$$i = \sqrt{-1}$$

$$\frac{d}{dt} = \dot{}$$

$$\frac{d}{dx} = {}'$$

$$\frac{d^2}{dt^2} = \ddot{}$$

$$\frac{d^2}{dx^2} = {}''$$

Notations

$$\frac{dx}{dt} = \dot{x}$$

$$\frac{dy}{dx} = y'$$

$$\frac{d^2\theta}{dt^2} = \ddot{\theta}$$

$$\frac{d^2V}{dx^2} = V''$$

Hooke's Law:

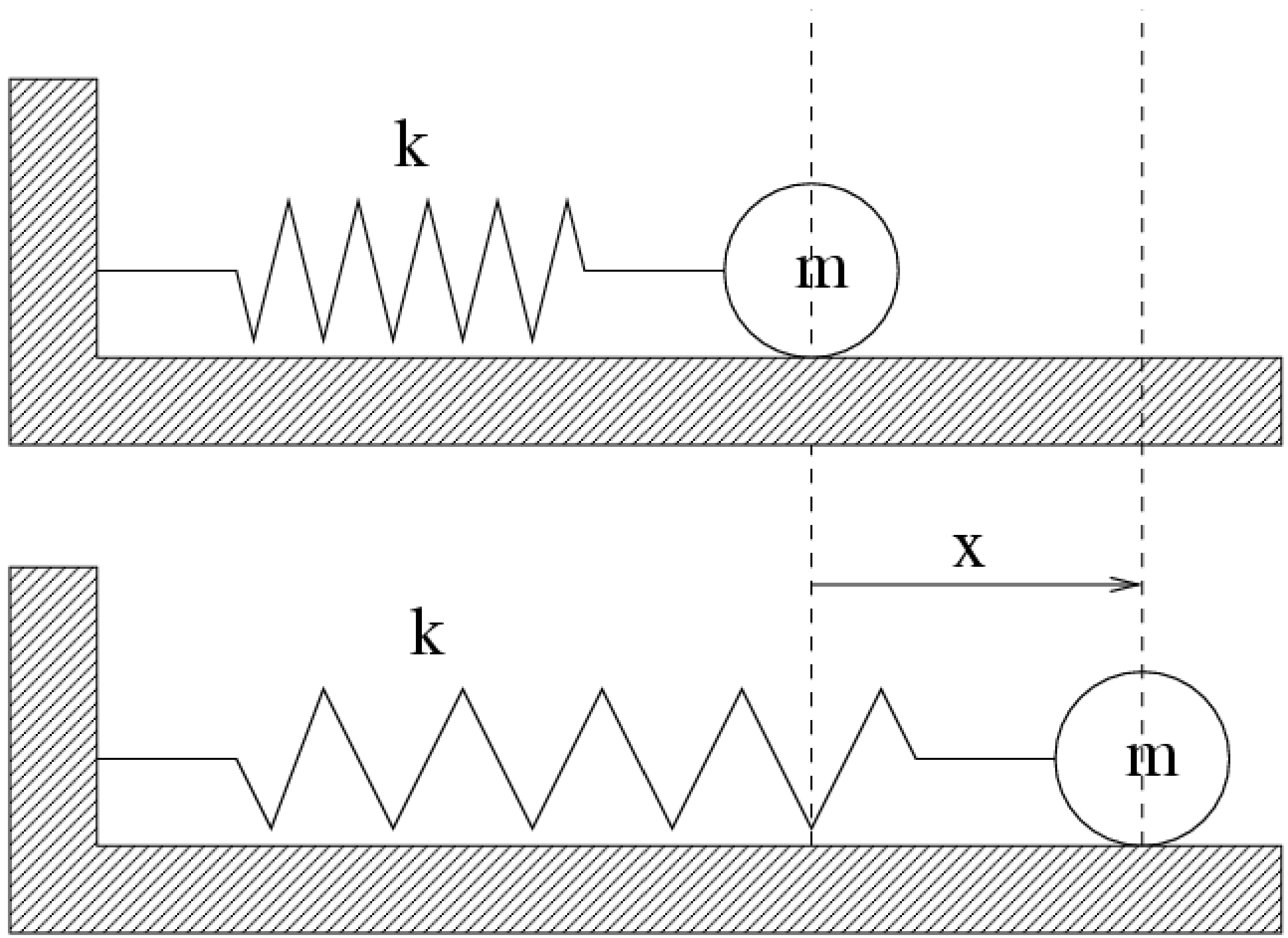
$$F = -kx$$

$$m\ddot{x} + kx = 0$$

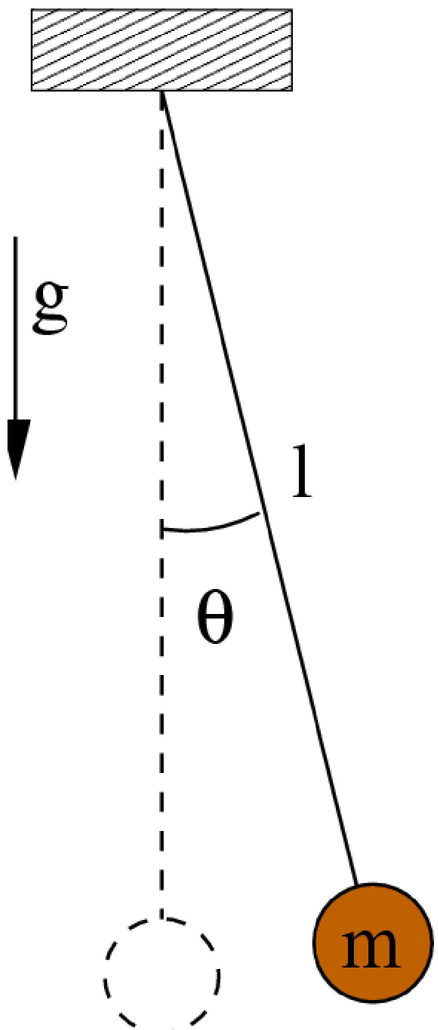
$$\ddot{x} + \omega_0^2 x = 0$$ SHM

