Resonance: $\omega = \omega_0 - \Delta\omega$

$$
\begin{aligned}
x(t) &= A(\sin\omega_0 t \cos\Delta\omega t \\
&\quad - \cos\omega_0 t \sin\Delta\omega t - \frac{\omega}{\omega_0}\sin\omega_0 t)
\end{aligned}
$$

$$
x(t) = A\left(\frac{(\omega_0 - \omega)}{\omega_0}\sin\omega_0 t - \Delta\omega t\cos\omega_0 t\right)
$$

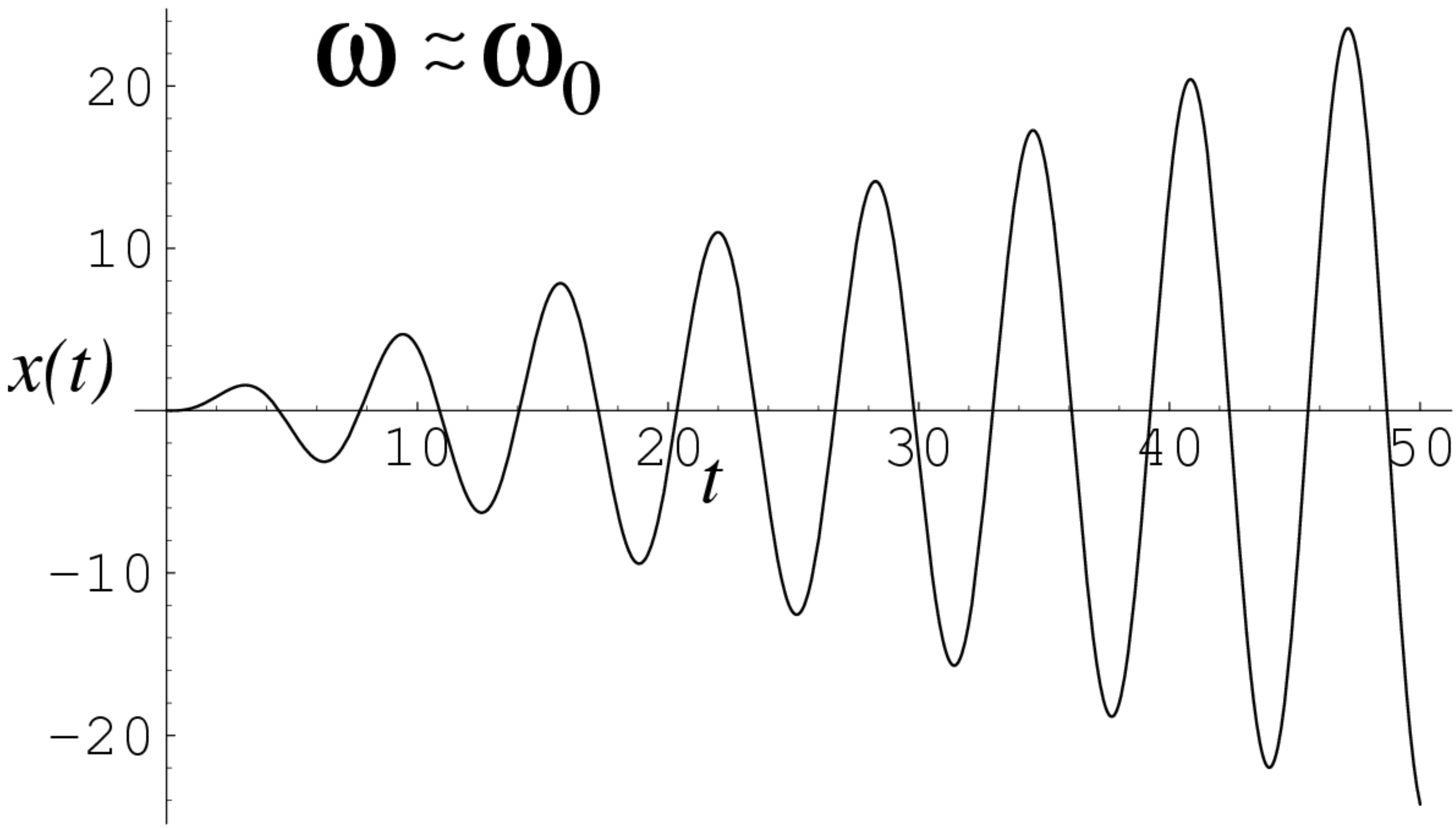
$\cos\Delta\omega t \approx 1$ and $\sin\Delta\omega t \approx \Delta\omega t$

Substituting, $A = \frac{f_0}{(\omega_0^2 - \omega^2)}$

$$x(t) = \frac{f_0}{\omega_0(\omega_0 + \omega)}(\sin \omega_0 t - \omega_0 t \cos \omega_0 t)$$

$$\approx \frac{f_0}{2\omega_0^2}(\sin \omega_0 t - \omega_0 t \cos \omega_0 t)$$

Near resonance:

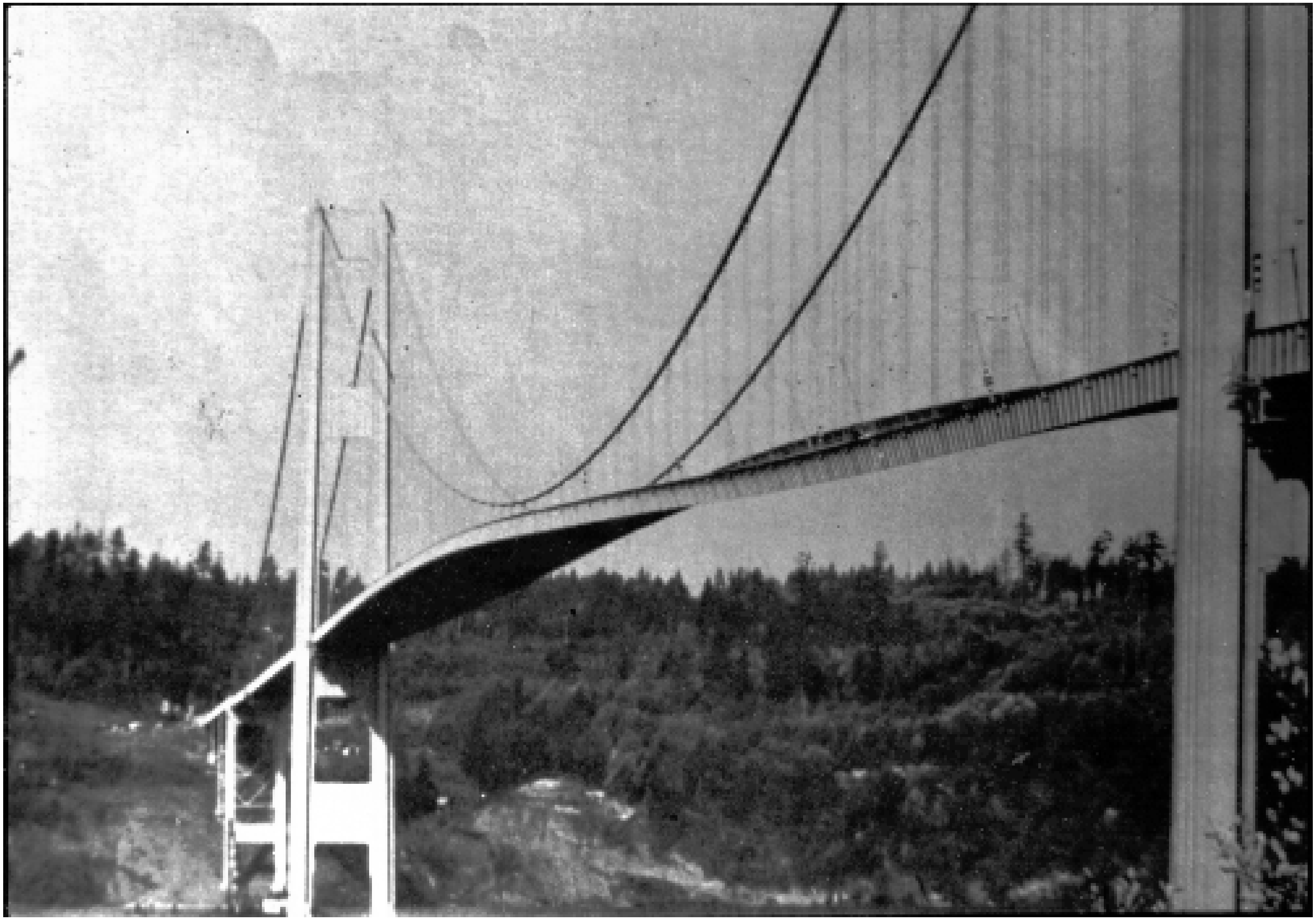


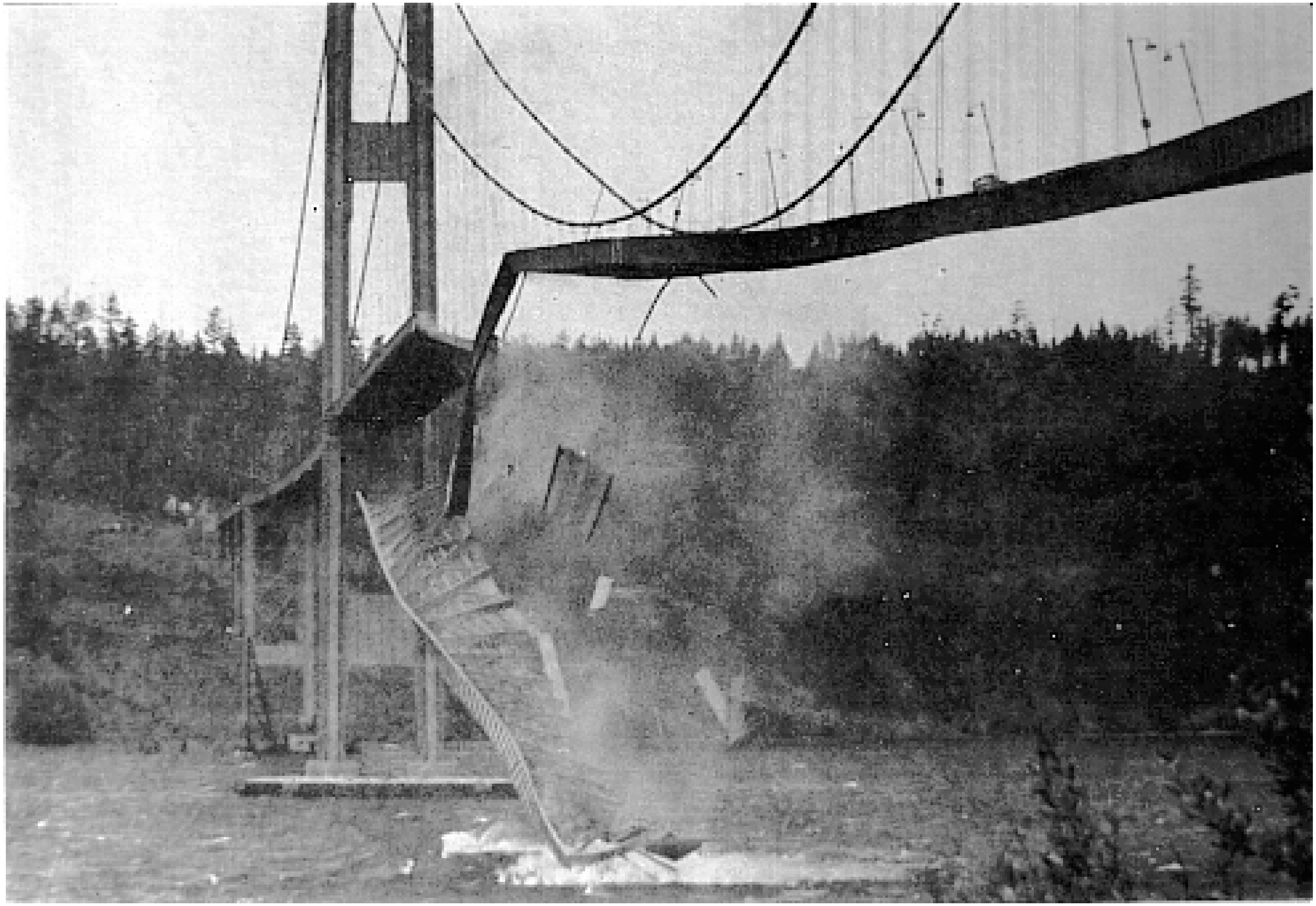
Tacoma Narrows bridge pictures and movies are taken from the following sites. These are used for purely educational purpose.

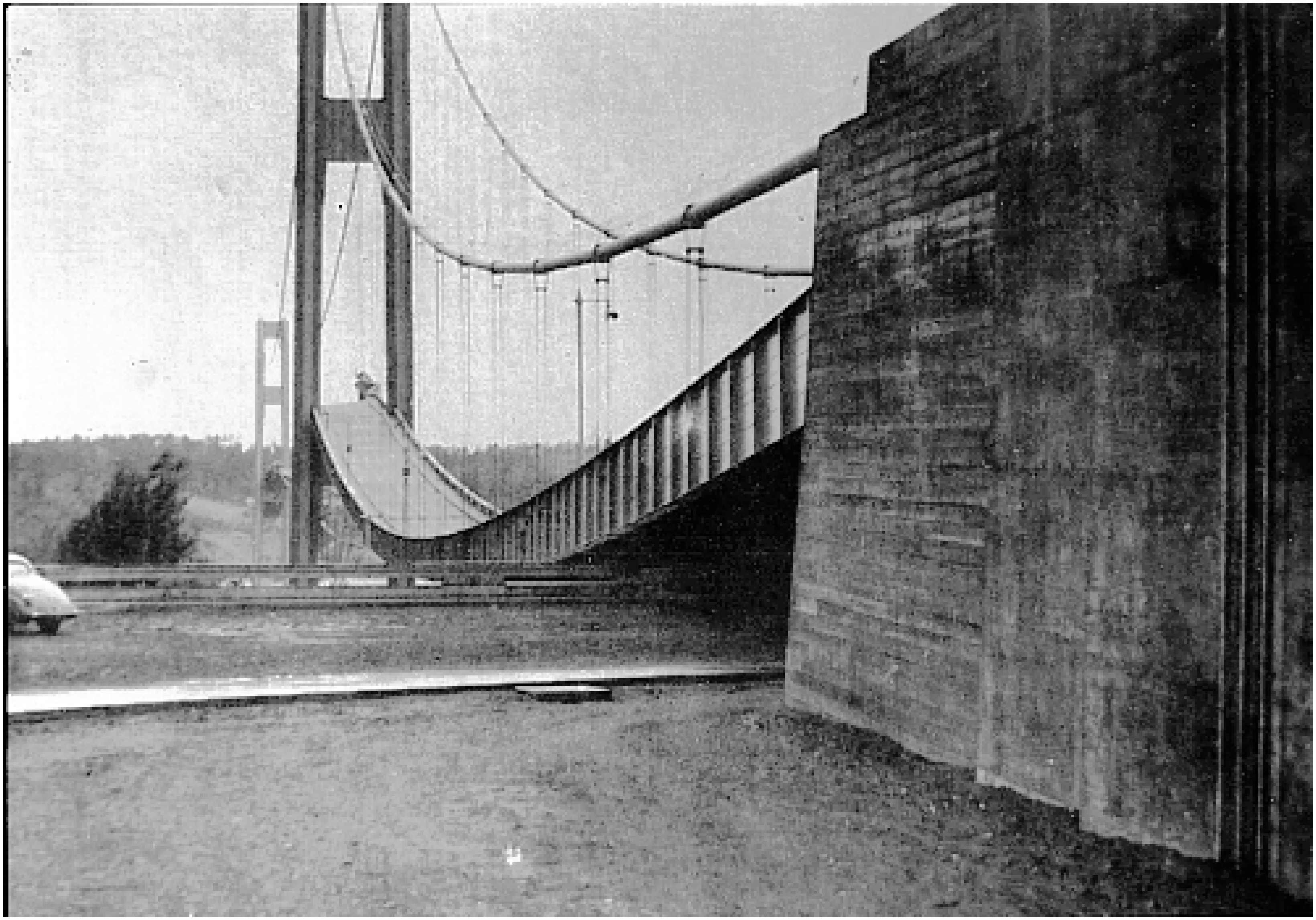
