line-of-sight component of magnetic field in a hydrogen cloud. [Adapted from H. Bradt, *Astronomy Methods*, Cambridge, 2004, Fig. 101, with permission]

Astrophysics Processes – Bradt 2008

Ground State n=1



Has zero orbital angular momentun

Total angular momentun is sum of electron and proton spins

$$\boldsymbol{F} = \boldsymbol{S} + \boldsymbol{I}.$$

Electron $S = 1/2$

$$\boldsymbol{S} \cdot \boldsymbol{S} \Rightarrow S(S+1)\hbar^2.$$

$$S_z \Rightarrow m_S \hbar; m_S = +1/2, -1/2.$$

Proton $I = 1/2$

$$\boldsymbol{I} \cdot \boldsymbol{I} \Rightarrow I(I+1)\hbar^2$$

$$I_z \Rightarrow m_I \hbar; m_I = +1/2, -1/2.$$

Triplet

UU	$\frac{\text{UD} + \text{DU}}{\sqrt{2}}$	DD
$F = 1$	$F = 1$	$F = 1$
$m_F = +1$	$m_F = 0$	$m_F = 1$

$$\frac{\text{UD} - \text{DU}}{\sqrt{2}}$$

$$F = 0$$

$$m_F = 0.$$

Singlet

Magnetic Moments

Electron

$$\boldsymbol{\mu}_{\mathrm{e}} = g_{\mathrm{e}} \frac{\mu_{\mathrm{B}}}{\hbar} \boldsymbol{S},$$

Bohr Magneton

$$\mu_{\mathrm{B}} \equiv \frac{e\hbar}{2m_{\mathrm{e}}} = 9.27401 \times 10^{-24}\ \mathrm{J/T}$$

$$g_{\mathrm{e}} = -2.00232.$$

Proton

$$\boldsymbol{\mu}_{\mathrm{p}} = g_{\mathrm{p}} \frac{\mu_{\mathrm{N}}}{\hbar} \boldsymbol{I}$$

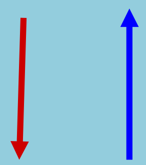
Nuclear Magneton

$$\mu_{\mathrm{N}} \equiv \frac{e\hbar}{2m_{\mathrm{p}}} = 5.050783 \times 10^{-27}\ \mathrm{J/T}.$$