$$\omega_0 = \sqrt{k/m}$$ Angular frequency

$$T = 2\pi/\omega_0$$ Time period



$$\omega_0 = \sqrt{k/m}$$



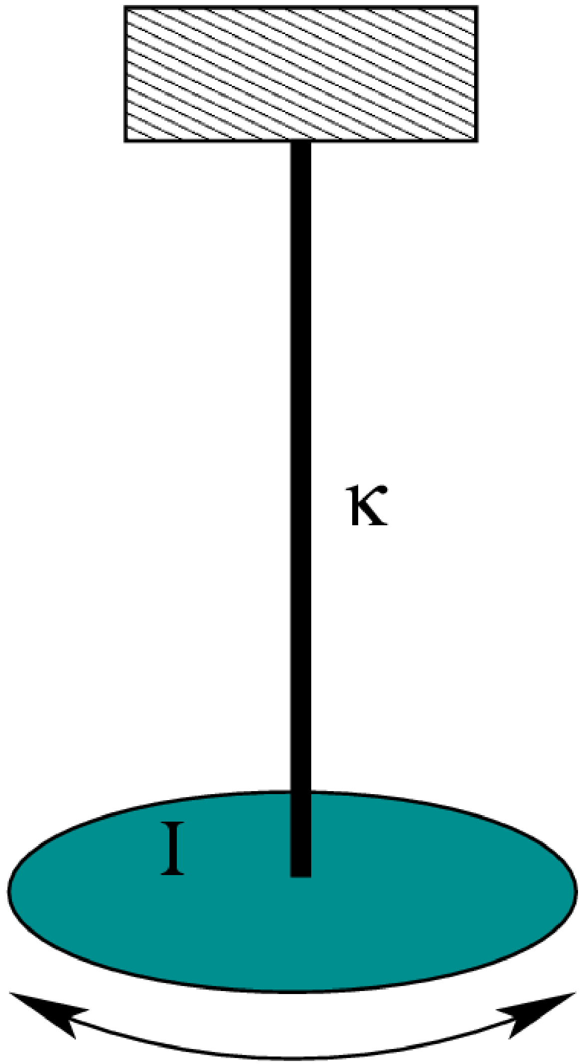
$$ml\ddot{\theta} = -mg\sin\theta$$

$$\theta \leq 4^\circ, \qquad \sin\theta \approx \theta$$

$$\ddot{\theta} + \frac{g}{l}\theta = 0$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

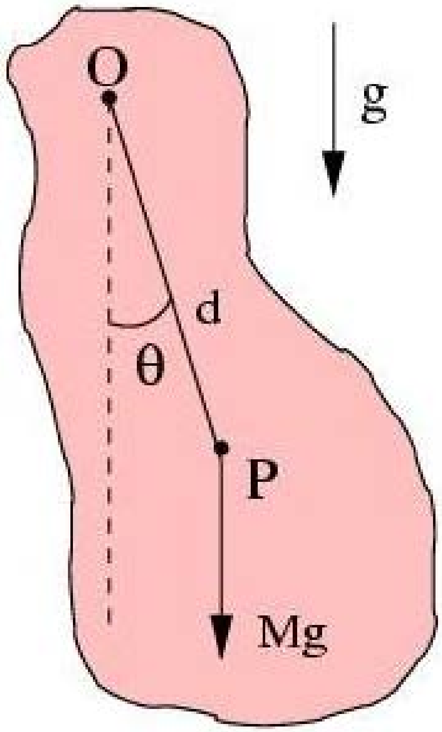
$$\sin 4^\circ = 0.0698, \qquad 4^\circ = 0.0698 \text{ rad}$$