http://www.enm.bris.ac.uk/anm/tacoma/tacoma.html

http://www.archive.org/details/Pa2096Tacoma

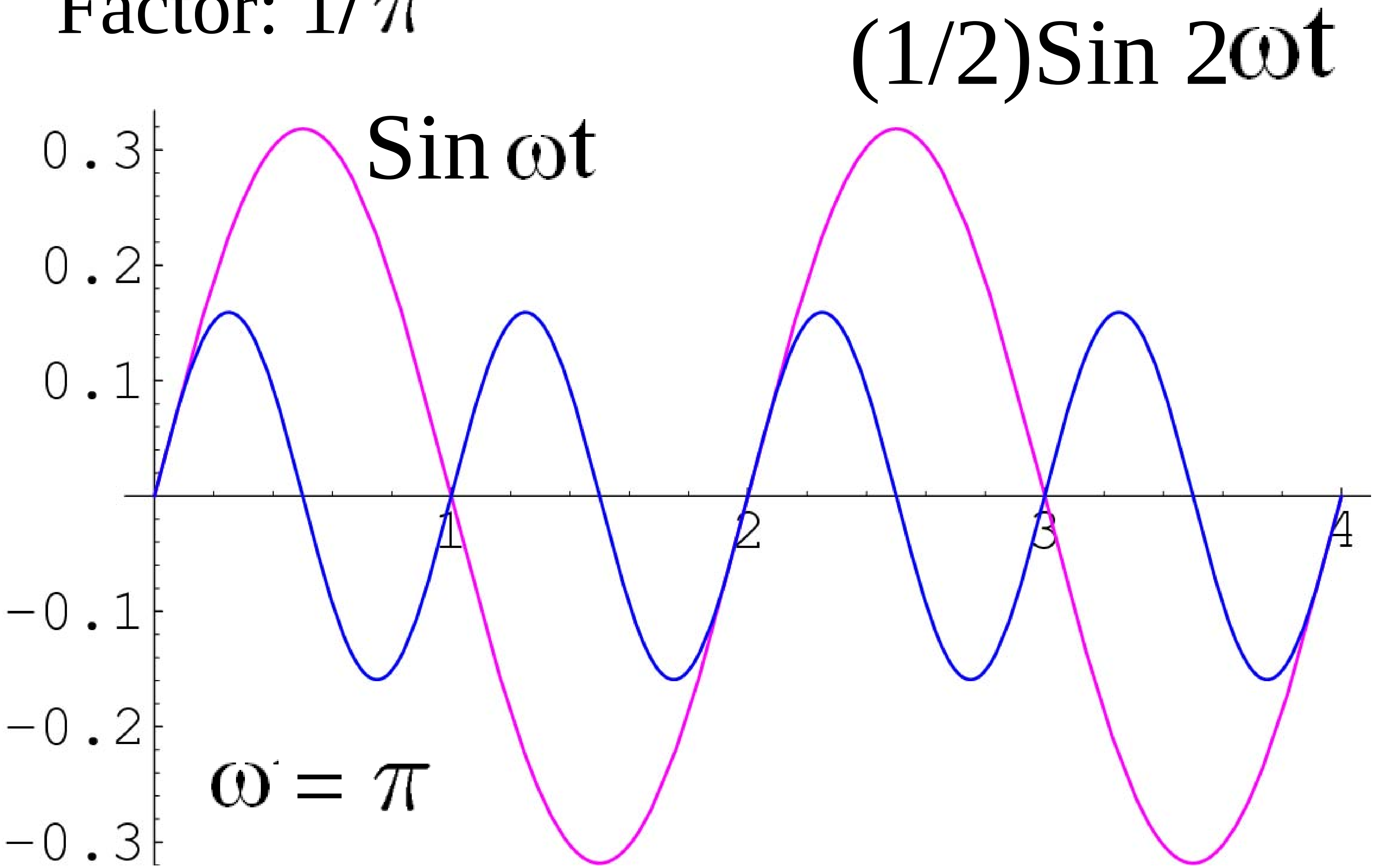
http://www.blogtelevision.net/index.php?p=
Videos-Browse-by-Video-Screen-Shots___1,62&offset=32



