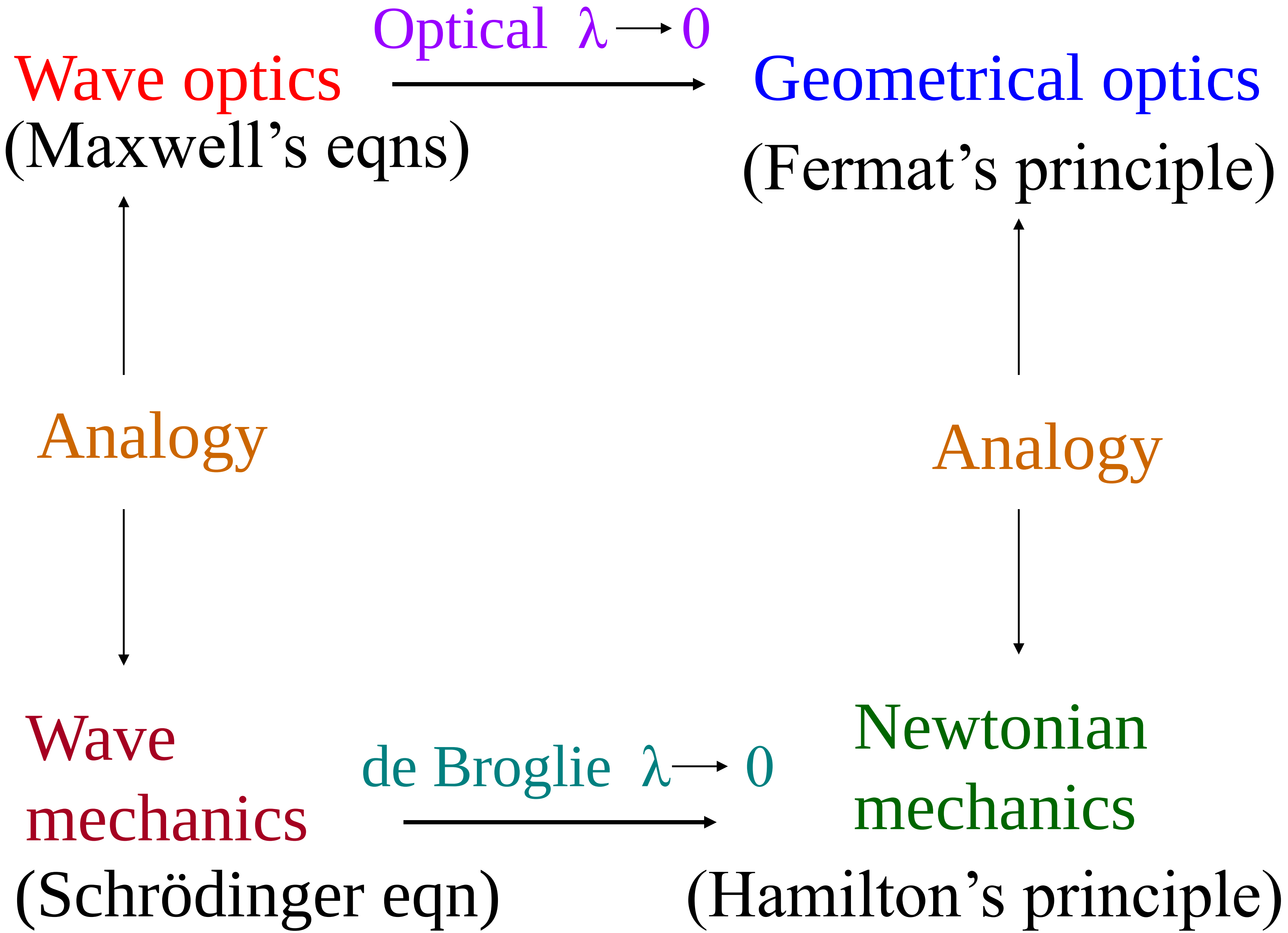

Wave optics
(Maxwell's eqns)
Optical $\lambda \rightarrow 0$
Geometrical optics
(Fermat's principle)
Analogy
Analogy
Wave
mechanics
(Schrödinger eqn)
de Broglie $\lambda \rightarrow 0$
Newtonian
mechanics
(Hamilton's principle)