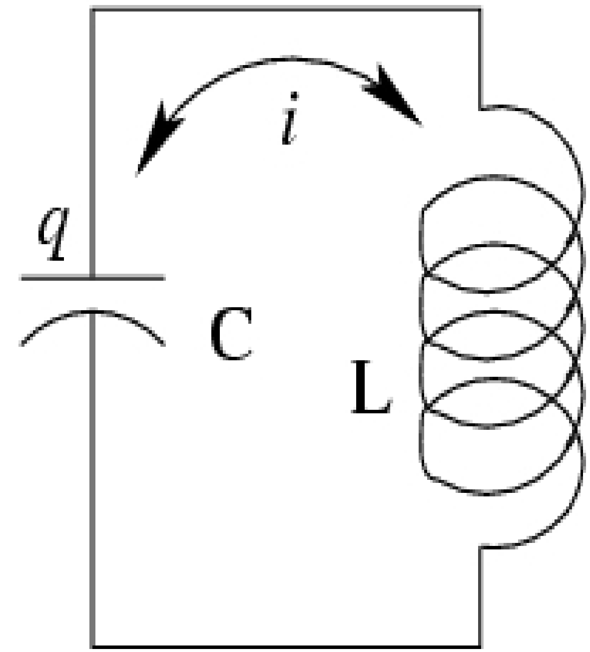


$$I\ddot{\theta} = -\kappa\theta,$$

$$\ddot{\theta} + \frac{\kappa}{I}\theta = 0,$$

$$T = 2\pi\sqrt{\frac{I}{\kappa}}.$$





$$I\ddot{\theta} = -Mgd\sin\theta \qquad L\ddot{q} + q/C = 0$$

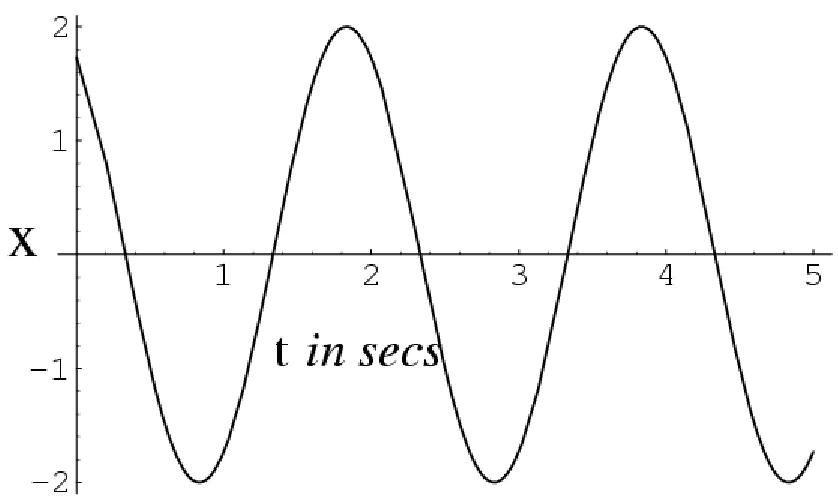
$$\omega_0 = \sqrt{\frac{Mgd}{I}} \qquad \omega_0 = \sqrt{\frac{1}{LC}}$$

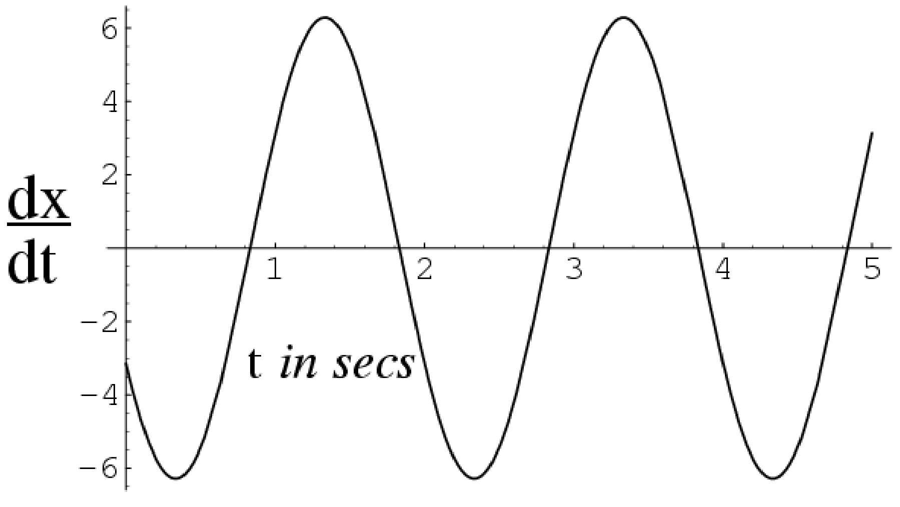
Solution:

$$x(t) = A\cos(\omega_0 t + \phi)$$

$$\dot{x}(t) = -A\omega_0 \sin(\omega_0 t + \phi)$$

A=Amplitude, ϕ =Phase





A=2 units $\phi = 30^\circ$ $\omega_0 = \pi \text{ rad/sec}$

Exponential solution:

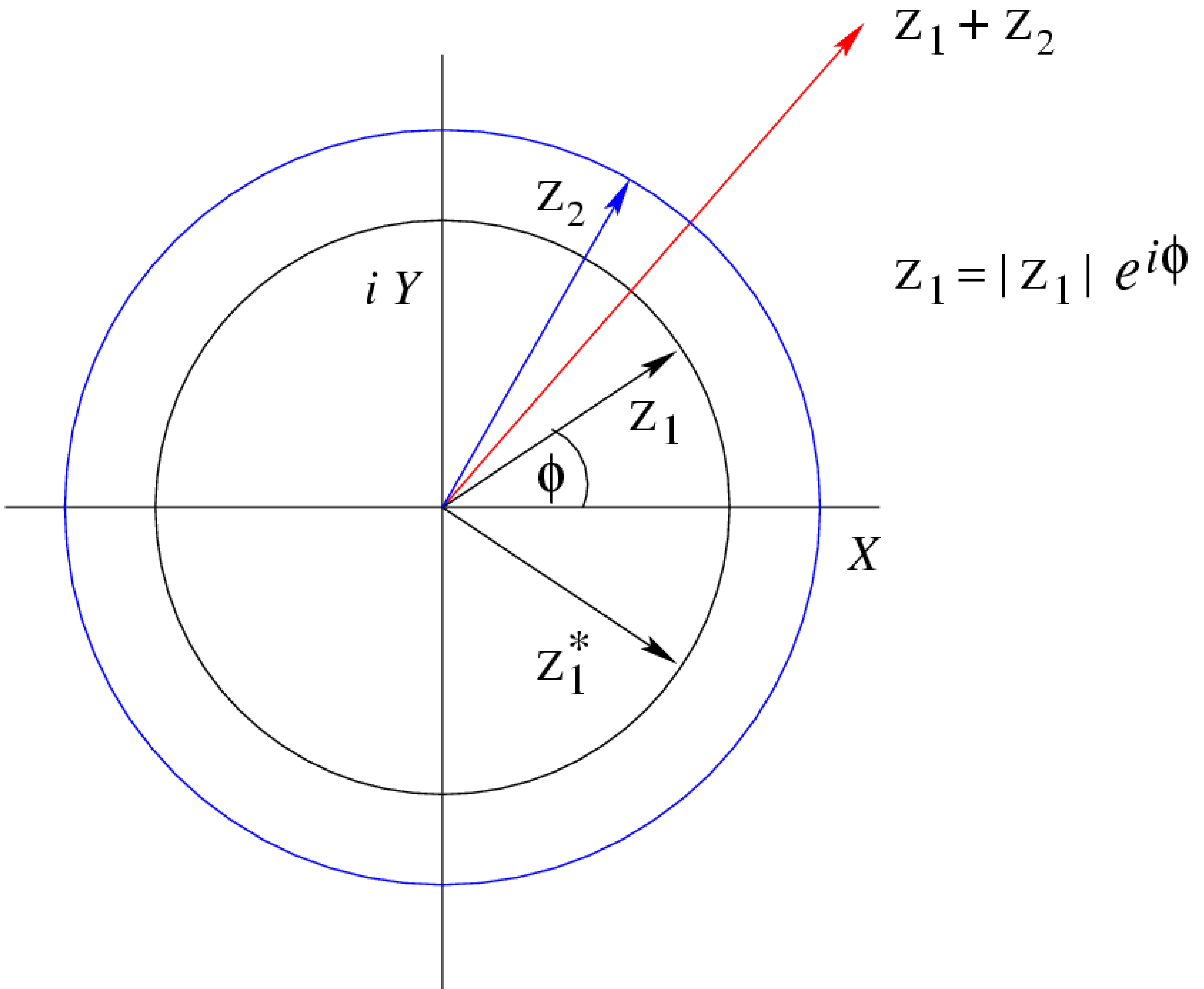
$$z(t) = A\exp(i\omega_0 t)$$

A=Complex amplitude

$$A = |A|\exp(i\phi)$$

Real and imaginary parts of z(t) satisfy simple harmonic equation of motion

$x(t)$=Re $z(t)$



$z_1 + z_2$
z_2
$z_1 = |z_1|\ e^{i\phi}$
$i\ Y$
z_1
ϕ
X
z_1^*

Additions of two SHMs become convenient

$$z_1(t) = A_1 \exp(i\omega_0 t)$$
$$z_2(t) = A_2 \exp(i\omega_0 t)$$

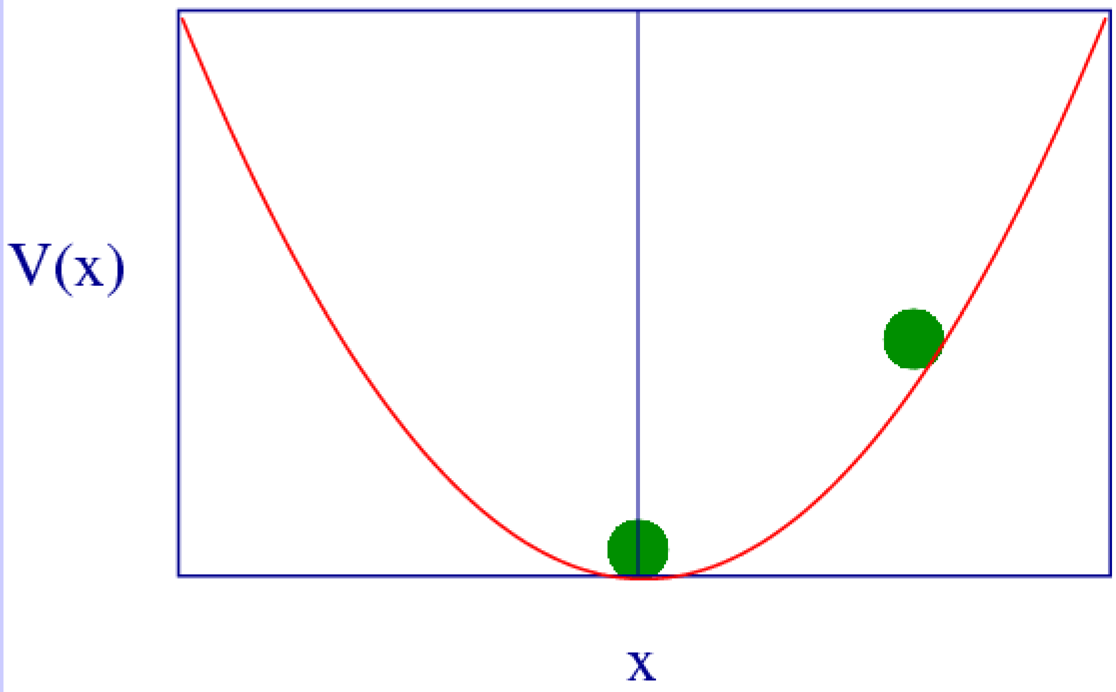
$$z(t) = z_1(t) + z_2(t) = (A_1 + A_2) \exp(i\omega_0 t)$$