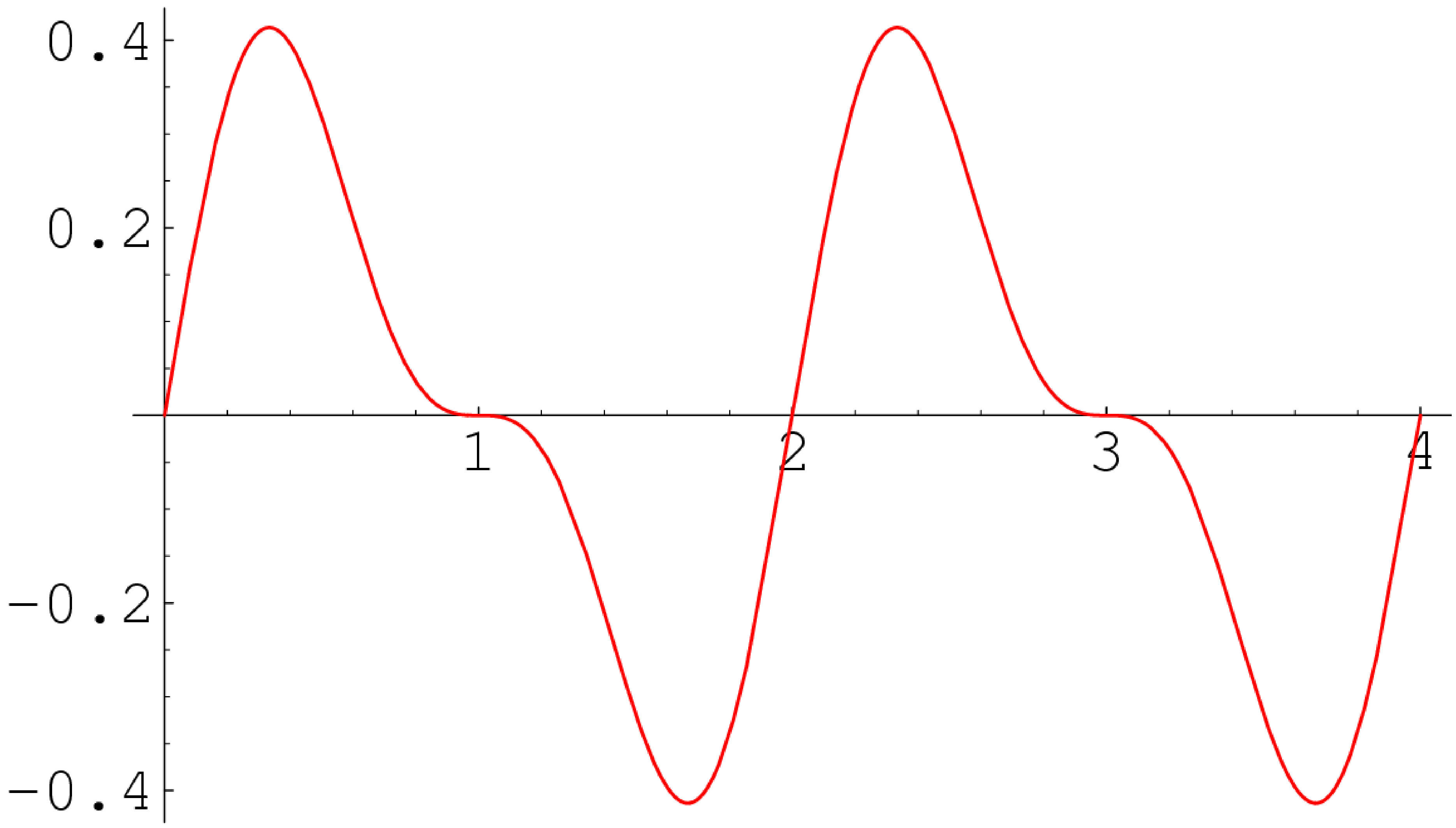




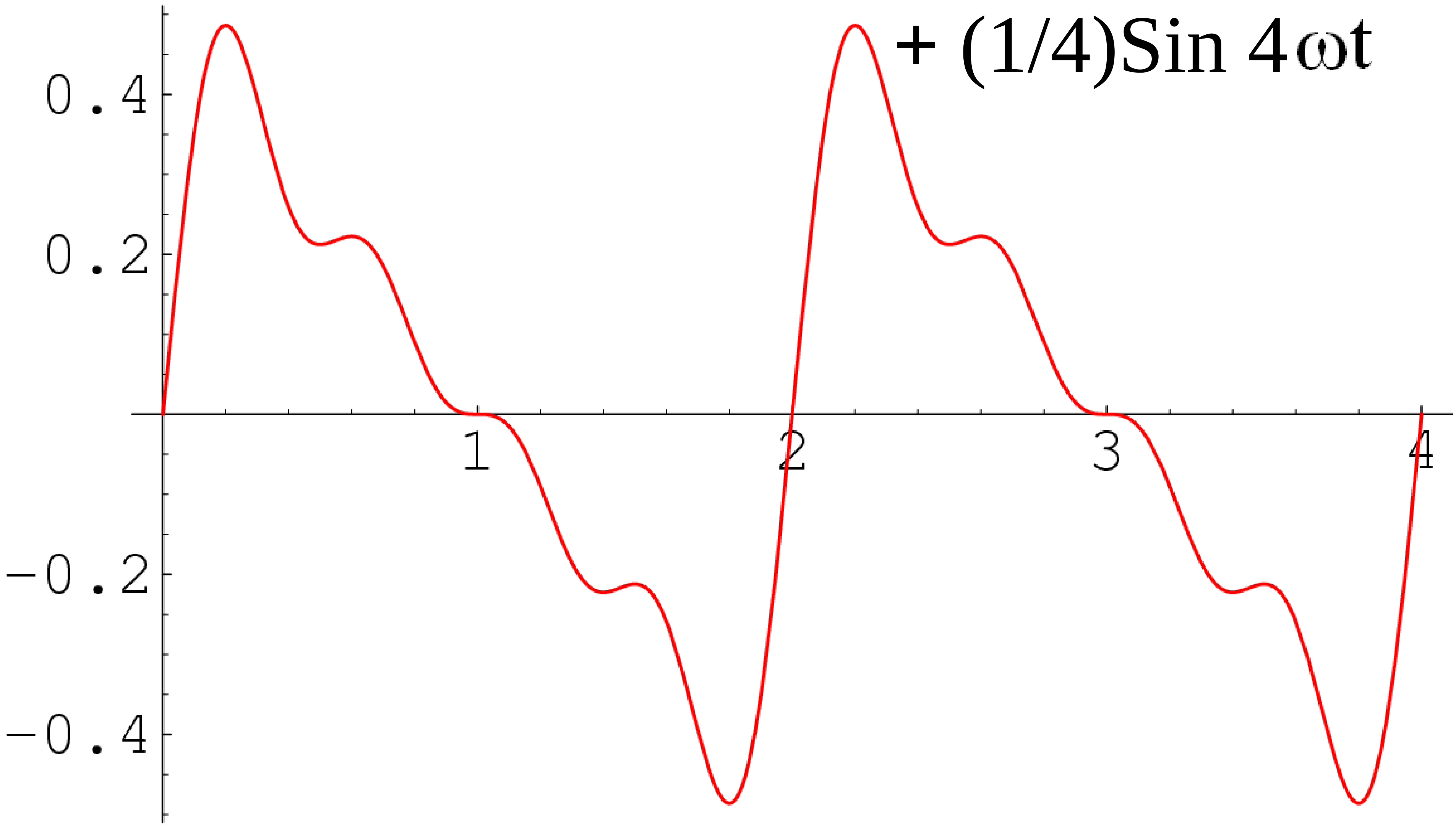
Factor: $1/\pi$



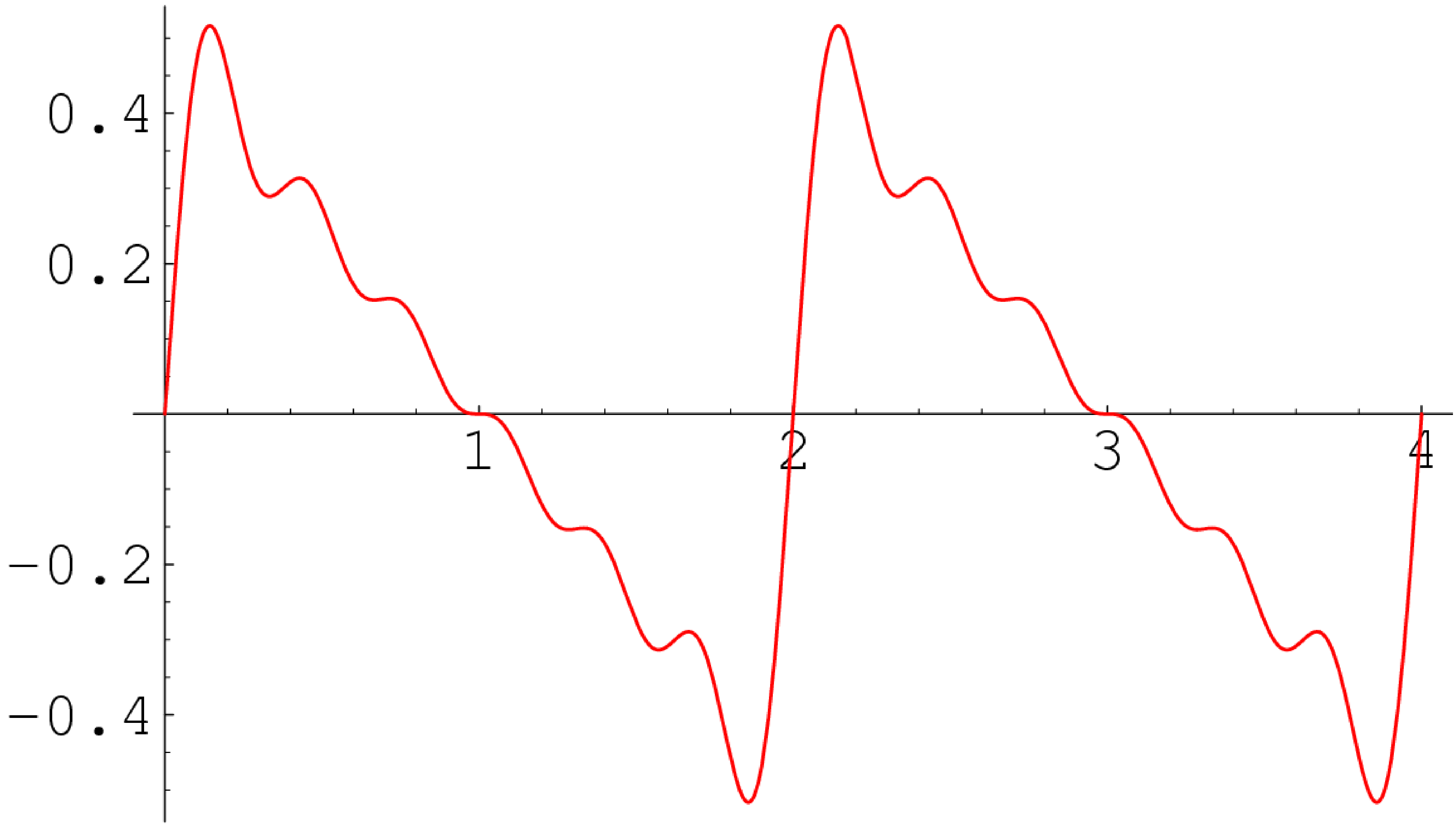
$$\sin \omega t + (1/2)\sin 2\omega t$$



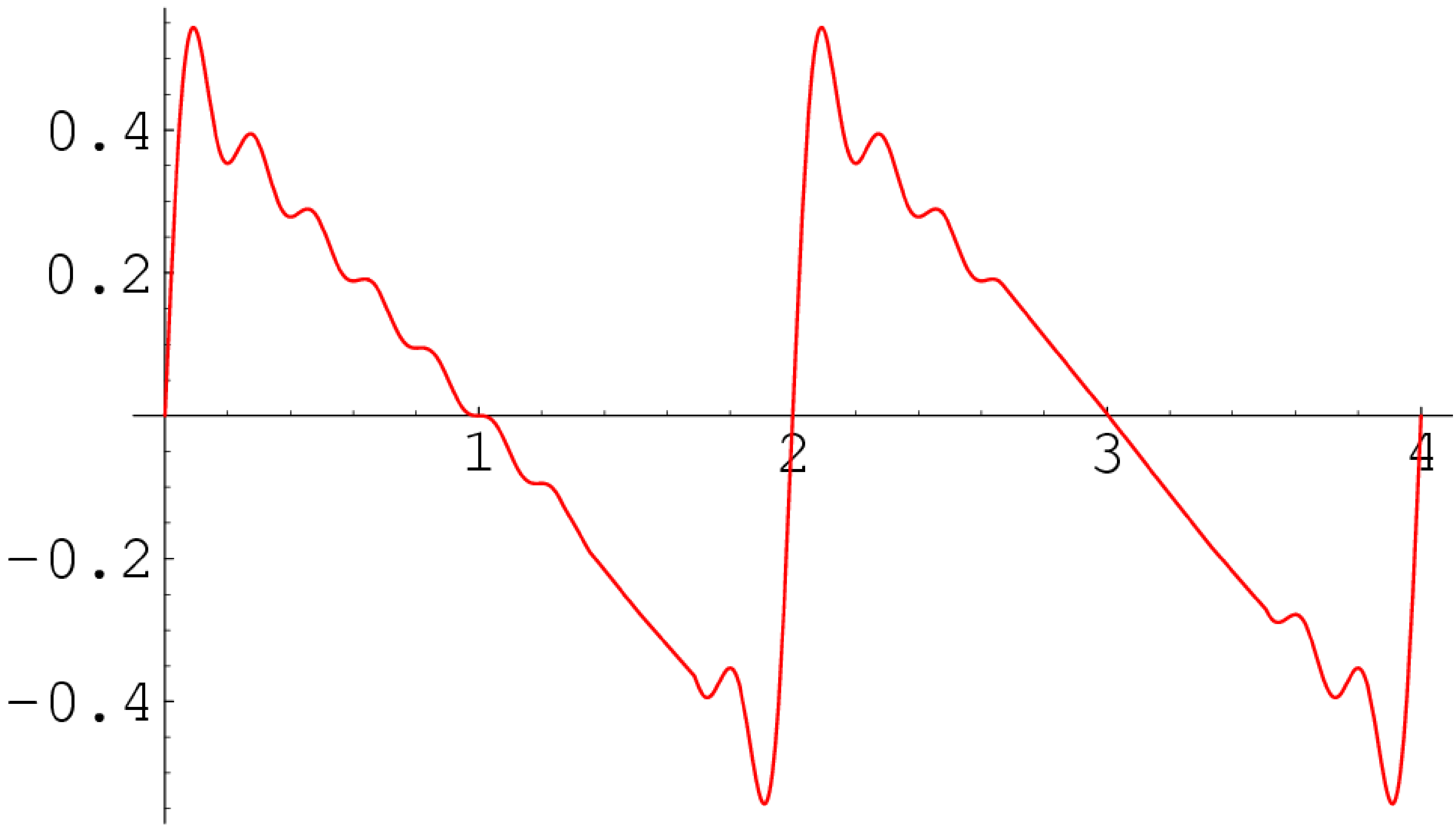
$$\sin\omega t + (1/2)\sin 2\omega t + (1/3)\sin 3\omega t + (1/4)\sin 4\omega t$$