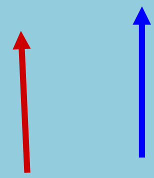
$$g_{\mathrm{p}} = +5.586$$

Proton magnetic moment is ~650 times smaller than electron

Triplet



Singlet

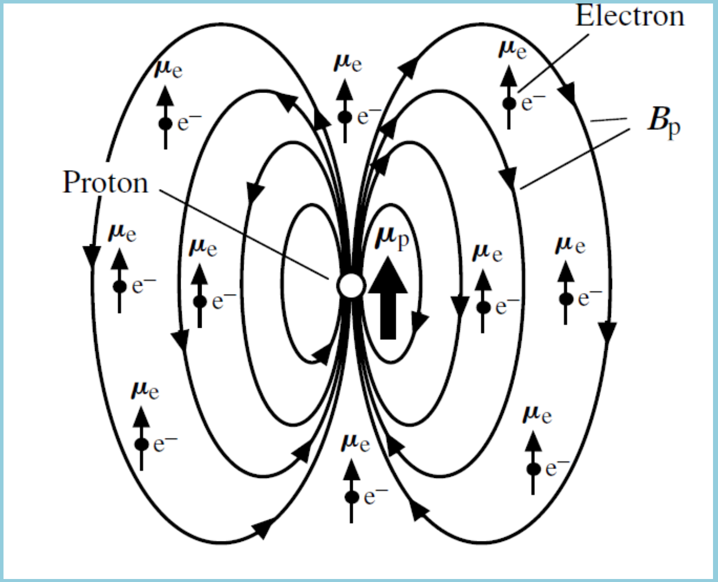


Interaction Energy

$$E_{\mathrm{pot}} = -\boldsymbol{\mu} \cdot \boldsymbol{B}$$

1. Proton's Magnetic Filed and Electron's Orbital Magnetic Moment – Not there in ground state.

2. Proton's Magnetic Field and Electron's Spin Magnetic Moment – Zero when averaged over Electron's Spatial Distribution



$$\psi(r) = \frac{1}{\sqrt{4\pi}} \frac{2}{a_0^{3/2}} \exp(-r/a_0).$$

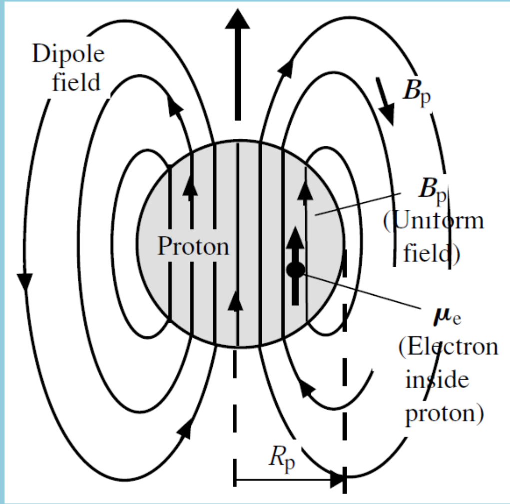
Bohr Radius

$$a_0 = 5.291772 \times 10^{-11}\ \mathrm{m},$$

Contact Term

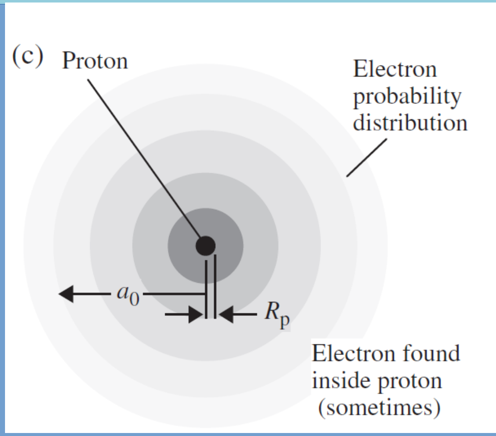
$$B_p = \frac{\mu_0}{2\pi R_p^3}\boldsymbol{\mu}_p,$$

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T\,m\,A^{-1}}$$



$$E_{pot} = -\boldsymbol{\mu}\cdot\boldsymbol{B}$$

$$= -\frac{4}{3}\frac{R_p^3}{a_0^3}\boldsymbol{\mu}_e\cdot\boldsymbol{B}_p$$



$$-\frac{4}{3}\frac{\mu_0}{2\pi}\frac{g_e g_p \mu_B \mu_N}{a_0^3}\frac{\boldsymbol{S}\cdot\boldsymbol{I}}{\hbar^2} \Rightarrow E_{pot}.$$

$$\boldsymbol{S}\cdot\boldsymbol{I} \Rightarrow \frac{1}{2}\hbar^2[F(F+1) - S(S+1) - I(I+1)].$$

$$\boldsymbol{S}\cdot\boldsymbol{I} \Rightarrow \frac{1}{2}\hbar^2\left[0 - \frac{3}{4} - \frac{3}{4}\right] = -\frac{3}{4}\hbar^2$$ $F = 0$

$$\boldsymbol{S}\cdot\boldsymbol{I} \Rightarrow \frac{1}{2}\hbar^2\left[2 - \frac{3}{4} - \frac{3}{4}\right] = -\frac{1}{4}\hbar^2.$$ $F = 1$

Hyperfine Splitting

$$\Delta E_{pot} = E_{pot,\,F=1} - E_{pot,\,F=0} = 9.42762 \times 10^{-25}\left[\frac{1}{4} - \left(-\frac{3}{4}\right)\right]$$