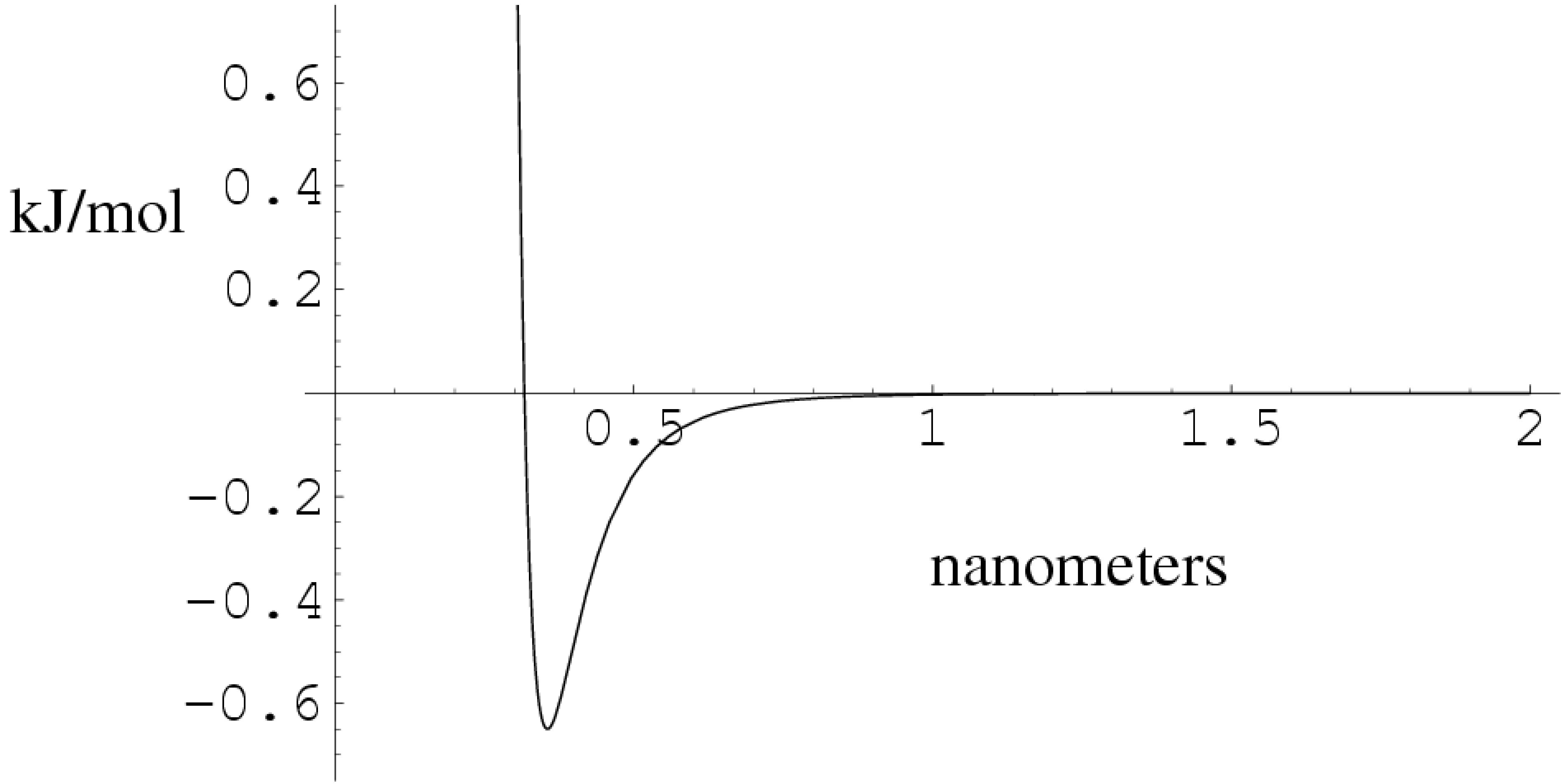
For,
$$A_1 = |A| \exp(i\phi_1)$$
$$A_2 = |A| \exp(i\phi_2)$$

$$z(t) = z_1 + z_2$$
$$= |A| \left[\ \exp(i\phi_1) + \exp(i\phi_2)\right]\ \exp(i\omega_0 t)$$

Harmonic oscillator potential:

$$V(x) = \frac{1}{2}kx^2 \qquad k = +\text{ve}$$





Lennard Jones potential, $V(r) \sim (\frac{a}{r})^{12} - (\frac{a}{r})^6$, for water

Arbitrary potentials can be well approximated by harmonic oscillator potentials near minima

Consider a potential *V(x)*. Let us assume a minimum at $x=x_0$, Taylor expand around $x=x_0$.

$$V(x) = V(x_0) + (x - x_0)V'(x_0) + \frac{1}{2!}(x - x_0)^2 V''(x_0) + \cdots,$$

$V'(x_0)$ is zero and $V''(x_0)$ is positive since x_0 is a minimum of $V(x)$.