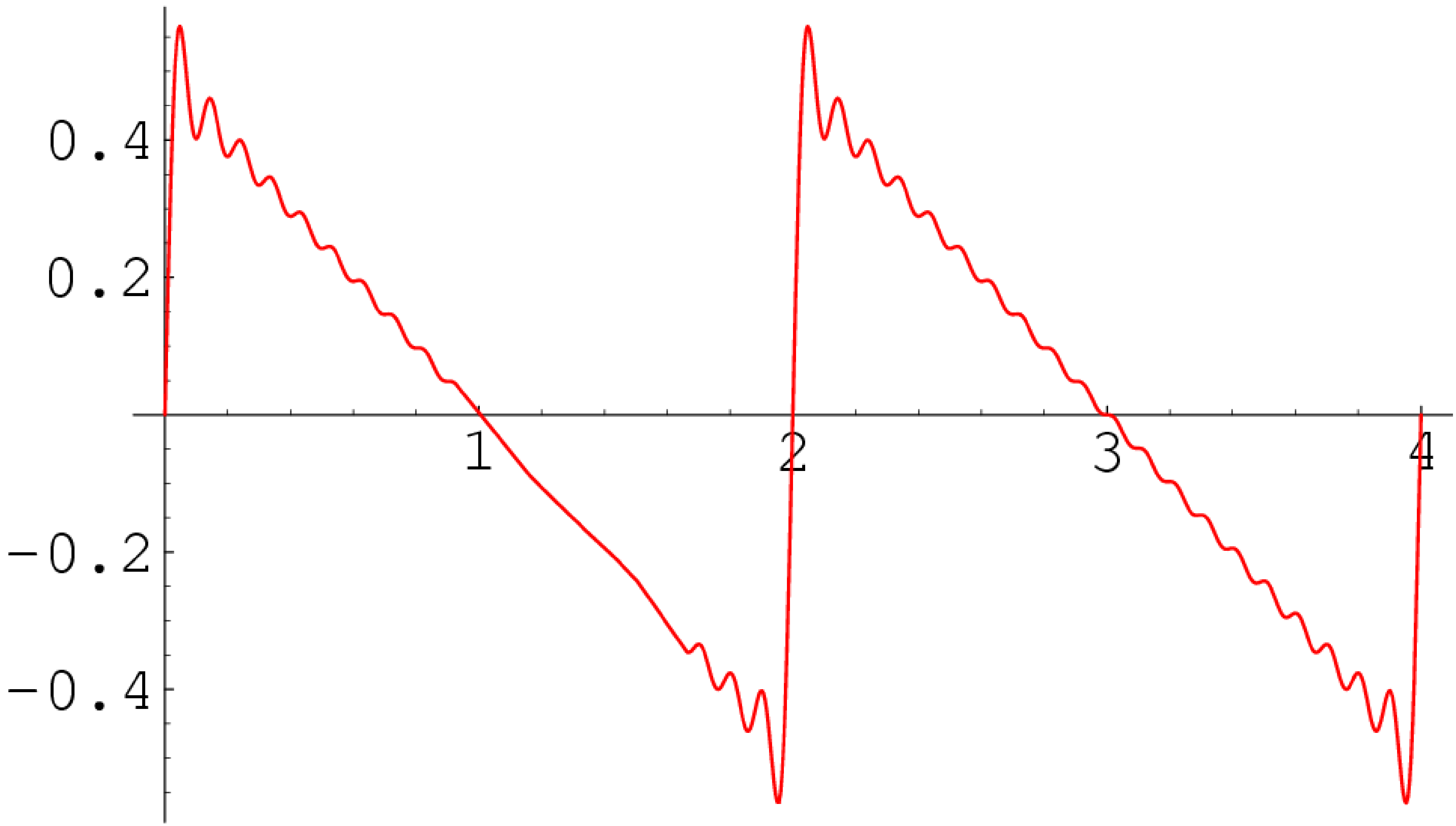
6 terms of the series

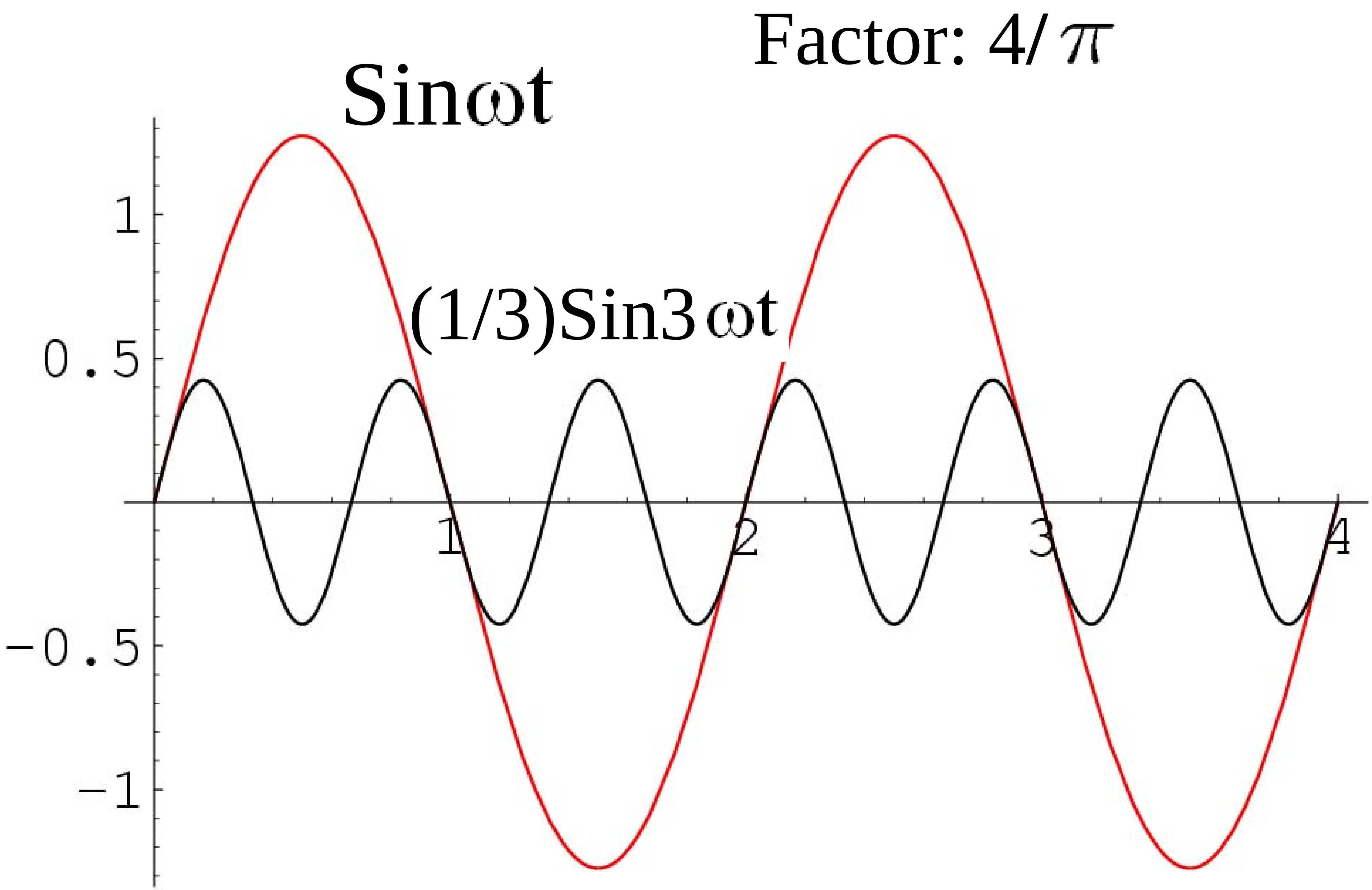


10 terms of the series

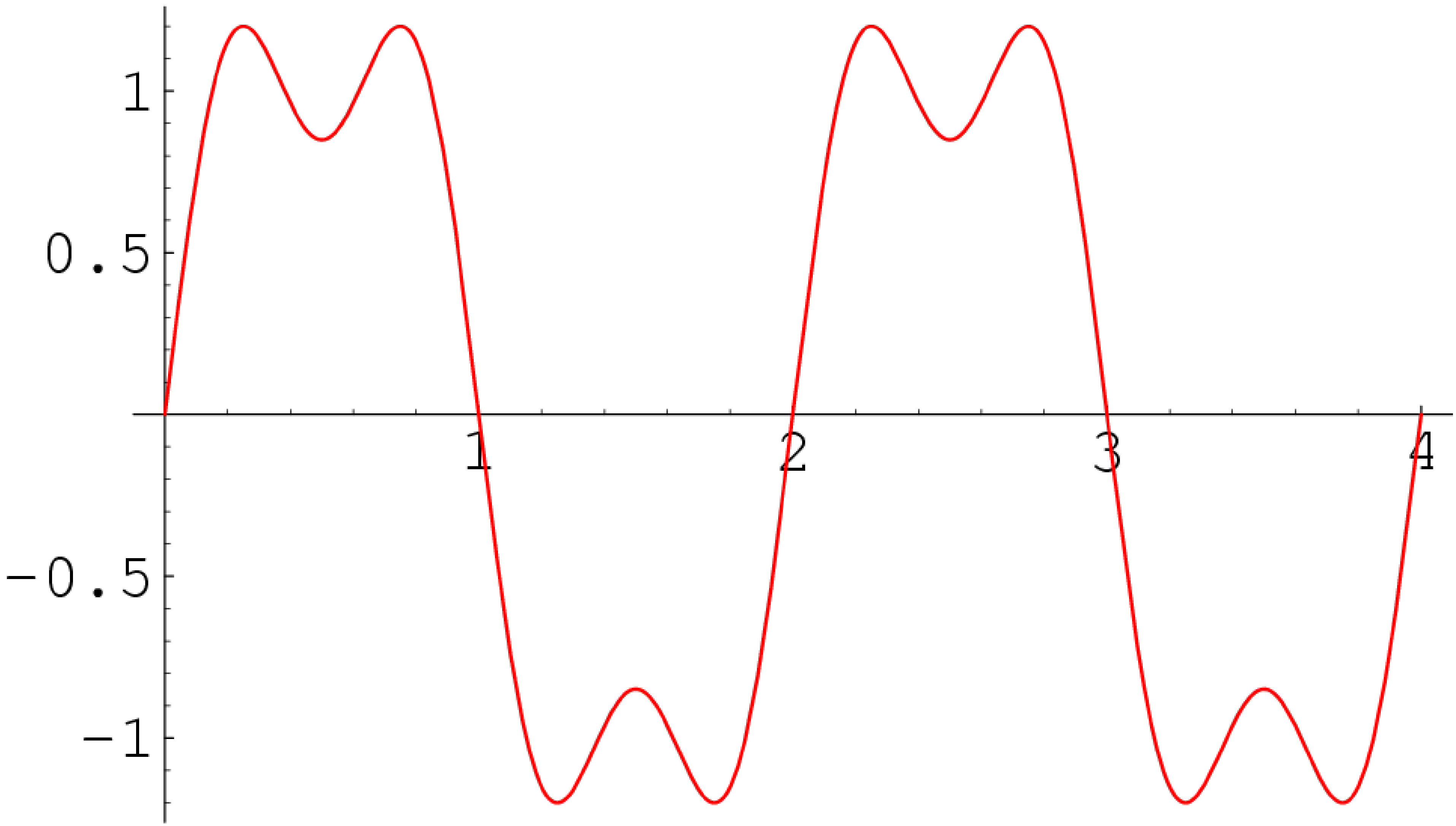


20 terms of the series

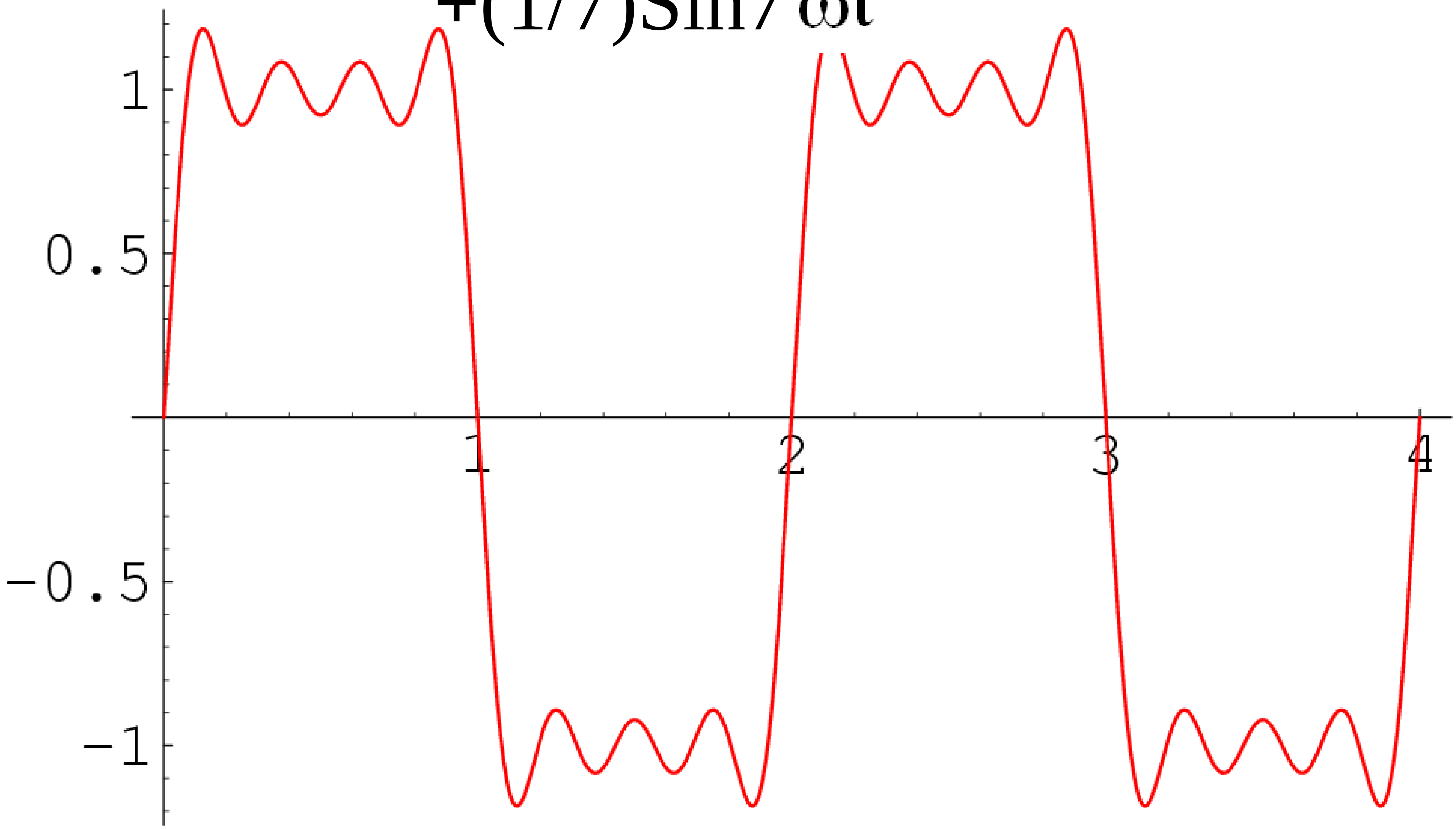



Factor: 4/π
Sinωt