1422.8 MHz

More precise QED calculation

$$\nu = 1\,420\,405\,752 \text{ Hz}$$

1420.4 MHz.

λ= 21.11 cm

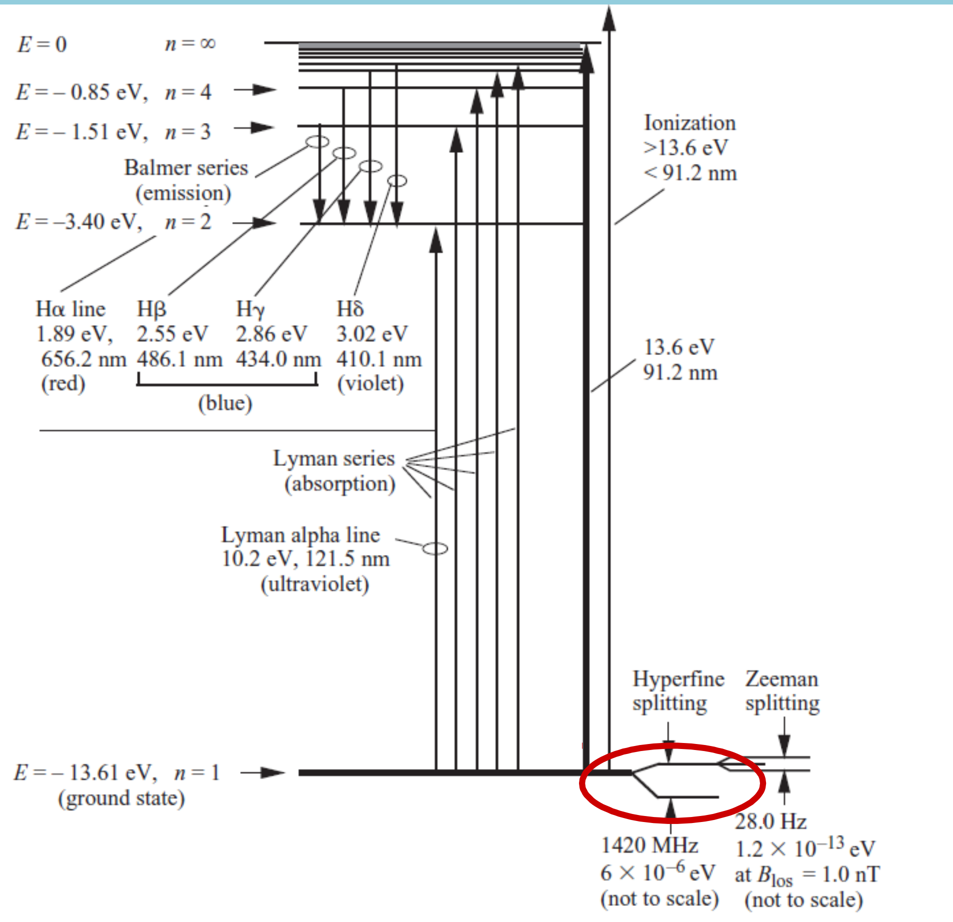
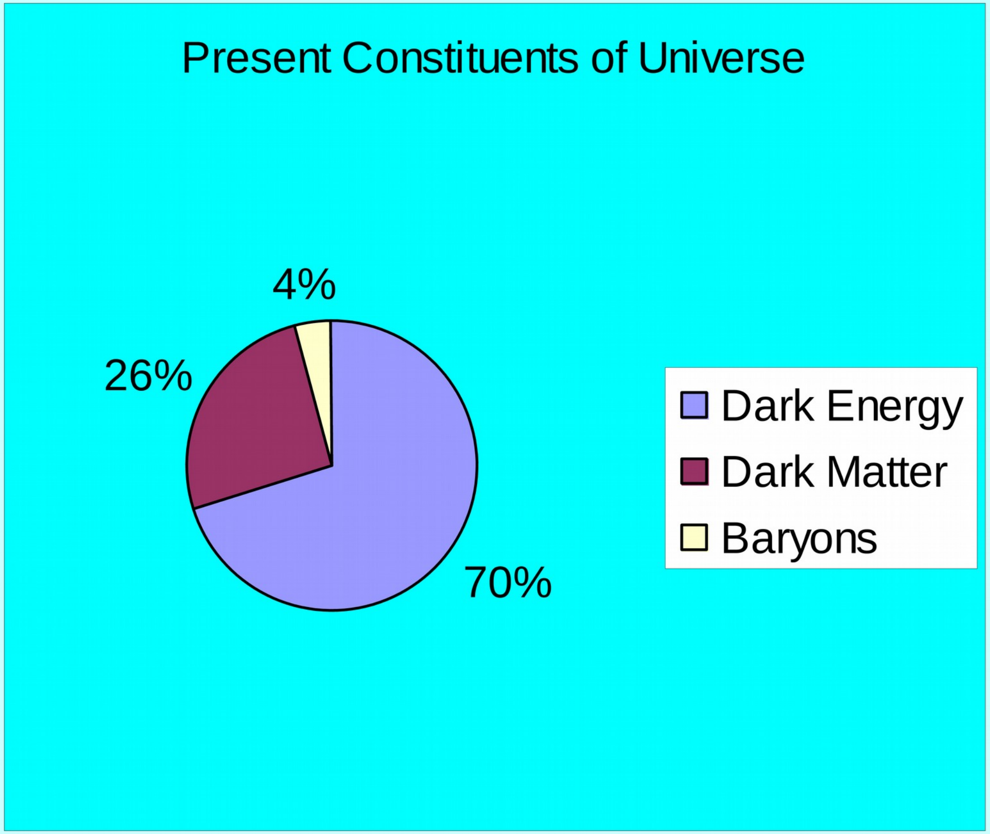


Fig. 10.3: Energy levels of the hydrogen atom. Downward Balmer and upward Lyman transitions are shown. The minuscule hyperfine splitting of the ground state is shown with greatly exaggerated spacing together with the even finer Zeeman splitting of the upper hyperfine level. The energy difference between the upper and lower Zeeman states is given for a 1-nT line-of-sight component of magnetic field in a hydrogen cloud. [Adapted from H. Bradt, *Astronomy Methods*, Cambridge, 2004, Fig. 101, with permission]