TAYLOR EXPANSION

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$$

$$f(0) = a_0$$

$$f'(x) = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots$$

$$f'(0) = a_1$$

$$f''(x) = 2a_2 + 3 \cdot 2a_3 x + 4 \cdot 3a_4 x^2 + \cdots$$

$$f''(0) = 2a_2$$

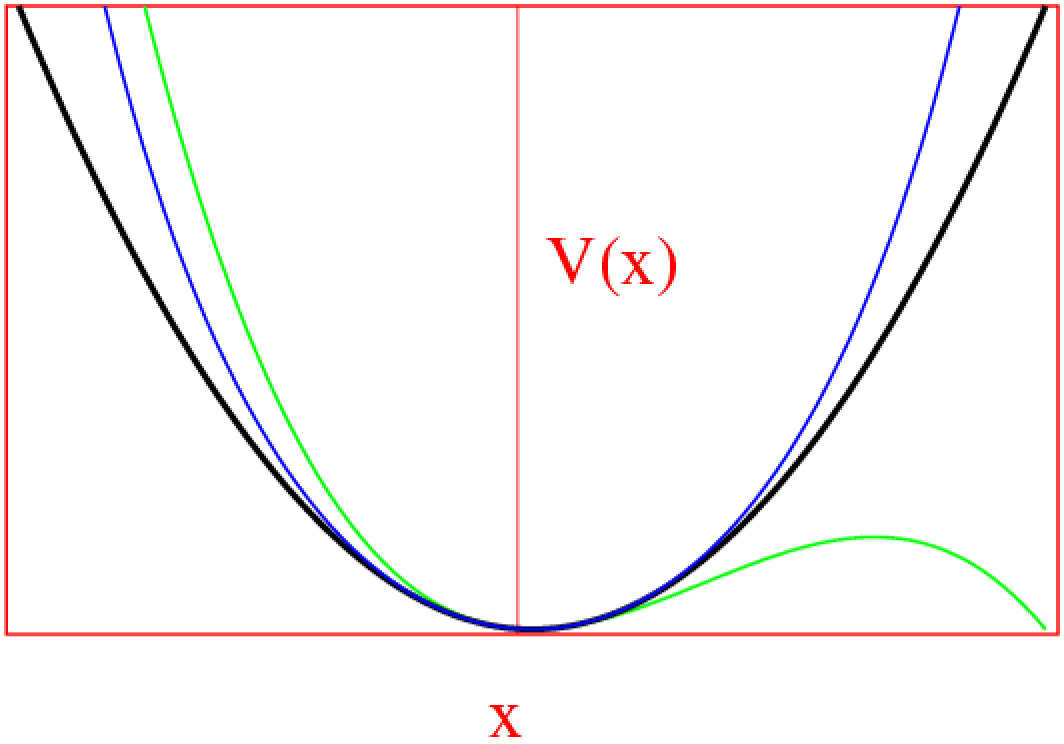
$$a_n = f^{(n)}(0)/n!$$

TAYLOR EXPANSION

$$f(x) = f(0) + x f'(0) + \frac{1}{2!} x^2 f''(0) + \frac{1}{3!} x^3 f'''(0) + \cdots$$

Problem: Show in general:

$$f(a+h) = f(a) + h f'(a) + \frac{1}{2!} h^2 f''(a) + \frac{1}{3!} h^3 f'''(a) + \cdots \frac{1}{n!} h^n f^{(n)}(a) \cdots$$