(1/3)Sin3ωt
1
0.5
−0.5
−1
1
2
3
4

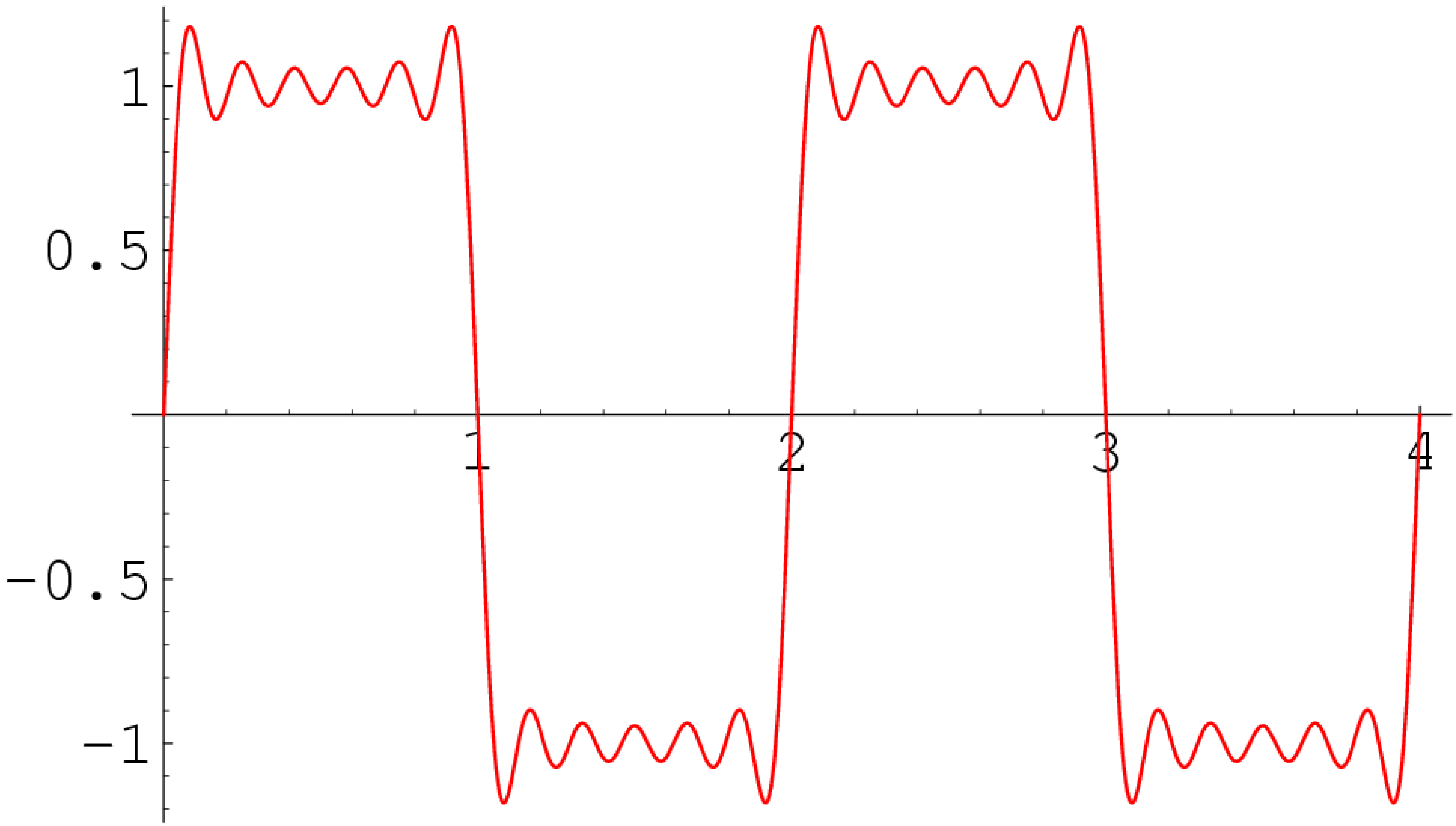
Sin ωt + (1/3)Sin3 ωt



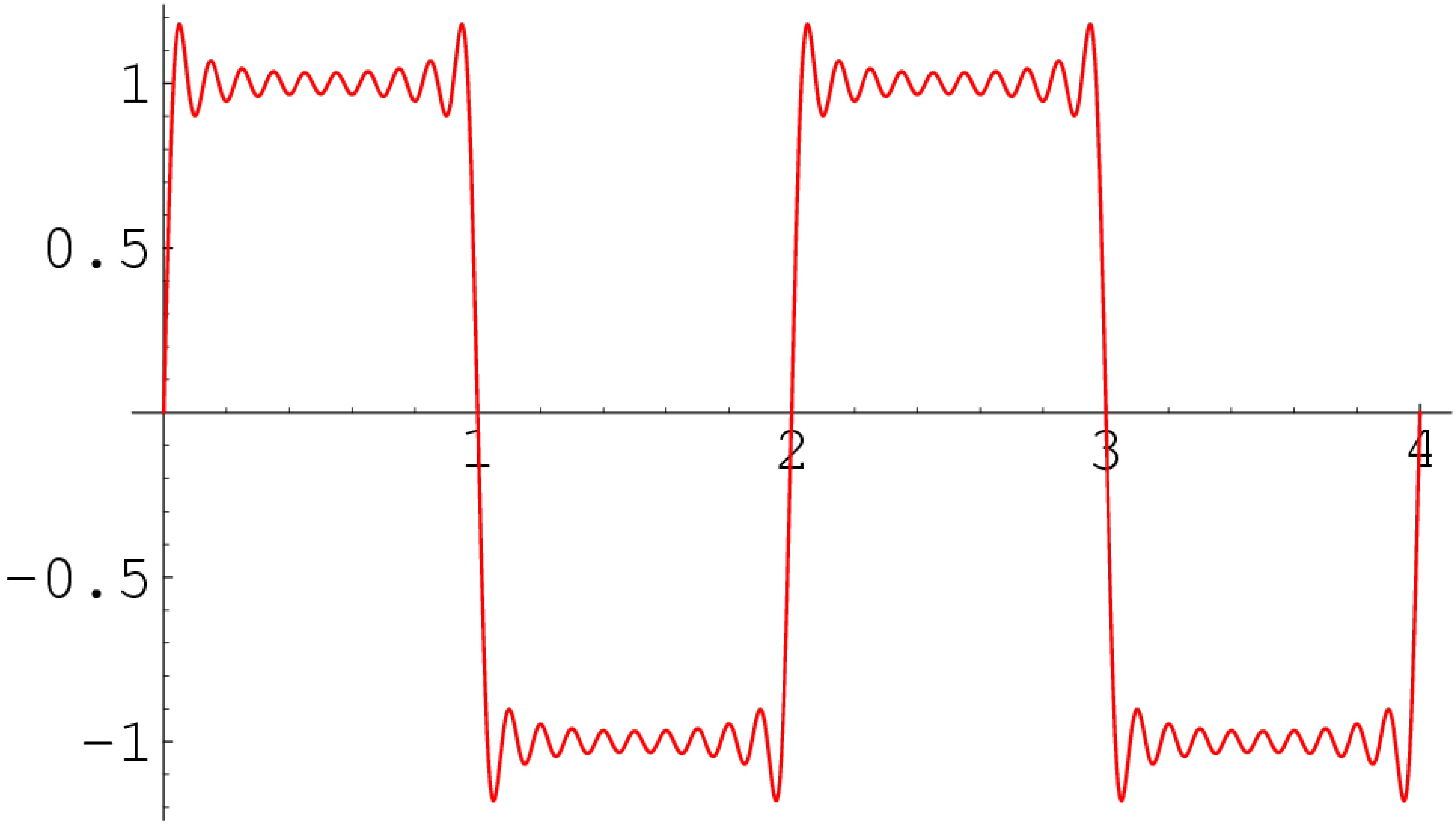
$$\mathrm{Sin}\,\omega t\ +(1/3)\mathrm{Sin}3\omega t +(1/5)\mathrm{Sin}5\,\omega t +(1/7)\mathrm{Sin}7\,\omega t$$