Constituents of the Universe



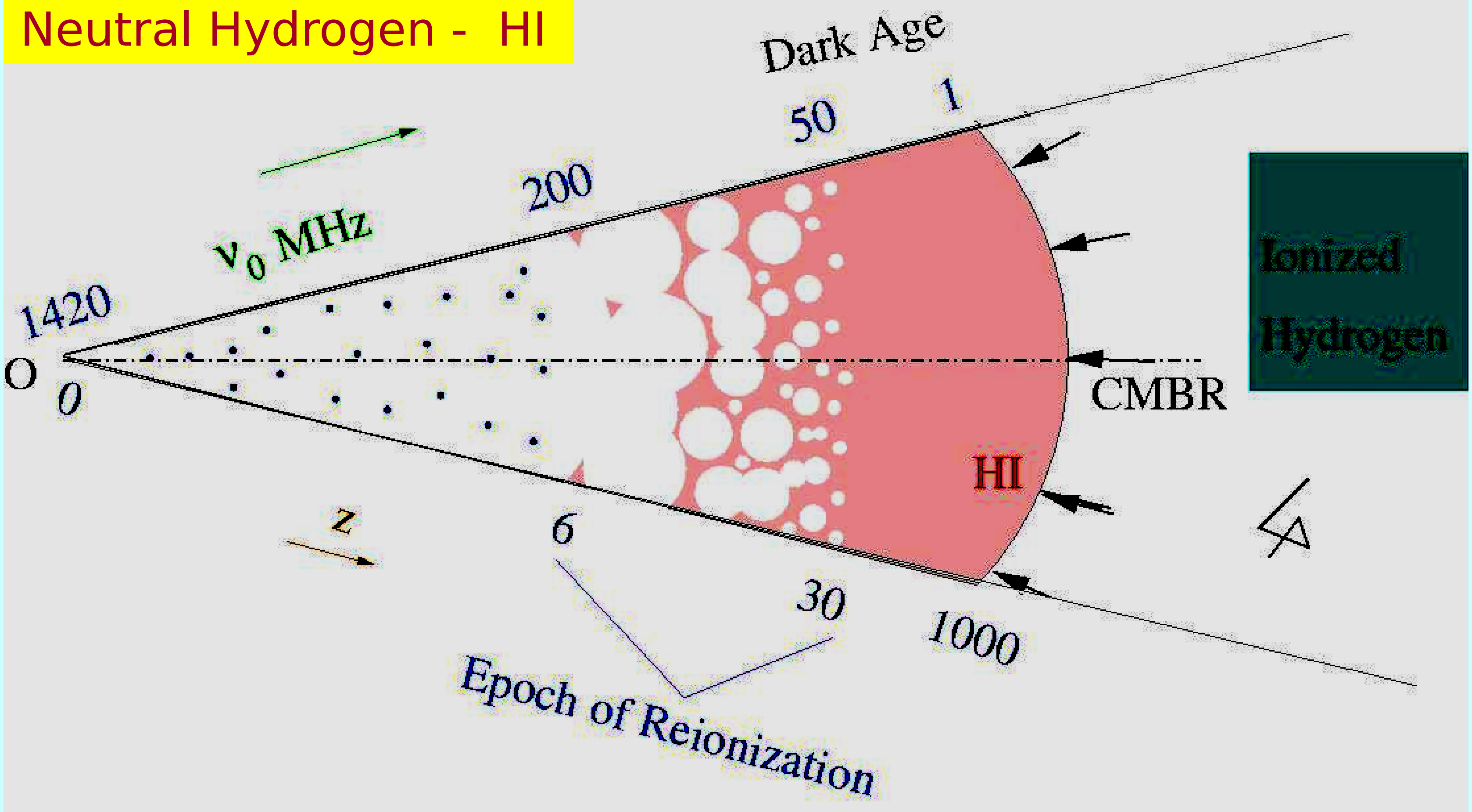
Baryons

75 % Hydrogen

25% Helium

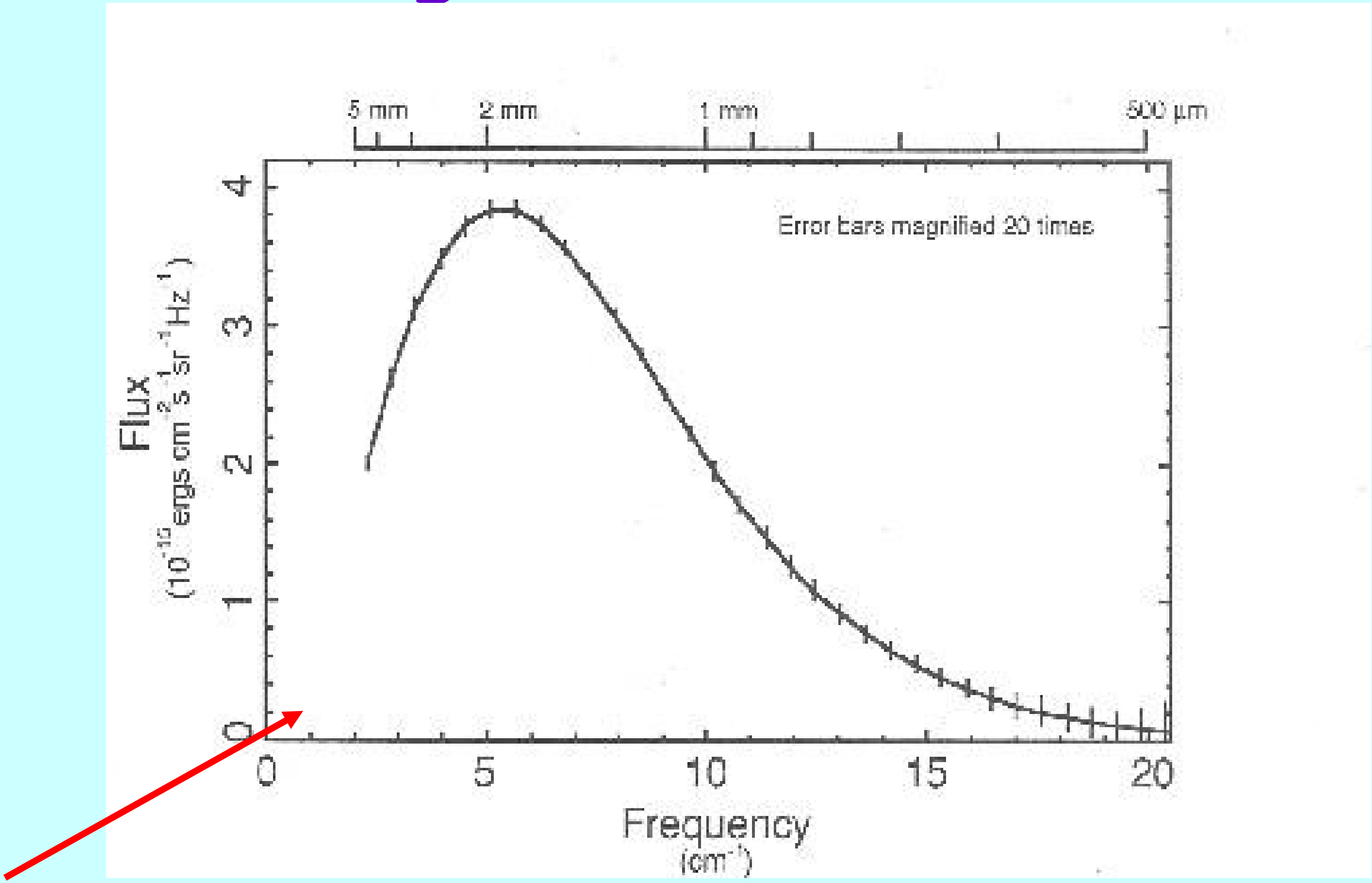
HI Evolution

Neutral Hydrogen - HI

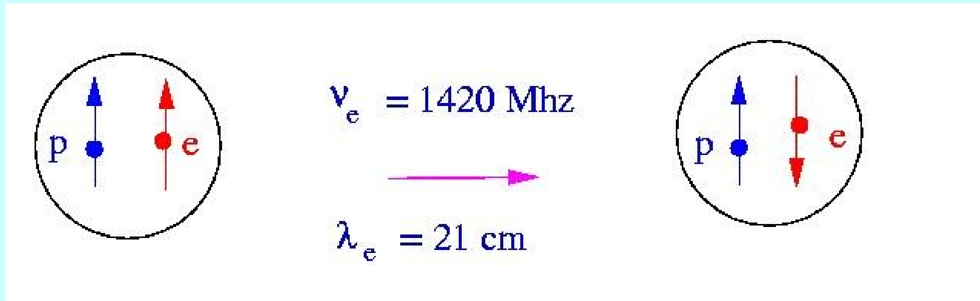


$\nu_o = 1420 \text{ Mhz} / (1+z)$ $\lambda_o = 21 \text{ cm } (1+z)$