$$V(x) = x^2$$

$$V(x) = \exp(x^2/2) - 1$$

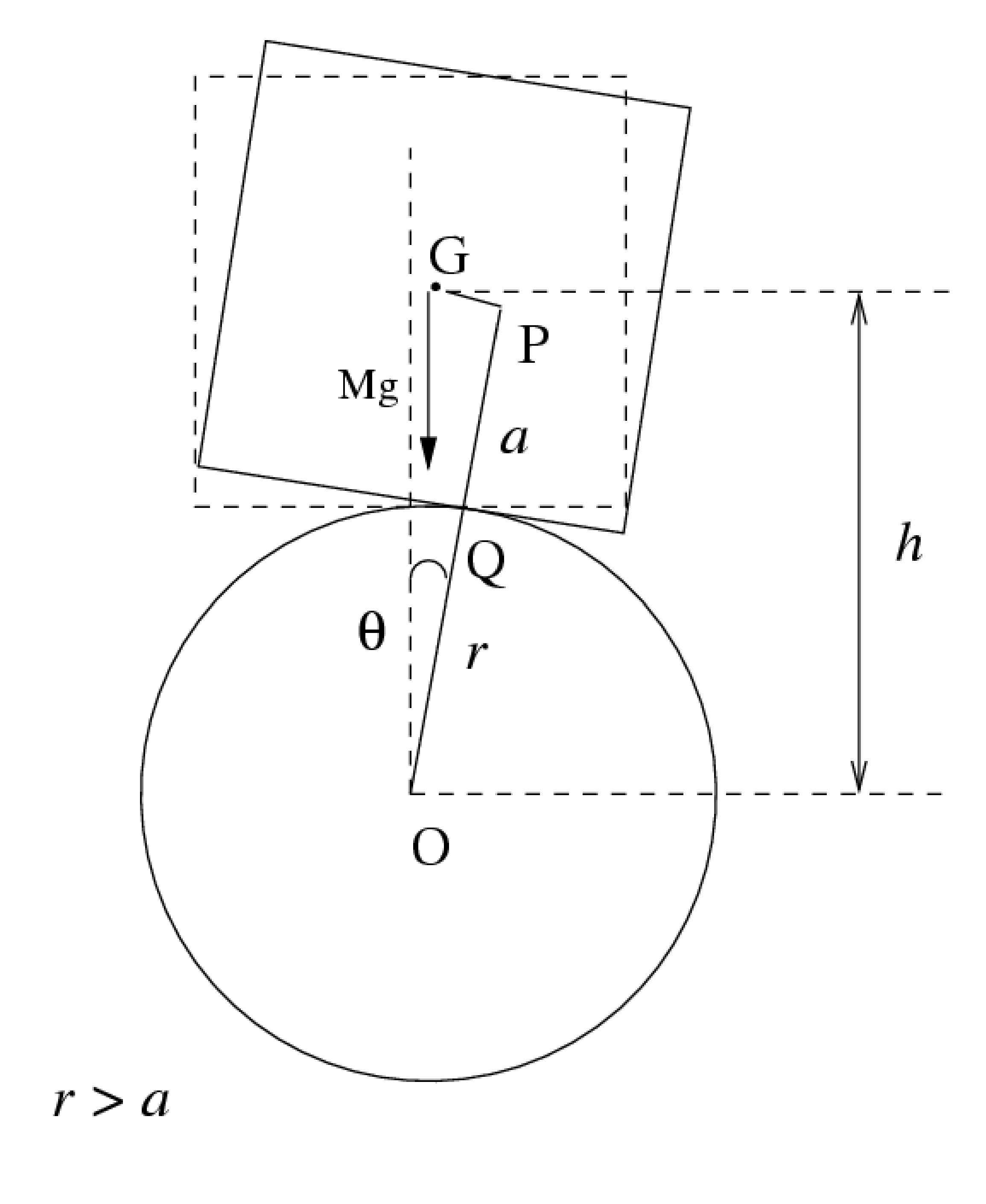
$$V(x) = x^2 - x^3$$

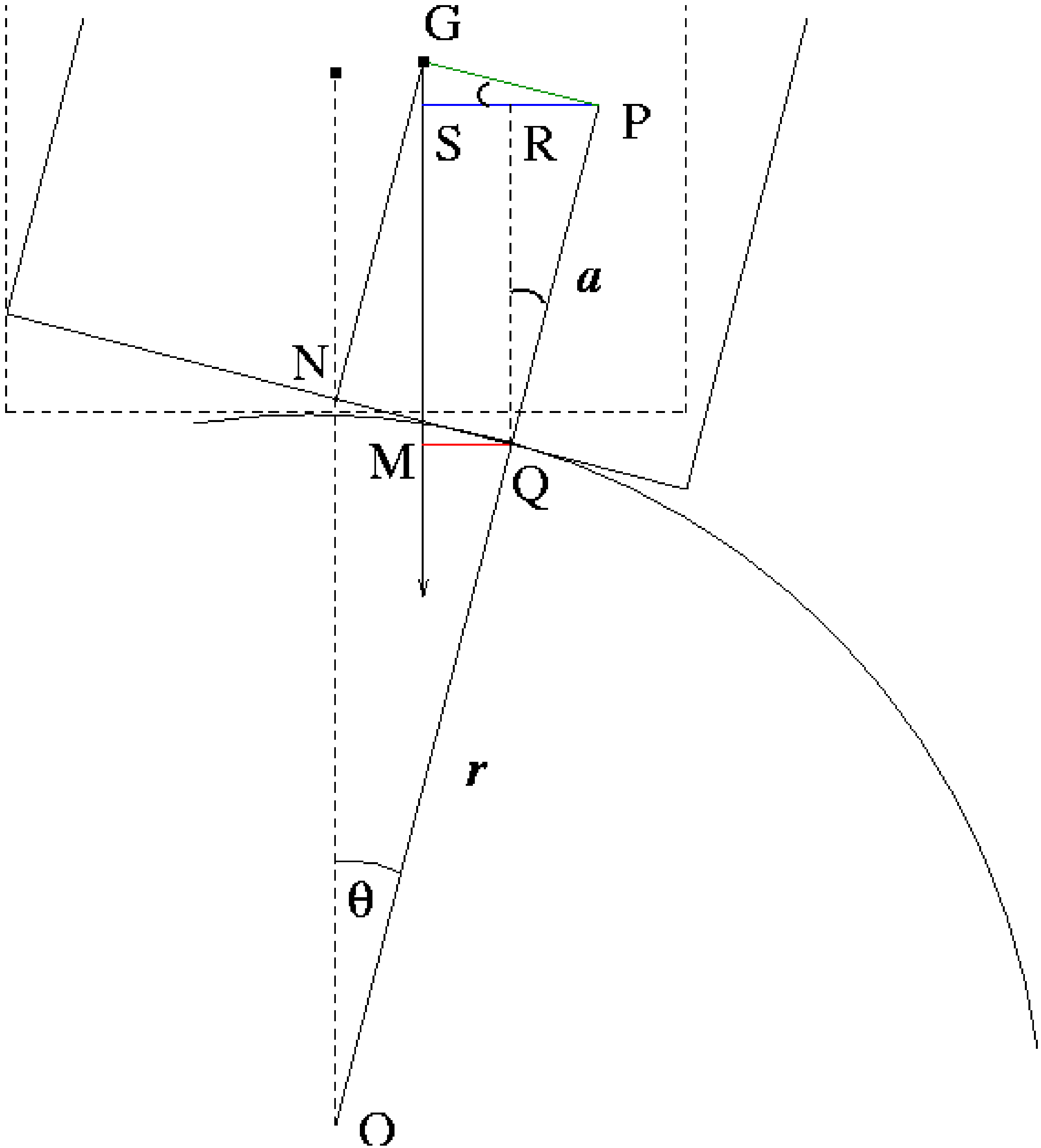
Cube of mass M and side *2a*

Cylinder of radius *r*, fixed along a horizontal axis

Cube rocks on the cylinder for small angles, does not slip

Find time period





Torque about Q

= Mg×QM

= Mg×(PS-PR)

PS=$r\theta\cos\theta$

PR=$a\sin\theta$

Torque about Q:

$$I\ddot{\theta} = -Mg(r\theta\cos\theta - a\sin\theta)$$

Moment of inertia of the cube about a horizontal axis passing through Q is I.