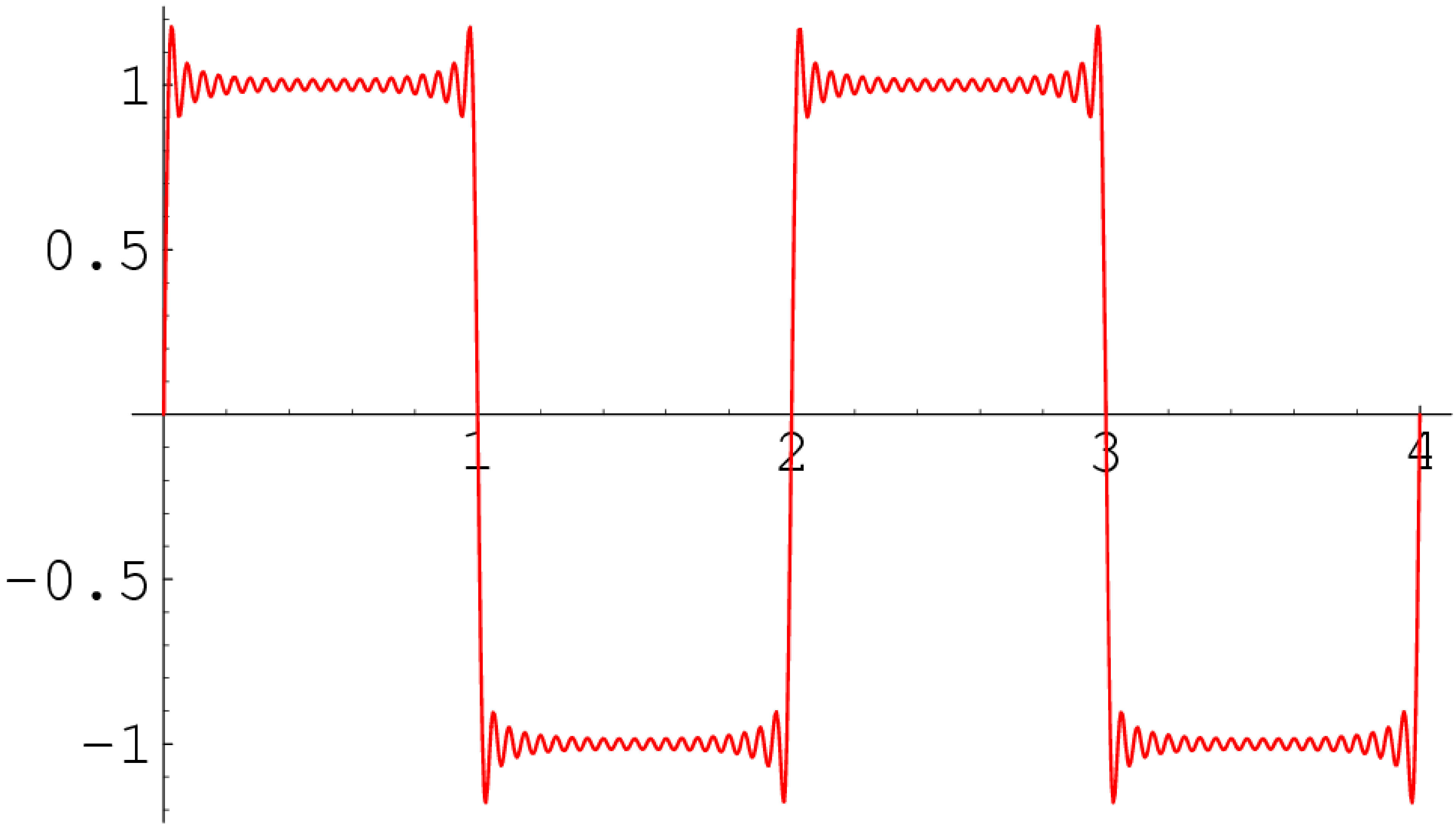
6 terms of the series



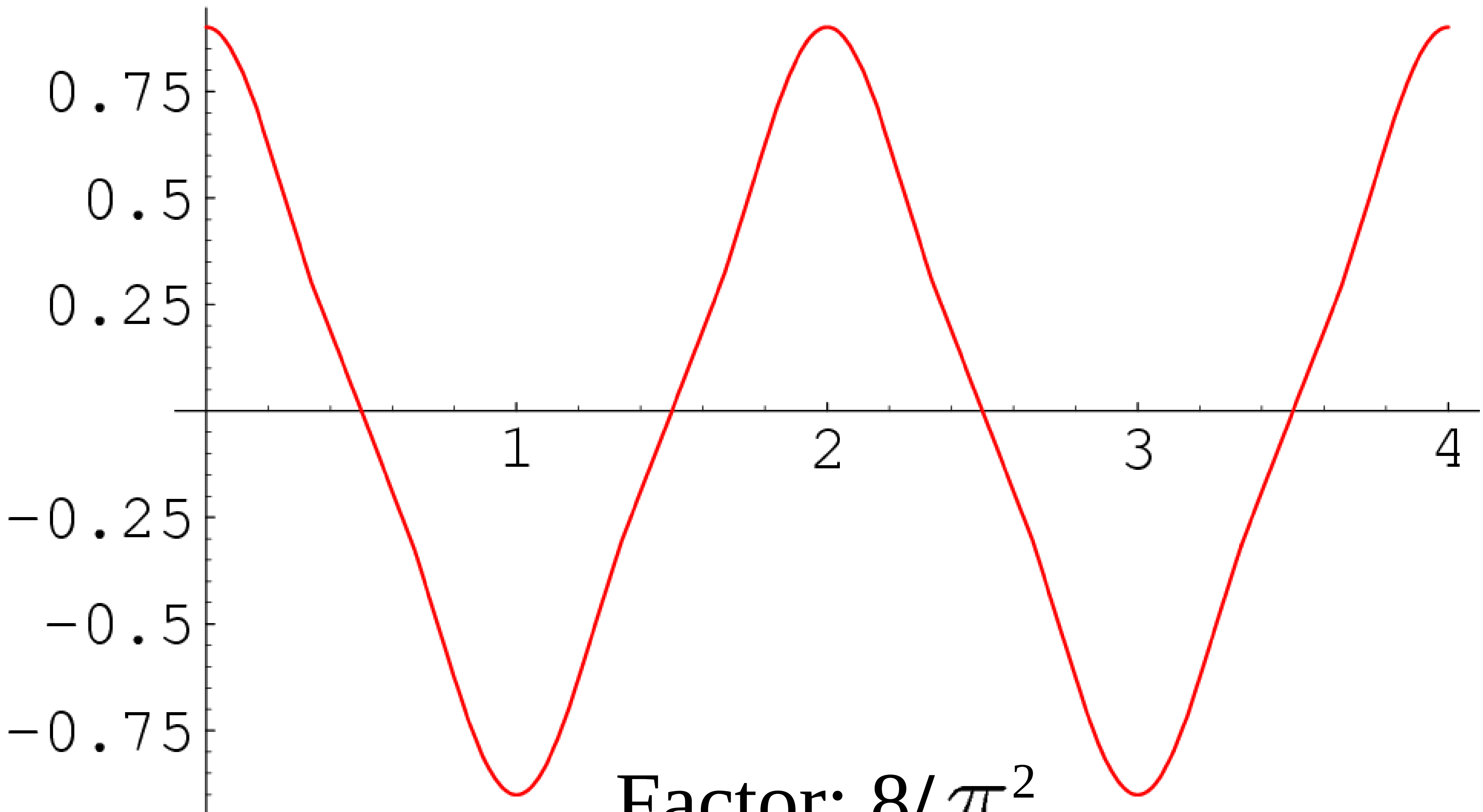
10 terms of the series



20 terms of the series

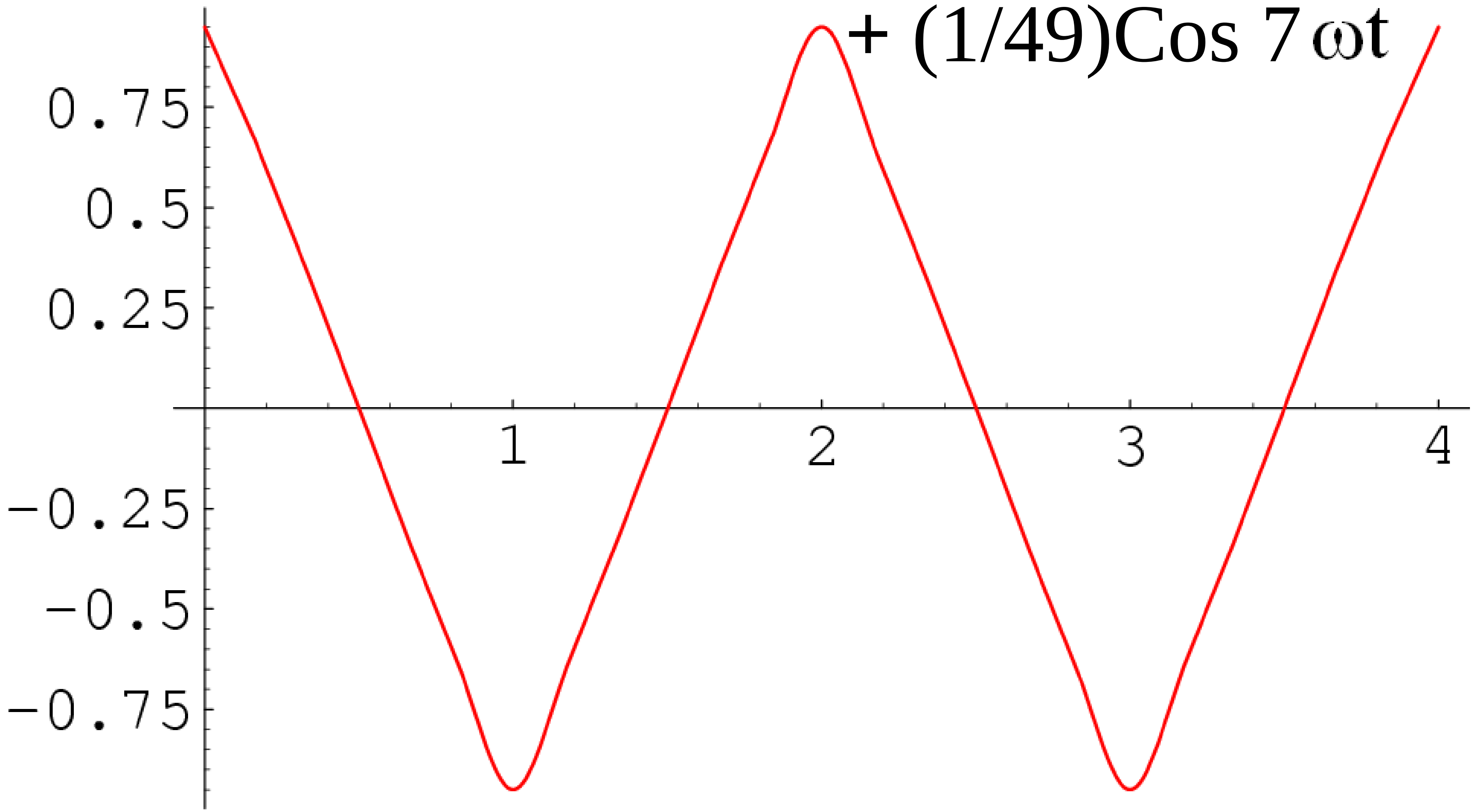


$\cos\omega t + (1/9)\cos 3\omega t$

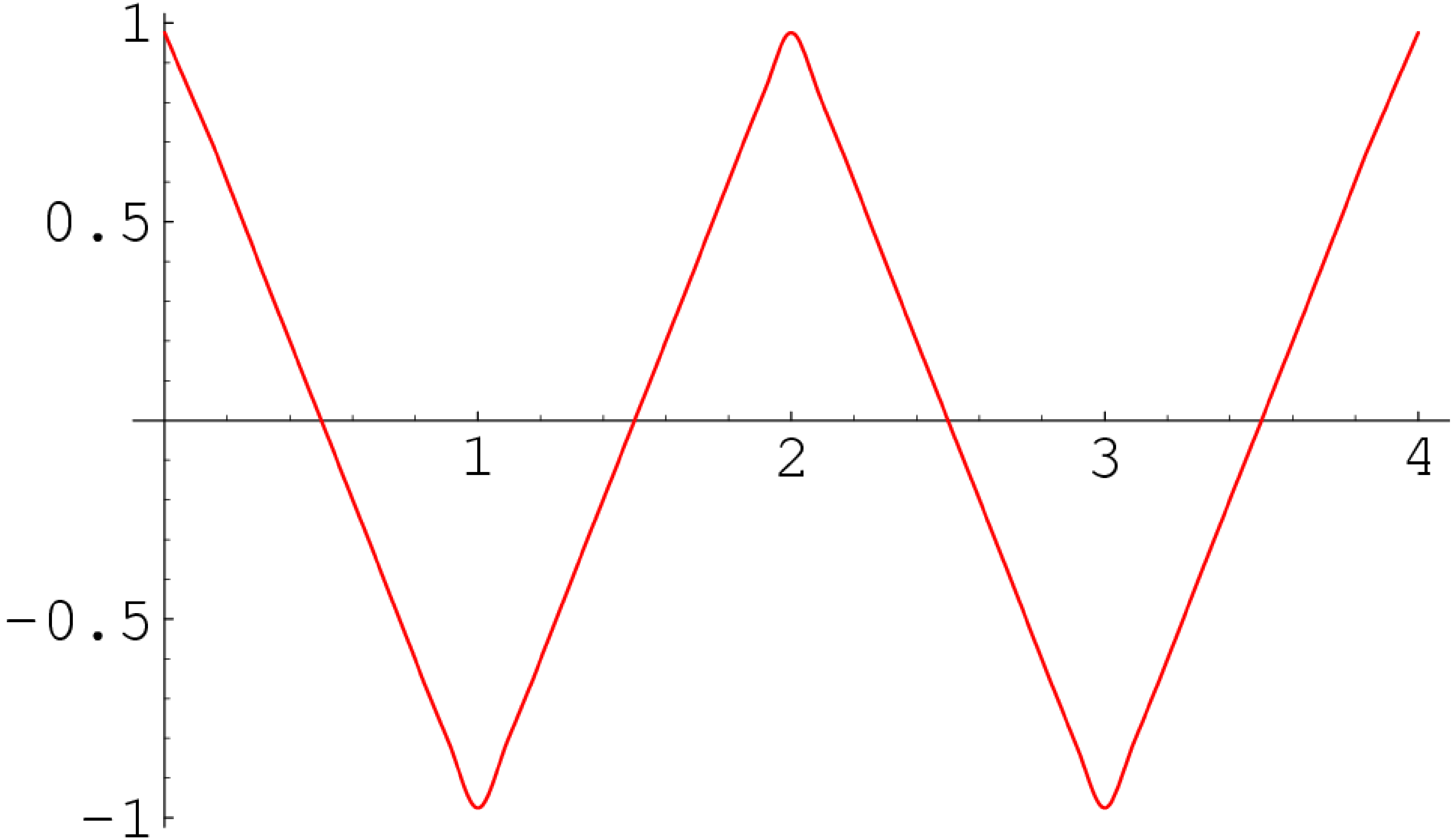


Factor: $8/\pi^2$

$$\text{Cos}\,\omega t + (1/9)\text{Cos}\,3\omega t + (1/25)\text{Cos}\,5\omega t + (1/49)\text{Cos}\,7\omega t$$



8 terms of the series



Fourier Series: $f(t), \quad 0 < t < T$

$$f(t) = A_0 + A_1 \text{Cos}\, \omega t + A_2 \text{Cos}\, 2\omega t + A_3 \text{Cos}\, 3\omega t + A_4 \text{Cos}\, 4\,\omega t + \ldots\ldots.$$

$$+ B_1 \text{Sin}\, \omega t + B_2 \text{Sin}\, 2\,\omega t + B_3 \text{Sin}\, 3\omega t + \ldots$$

$$A_0 = \frac{1}{T}\int_0^T dt \quad f(t)$$

$$A_n / 2 = \frac{1}{T}\int_0^T dt \; f(t)\, \text{Cos}\, n\,\omega t$$