Cosmic Microwave Background Radiation



Spin Temperature

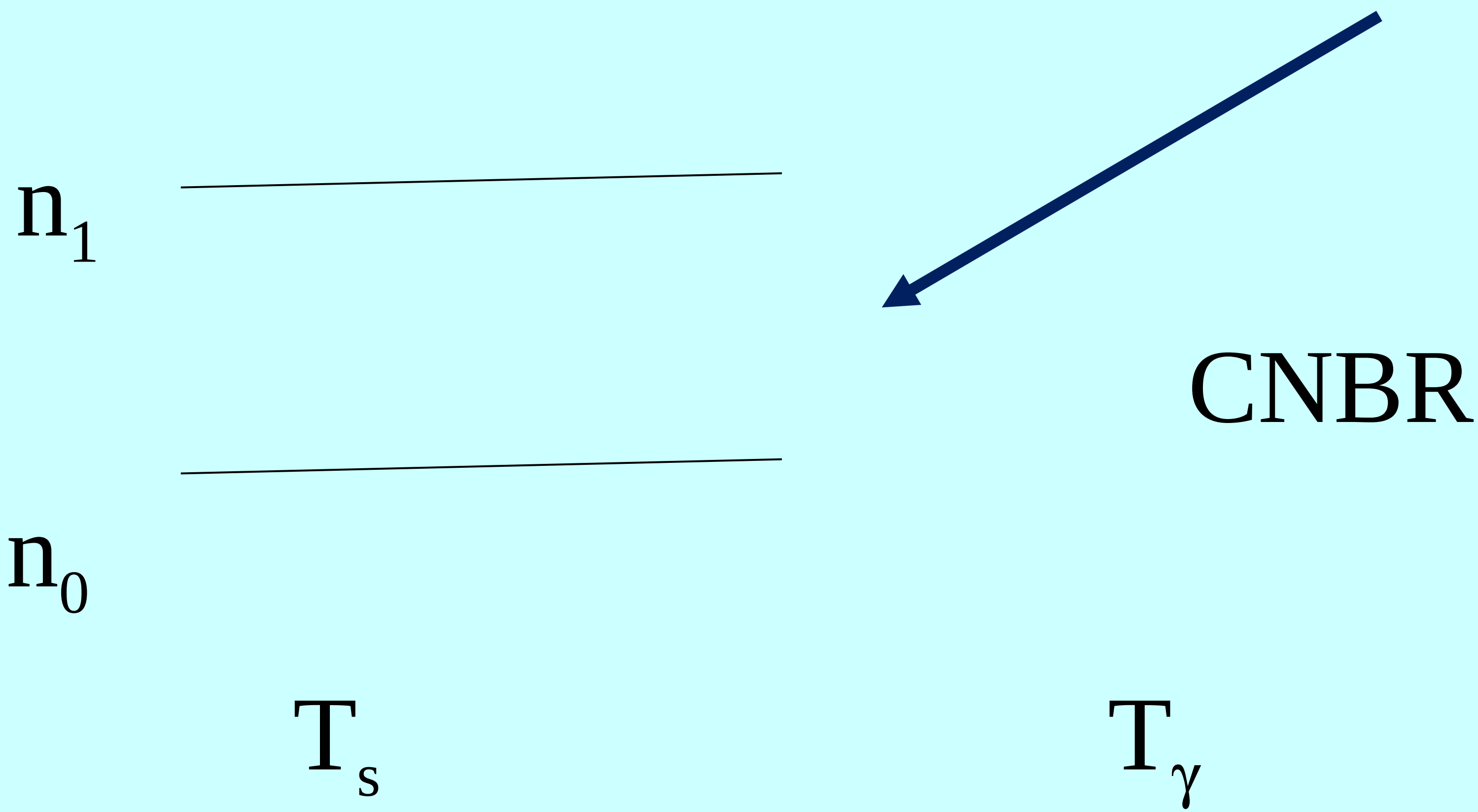


$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-T_\star / T_s}$$

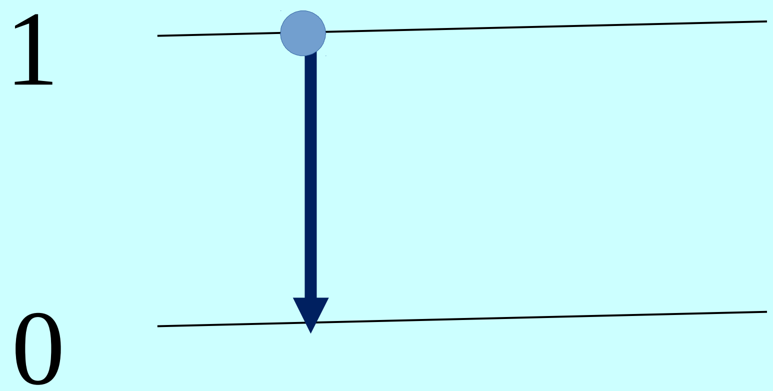
3

$$T_\star = h_p \nu_e / k_B = 0.068 \text{ K}$$