I = $\frac{5}{3}Ma^2$. Using small angle approximation,

$$\frac{5}{3}Ma^2\ddot{\theta} + Mg(r-a)\theta = 0$$

$$T = 2\pi\sqrt{\frac{5a^2}{3(r-a)g}}$$

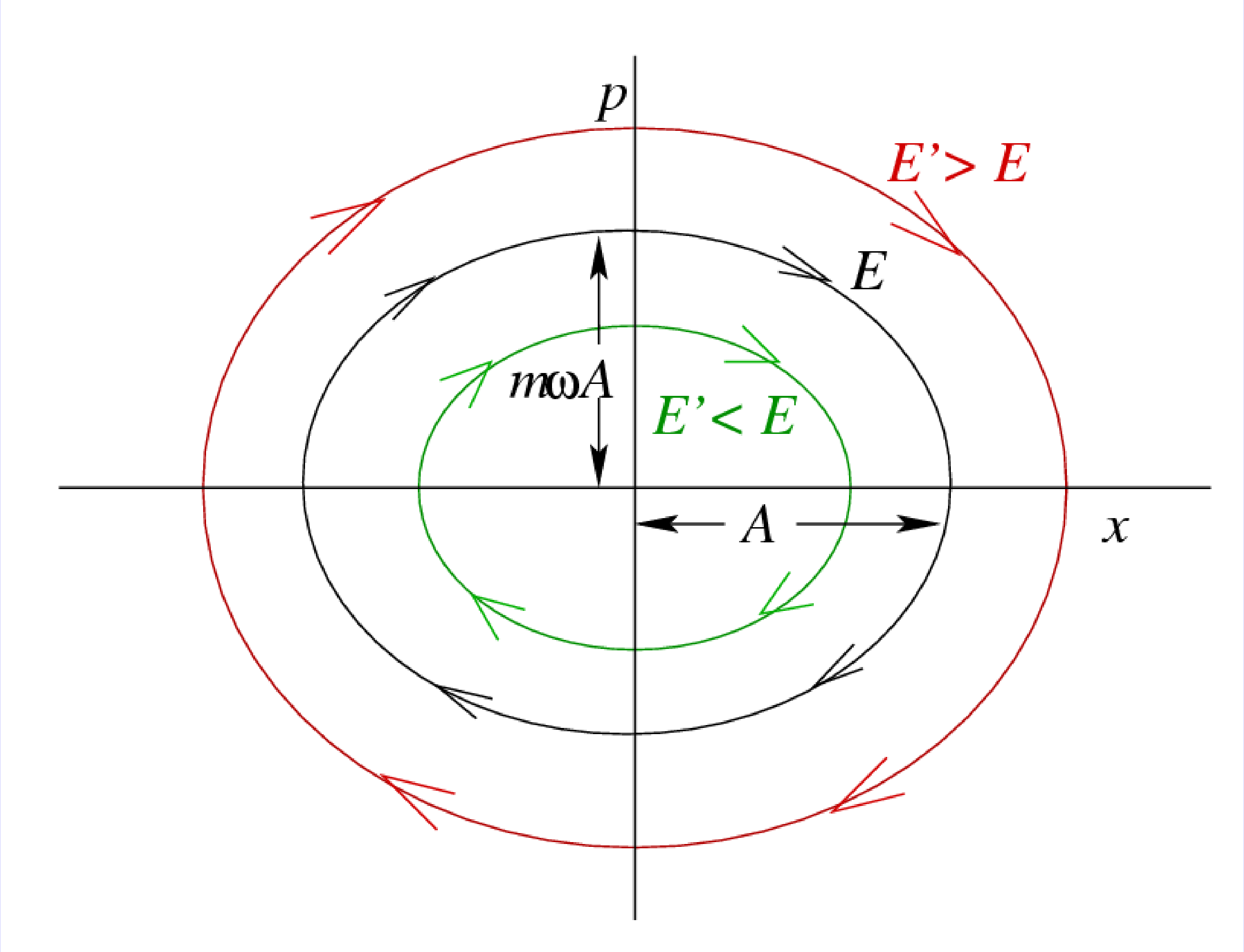
Energy of simple harmonic oscillator:

$$k.e. = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}m\omega_0^2 A^2 \sin^2(\omega_0 t + \phi) = \frac{1}{2}kA^2 \sin^2(\omega_0 t + \phi)$$

$$p.e. = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega_0 t + \phi) = \frac{1}{2}m\omega_0^2 A^2 \cos^2(\omega_0 t + \phi)$$

$$E = k.e. + p.e. = \frac{1}{2}kA^2 = \frac{1}{2}m\omega_0^2 A^2$$

Phase space:



$$E = \frac{1}{2}kA^2 \quad = \quad p^2/2m + \frac{1}{2}kx^2$$

$$\text{or, } 1 \quad = \quad \frac{p^2}{m^2\omega_0^2 A^2} + \frac{x^2}{A^2}$$