$$B_m / 2 = \frac{1}{T} \int_0^T dt\, f(t)\, \text{Sin}\, m\,\omega t$$

$$\int_0^T dt\ \text{Cos}\, n\omega t\ \text{Sin}\, m\,\omega t = 0$$

$$\int_0^T dt\ \text{Cos}\, n\omega t\ \text{Cos}\, m\,\omega t = 0$$

$$\int_0^T dt\ \text{Sin}\, n\,\omega t\ \text{Sin}\, m\ \omega t = 0$$

Orthogonality relations

n and m
are different

$$
\begin{aligned}
A_0 \; &+ \; \frac{A_1}{2}[e^{i\omega t} + e^{-i\omega t}] + \frac{A_2}{2}[e^{i2\omega t} + e^{-i2\omega t}] \\
&+ \; \frac{A_3}{2}[e^{i3\omega t} + e^{-i3\omega t}] \qquad \ldots\ldots \\
&+ \; \frac{B_1}{2i}[e^{i\omega t} - e^{-i\omega t}] + \frac{B_2}{2i}[e^{i2\omega t} - e^{-i2\omega t}] \\
&+ \; \frac{B_3}{2i}[e^{i3\omega t} - e^{-i3\omega t}] \qquad \ldots\ldots \\
&= \; \sum_{n=0}^{+\infty}[C_n e^{in\omega t} + C_n^* e^{-in\omega t}]
\end{aligned}
$$

$$
C_n = (A_n - iB_n)/2
$$