21-signal



Spontaneous Emission

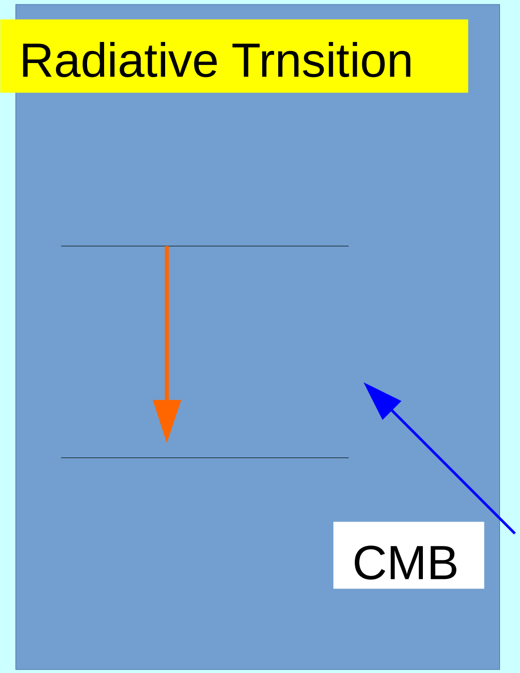


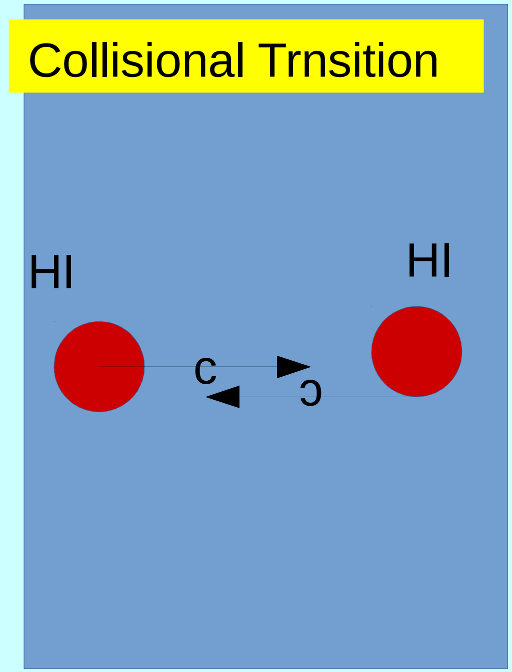
Einstein Coefficient
Transition probability per unit time

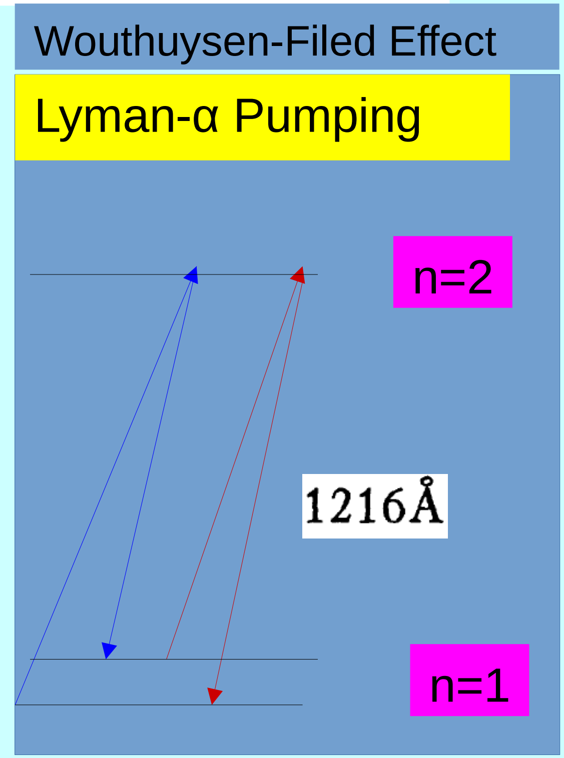
$$A_{10} = 2.85 \times 10^{-15}\ \mathrm{s}^{-1}$$

