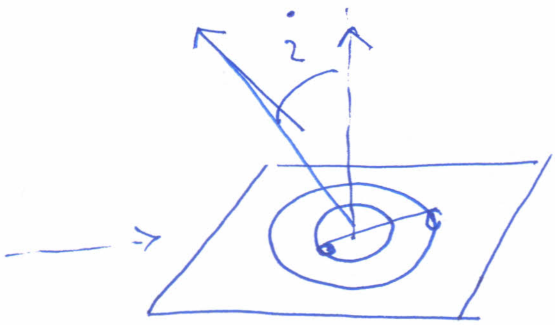
19/6/20..

Mass determination.

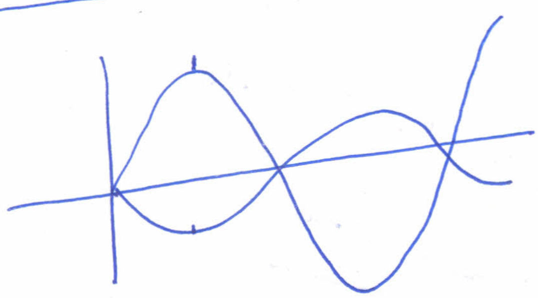


spectroscopic binary

Circular orbit.

$V_1(t) = a_1 \omega \sin(\omega t) \sin i = v_1 \sin \omega t \sin i$

$V_2(t) = -a_2 \omega \sin \omega t \sin i = -v_2 \sin \omega t \sin i$



V_1 ~~max~~ $= a_1 \omega \sin i$

V_2 ~~max~~ $= a_2 \omega \sin i$

] observation give these and P the period

Kepler's Law III

$GM P^2 = 4\pi^2 a^3$

$\Rightarrow$ ~~GM~~ ~~GM (~~

$$G(m_1+m_2)\left(\frac{2\pi}{\omega}\right)^2 = 4\pi^2 \frac{(m_1+m_2)^3 a_1^3}{m_2^3}$$

$$\frac{m_2^3}{(m_1+m_2)^2} = \frac{\omega^2 a_1^3}{G} = \frac{P\,\omega^3 a_1^3}{2\pi G}$$

~~$= \frac{P}{2\pi} v_1$~~

$$\frac{m_2^3}{(m_1+m_2)^2}\sin^3 i = \frac{P(v_1 \sin i)^3}{2\pi G}$$

$$\Rightarrow f_1 = \frac{m_2^3}{(m_1+m_2)^2}$$

this is the mass function of object 1.

Observations of the Doppler shift of the line from object 1 yield f_1.

This has the dimensions of mass, but is not the mass.

$$f_1 = \frac{m_2^3}{(m_1+m_2)^3}\sin^3 i$$

In a situation where $m_2 >> m_1$

$f_1 = m_2 \sin^3 i$, it gives a lower limit

$m_2 \geq f_1$: if $i = 90^\circ$ then $m_2 = f_1$ else

$m_2 > f_1$.

Observations of object 2 similarly provide

$$f_2 = \frac{m_1}{(m_1 + m_2)^3} \sin^3 i .$$

Observations of ~~the~~ 2 Doppler shifts from both objects give f_1 and f_2.

3 unknowns – m_1, m_2 and inclination i.

A third condition is required to solve for the ~~masses~~ ~~individual~~ individual masses.

For example if also eclipsing – duration of eclipses puts constraints.

If we assume $i = 90°$ $\sin i = 1$.

we can solve for m_1 and m_2.

This gives lower limits ~~M~~ on masses.

Masses will be larger if orbit is inclined

$i < 90°$.

Example.

$\sin i \, v_1 = 30$ km/s. $\quad P = 30$ days.

$$f_1 = \frac{P (v_1 \sin i)^3}{2\pi G} =$$

$$= \frac{[30 \times 24 \times 60^2][3 \times 10^4]^3}{2\pi \times 6.67 \times 10^{-11}}$$

$$= 1.67 \times 10^{29} \text{ kg}$$

$$= 0.083 \, M_\odot = \frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2}$$

if $i = 90°$

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if ~~m~~

$\frac{v_2}{v_1} = \frac{m_1}{m_2}$ → ratio of velocities gives ratio of masses.

