

What are the eigen values for the die ? Calculate the expectation value.


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## Schrödinger Equation

$$
\begin{aligned}
i \hbar \frac{\partial \Psi(x, t)}{\partial t}= & -\frac{\hbar^{2}}{2 m} \frac{\partial^{2} \Psi(x, t)}{\partial x^{2}}+V(x, t) \Psi(x, t) \\
i \hbar \frac{\partial \Psi(x, t)}{\partial t}= & -\frac{\hbar^{2}}{2 m} \frac{\partial^{2} \Psi(x, t)}{\partial x^{2}}+V(x) \Psi(x, t) \\
& \Psi(x, t)=\psi(x) f(t)
\end{aligned}
$$

$$
\begin{gathered}
i \hbar \psi(x) \frac{\partial f(t)}{\partial t}=-\frac{\hbar^{2}}{2 m} f(t) \frac{\partial^{2} \psi(x)}{\partial x^{2}}+V(x) \psi(x) f(t) \\
i \hbar \frac{1}{f(t)} \frac{\partial f(t)}{\partial t}=-\frac{\hbar^{2}}{2 m} \frac{1}{\psi(x)} \frac{\partial^{2} \psi(x)}{\partial x^{2}}+V(x)=E \\
i \hbar \frac{1}{f(t)} \frac{\partial f(t)}{\partial t}=E
\end{gathered}
$$

$$
\begin{gathered}
\frac{1}{f(t)} \frac{\partial f(t)}{\partial t}=\frac{-i E}{\hbar} \\
\frac{\partial f(t)}{f(t)}=\frac{-i E}{\hbar} \partial t \\
f(t)=f_{0} \exp \left(\frac{-i E}{\hbar} t\right)
\end{gathered}
$$

## Time independent Schrödinger eqn.

$$
-\frac{\hbar^{2}}{2 m} \frac{\partial^{2} \psi(x)}{\partial x^{2}}+V(x) \psi(x)=E \psi(x)
$$

$$
\frac{d^{2} \psi(x)}{d x^{2}}+\frac{2 m}{\hbar^{2}}(E-V(x)) \psi(x)=0
$$

## Particle in a box of size L

$$
\begin{gathered}
\psi(x)=A \cos (k x)+B \sin (k x) \\
\psi(x)=0, \quad x=0, L
\end{gathered}
$$

$$
\psi_{n}(x)=B \sin \left(\frac{n \pi}{L} x\right), \quad n=1,2,3, \cdots
$$

$$
E_{n}=p^{2} / 2 m=n^{2} \hbar^{2} \pi^{2} / 2 m L^{2}
$$

$$
\int_{-\infty}^{\infty} \psi_{n}^{*}(x) \psi_{m}(x) d x=0
$$

$$
\int_{-\infty}^{\infty} \psi_{n}^{*}(x) \psi_{n}(x) d x=1
$$

$$
B=\sqrt{\frac{2}{L}}
$$




Find the probability that a particle trapped in one dimensional box of length L can be found between 0.45 L and 0.55 L for the ground state.

$$
\begin{aligned}
P & =\int_{0.45 L}^{0.55 L} \psi_{1}^{*}(x) \psi_{1}(x) d x \\
P & =\frac{2}{L} \int_{0.45 L}^{0.55 L} \sin ^{2}(\pi x / L) d x
\end{aligned}
$$

$$
\begin{gathered}
P=\frac{1}{L} \int_{0.45 L}^{0.55 L}(1-\cos (2 \pi x / L)) d x \\
P=\frac{1}{L}\left[x-\frac{L}{2 \pi} \sin (2 \pi x / L)\right]_{0.45 L}^{0.55 L} \\
P=\frac{1}{L}\left[0.1 L-\frac{L}{2 \pi}(\sin (1.1 \pi)-\sin (0.9 \pi))\right] \\
P=0.198
\end{gathered}
$$

## Electron trapped inside the atom

 $\mathrm{L}=10^{-10} \mathrm{~m} \quad$ Ground state energy$$
E_{1}=\left(10^{-34}\right)^{2} \times 10 /\left(2 \times 10^{-30} \times\left(10^{-10}\right)^{2}\right)
$$

$$
E_{1} \simeq\left(0.5 \times 10^{-67}\right) / 10^{-50}
$$

$$
E_{1} \simeq 0.5 \times 10^{-17} \mathrm{~J}
$$

$$
E_{1} \simeq 0.5 \times 10^{-17} /\left(1.6 \times 10^{-19}\right) \simeq 30 \mathrm{eV}
$$

## $\alpha$-particle inside the nucleus

## Ground state energy

$$
\mathrm{L}=2 \times 10^{-15} \mathrm{~m}
$$

$$
E_{1}=\left(10^{-34}\right)^{2} \times 10 / 2 \times 6.4 \times 10^{-27} \times\left(2 \times 10^{-15}\right)^{2}
$$

$$
E_{1} \simeq 10^{-10} / 50 \mathrm{~J} \simeq 10 \mathrm{MeV}
$$