Lifetime ~10 million yrs

Excitation - De-excitation







CMB Temperature

T_γ

Gas Kinetic Temperature

T_K

Effective Color Temperature
Of Lyman-α Radiation

$\tilde{T}_c \approx T_K$

Rate Equation

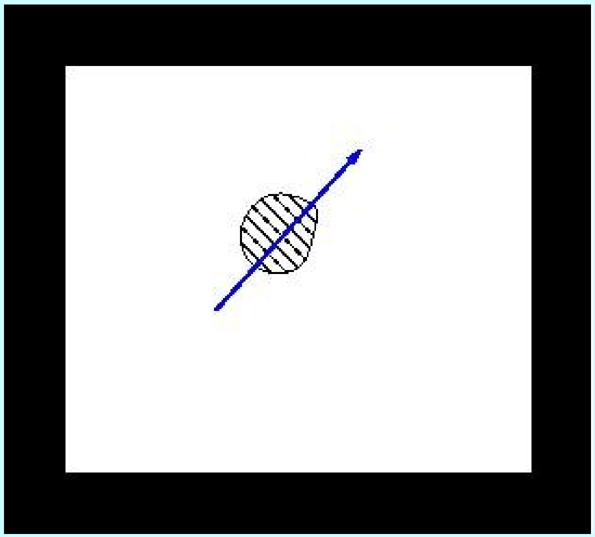
Excitation rate

De-excitation rate

$$\dot{n}_0 = -n_0 \, [\, P_{01}^{\gamma} + P_{01}^{c} + P_{01}^{\alpha} \,] + n_1 \, [\, P_{10}^{\gamma} + P_{10}^{c} + P_{10}^{\alpha} \,]$$

Radiative Collision Lyman-α

Detailed Balance



Consider a situation where radiative processes dominate – in equilibrium

$$T_s = T_\gamma \quad \dot{n}_0 = 0$$

$$n_0 P_{01}^{\gamma} = n_1 P_{10}^{\gamma}$$

$$(T_*/T_\gamma) \ll 1$$

$$\frac{P_{01}^{\gamma}}{P_{10}^{\gamma}} = \frac{n_1}{n_0} = 3\exp(-T_*/T_\gamma) \approx 3\left(1 - \frac{T_*}{T_\gamma}\right)$$

$$\frac{P_{01}^{c}}{P_{10}^{c}} = \approx 3\left(1 - \frac{T_*}{T_K}\right)$$

Collisions

$$\frac{P_{01}^{\alpha}}{P_{10}^{\alpha}} = \approx 3\left(1 - \frac{T_*}{T_c}\right)$$

Lyman-α

Equilibrium

If all three processes are active

$$n_0 \left[P_{01}^{\gamma} + P_{01}^{c} + P_{01}^{\alpha} \right] = n_1 \left[P_{10}^{\gamma} + P_{10}^{c} + P_{10}^{\alpha} \right]$$

$$\frac{n_1}{n_0} \approx 3\left(1 - \frac{T_*}{T_s}\right)$$

$$\frac{P_{01}^{c}}{P_{10}^{c}} = \approx 3\left(1 - \frac{T_*}{T_K}\right)$$

$$T_s^{-1} = \frac{T_\gamma^{-1} P_{10}^{\gamma} + T_K^{-1} P_{10}^{c} + T_c^{-1} P_{10}^{c}}{P_{10}^{\gamma} + P_{10}^{c} + P_{10}^{c}}$$

$$T_s^{-1} = \frac{T_\gamma^{-1} + x_c T_K^{-1} + x_\alpha T_c^{-1}}{1 + x_c + x_\alpha}$$

$$x_c = \frac{P_{10}^{c}}{P_{10}^{\gamma}}$$

$$x_\alpha = \frac{P_{10}^{\alpha}}{P_{10}^{\gamma}}$$

$$P_{10}^{\gamma} = A_{10} + B_{10} \bar{J} \approx \frac{A_{10} T_\gamma}{T_*}$$

Spin Temperature

$$T_S^{-1} = \frac{T_\gamma^{-1} + \tilde{x}_\alpha \tilde{T}_c^{-1} + x_c T_K^{-1}}{1 + \tilde{x}_\alpha + x_c}.$$

$$x_c = \frac{\kappa_{10} n_{\mathrm{H}} T_\star}{A_{10} T_\gamma}$$

k_{10} Tabulated in Zygelman 2005

$$k_{10} \approx \begin{cases} 1.19 \times 10^{-10} T_2^{0.74-0.20 \ln T_2}\ \mathrm{cm^3\,s^{-1}} & (20\,\mathrm{K} < T < 300\,\mathrm{K}) \\ 2.24 \times 10^{-10} T_2^{0.207} \mathrm{e}^{-0.876/T_2}\ \mathrm{cm^3\,s^{-1}} & (300\,\mathrm{K} < T < 10^3\,\mathrm{K}) \end{cases}$$

Physics of Interstellar and Intergalactic Medium -Draine, 2011

$$\tilde{x}_\alpha = \frac{8\pi \lambda_{\mathrm{Ly}\alpha}^2 \gamma T_\star}{9 A_{10} T_\gamma} \tilde{S}_\alpha J_\alpha = 1.81 \times 10^{11} (1+z)^{-1} \tilde{S}_\alpha J_\alpha$$