$m_2 = m_1/3$

$$\Rightarrow \quad 0.083 = \frac{\frac{1}{3^3} m_1^3}{(\frac{4}{3})^2 m_1^2} = \frac{m_1}{48}$$

$m_1 \simeq 4 M_\odot \qquad m_2 = 1.3 M_\odot$

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masses will be larger if $i < 90°$.

Exoplanets.

Planets outside the solar system

Currently a very active field of research

~~778 ex~~ more than 700 exoplanets known at present.

~~More~~ Estimated that more than 50% of Sun like stars host atleast one planet.

Most known ~~plane~~ exoplanets are giants like Jupiter and ~~Neptud~~ Neptune.

Some relatively light exoplanets are known - few times the mass of Earth - called Super Earths.

Planets are very faint compared to stars.

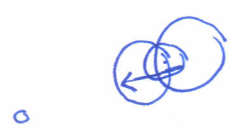
At visible wavelengths ~ 1 million times fainter.

How are they detected?

Doppler shift of absorption line from the star.

In a planet - star binary system, both the star and the planet orbit around the common center of mass.

Star is much more massive than the planet, Star appears to wobble around the center of mass.



Example. Sun - Jupiter.

Sun $m_1 = 2 \times 10^{30}$ kg Jupiter $m_2 = 1.9 \times 10^{27}$ kg

$r = 5.2$ AU $P = 11.9$ yr.

$$V = \frac{2\pi}{11.9 \times 365 \times 24 \times 60 \times 60} \times \frac{2 \times 10^{27}}{2 \times 10^{30}} \times 5.2 \times 1.5 \times 10^{11}$$

≈ 13 m/s

High resolution Spectrographs can make out Doppler shifts ~ 3 m/s

~~$\Delta\lambda = 5000 \times \frac{3}{3\times10^{8}}$~~

$=$

$$\left|\frac{\Delta\lambda}{\lambda}\right| \sim 10^{-8}$$

corresponding to 3 m/s.

Possible to measure the Doppler shift due to a planet like Jupiter around the Sun.

Example of exoplanet

M4 star named GJ 876 —

2 planets of mass

$\sim 0.79\ M_{Jup}$ and $\sim 2.5\ M_{Jup}$

Problem.

A comet is observed to have a perihelion $= 0.1$ AU and its speed is 10 km/s at perihelion.

Determine 'a' and 'p'.

$$E = \frac{1}{2} v^2 - \frac{GM}{r}$$

$$J = vr$$

$$E = \frac{1}{2}(10^4)^2 - \frac{6.67\times10^{-11}\times 2\times10^{30}}{0.1\times 1.5\times10^{11}}$$

$$= .5\times10^7 - 8.9\times10^9 \approx 8.9\times10^9$$

$$J = 10^4\times .1\times 1.5\times10^{11} = 1.5\times10^{14}$$

$$e = \left[1 + \frac{2J^2E}{G^2M^2}\right]^{1/2}$$

$$= \left[1 - \frac{2\, v^2 r_{min}^2\, GM/r_{min}}{G^2M^2}\right]^{1/2}$$

$$= \left[1 - \frac{2 \times (1.5\times10^{14})^2 \times 8.9\times10^{9}}{[6.67\times10^{-11} \times 2\times10^{30}]^2}\right]^{1/2}$$

$$e = [0.977]^{1/2} = 0.9887$$

$$r_{min} = 0.1\ \text{AU} = a \times 0.99\ (1-e)$$

$$\Rightarrow a = 8.85\ \text{AU}.$$

$$e = \left[1 - 2\frac{r_{min}^2 v^2}{G^2M^2} \times \frac{GM}{r_{min}}\right]^{1/2}$$

$$= \left[1 - \frac{2v^2}{E}\right]^{1/2} = \left(1 - \frac{2\times10^{8}}{8.9\times10^{9}}\right)^{1/2}$$

$e = 0.9887$

$.1 = a \times (1 - e)$

$\Rightarrow a = 8.85$ AU.

Period.

$P^2 = K a^3$

$\left(\frac{P}{1\,yr}\right)^2 = \left(\frac{8.85}{1\,AU}\right)^3$

$P = 8.85^{3/2}$ yr

$= 26.3$ yr.

Problem. Halley has period of 75 yrs.

Determine semi-major axis.

$\left(\frac{75\,yr}{1\,yr}\right)^2 = \left(\frac{a}{1\,AU}\right)^3$

$\Rightarrow a = (75)^{2/3} = 17.8$ AU.