Hirata 2005

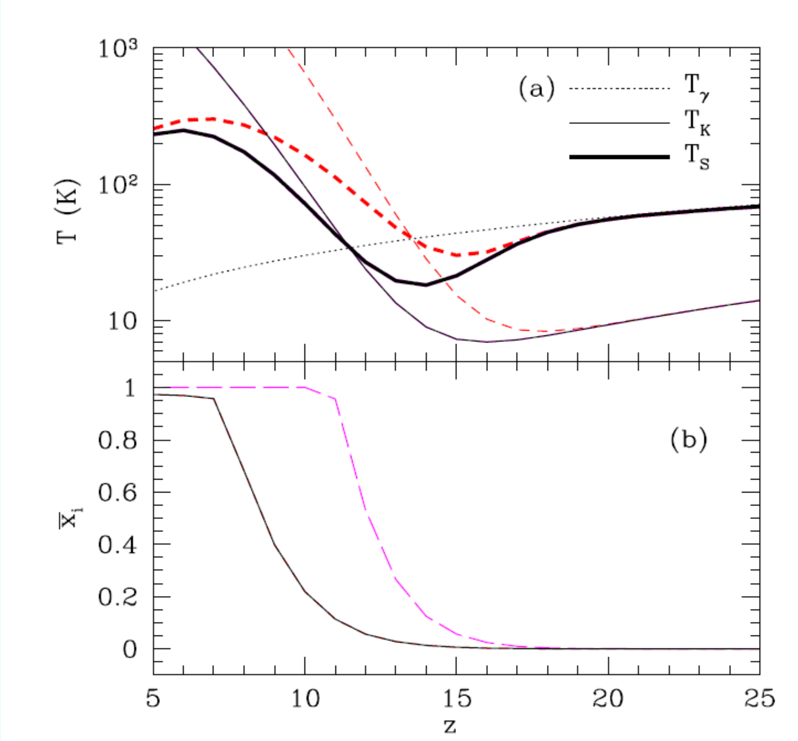
$\gamma = 50\,\mathrm{MHz}$

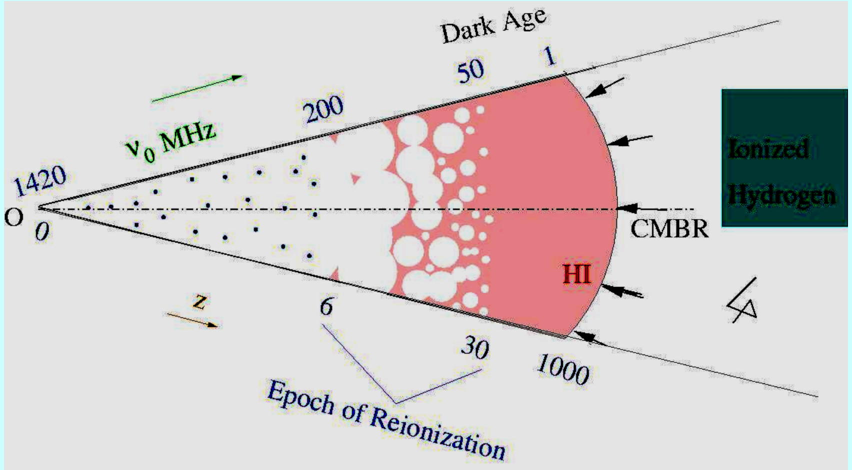
J_α is the flux of Lyα photons (in $\mathrm{cm^{-2}\ s^{-1}\ Hz^{-1}\ sr^{-1}}$),

$\tilde{S}_\alpha$ is a factor of order unity

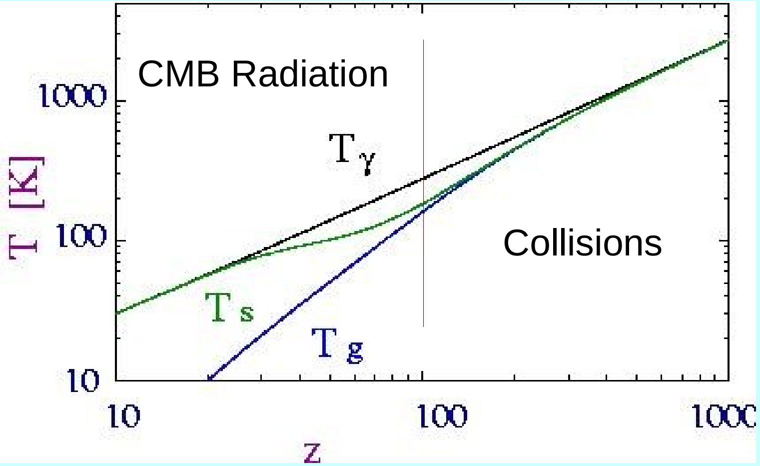
Evolution of Spin Temperature

Cosnic Dawn and Epoch of Reionization
First stars and AGNs heat IGM
Emit Lyman-α photons - Couple T_S to T_K





Dark Ages – No Luminous Sources



Furlanetto 2005

Bharadwaj and Ali 2004

Summary

- HI 21-cm line - spin-flip in ground state of neutral Hydrogen
- Potential probe of Universe over large redshift range
- Relative population ratio – quantified by spin temperature Ts
- $z>100$ - collisions couple Ts to T_K gas kinetic temperature
- $z < 100$ – Ts coupled to $T\gamma$ CMBR temperature
- After first stars and AGNs formed Lyman-α couples Ts to T_K