Classical physics is deterministic

Quantum physics is probabilistic

Basic postulates of Quantum mechanics

Probabilities in QM are determined by wave functions (not observable)

$$\psi(x,t)$$ Complex

$$\left|\psi(x,t)\right|^2$$ Observable

$$\psi(\pm\infty,t)\rightarrow 0$$ Well behaved

Shrödinger Equation

(Time dependent)

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t)$$

$$i\hbar\frac{\partial\Psi}{\partial t} = H\Psi$$

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t)$$

Hamiltonian

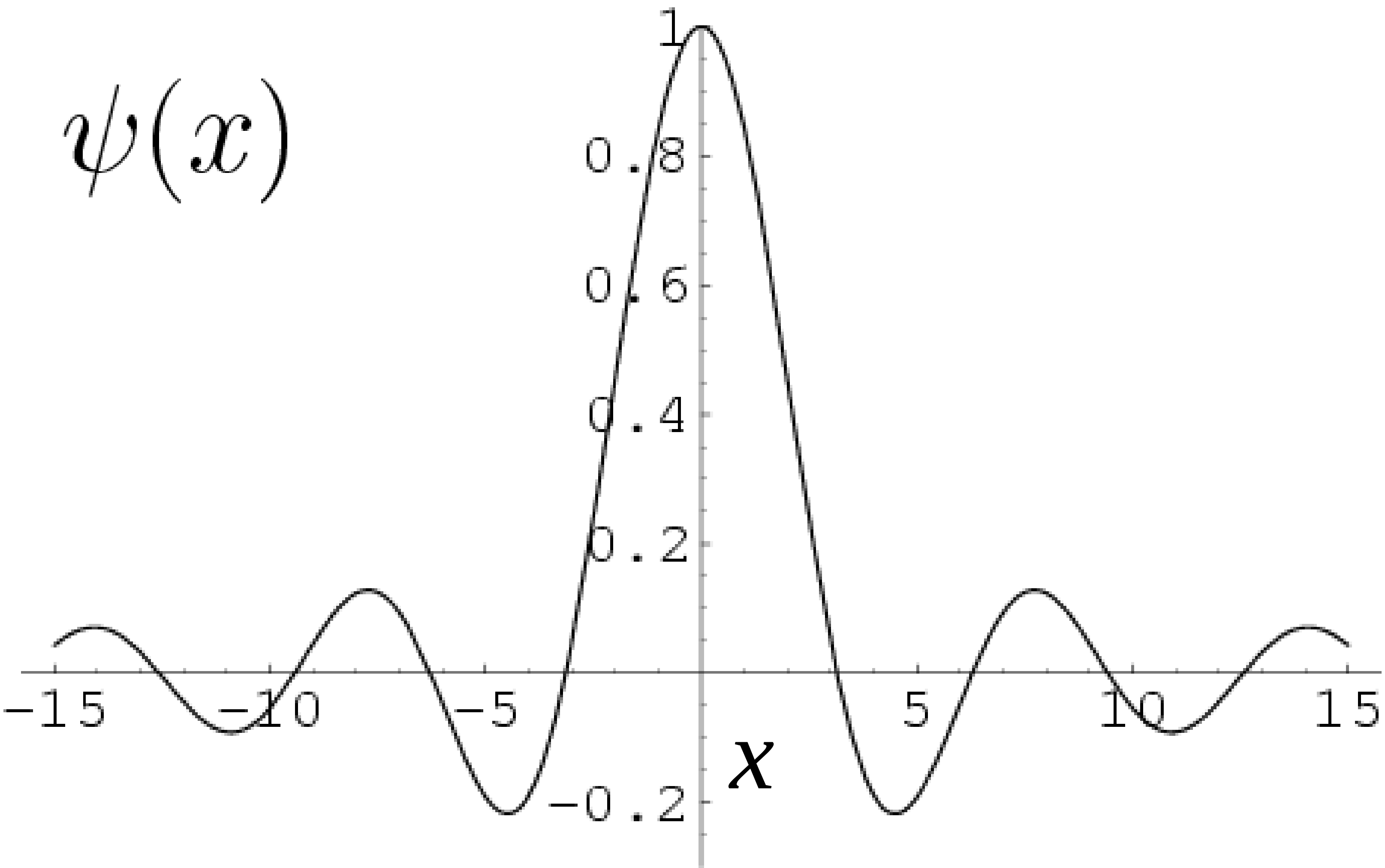
Probability of finding the particle between

x and $x + dx$

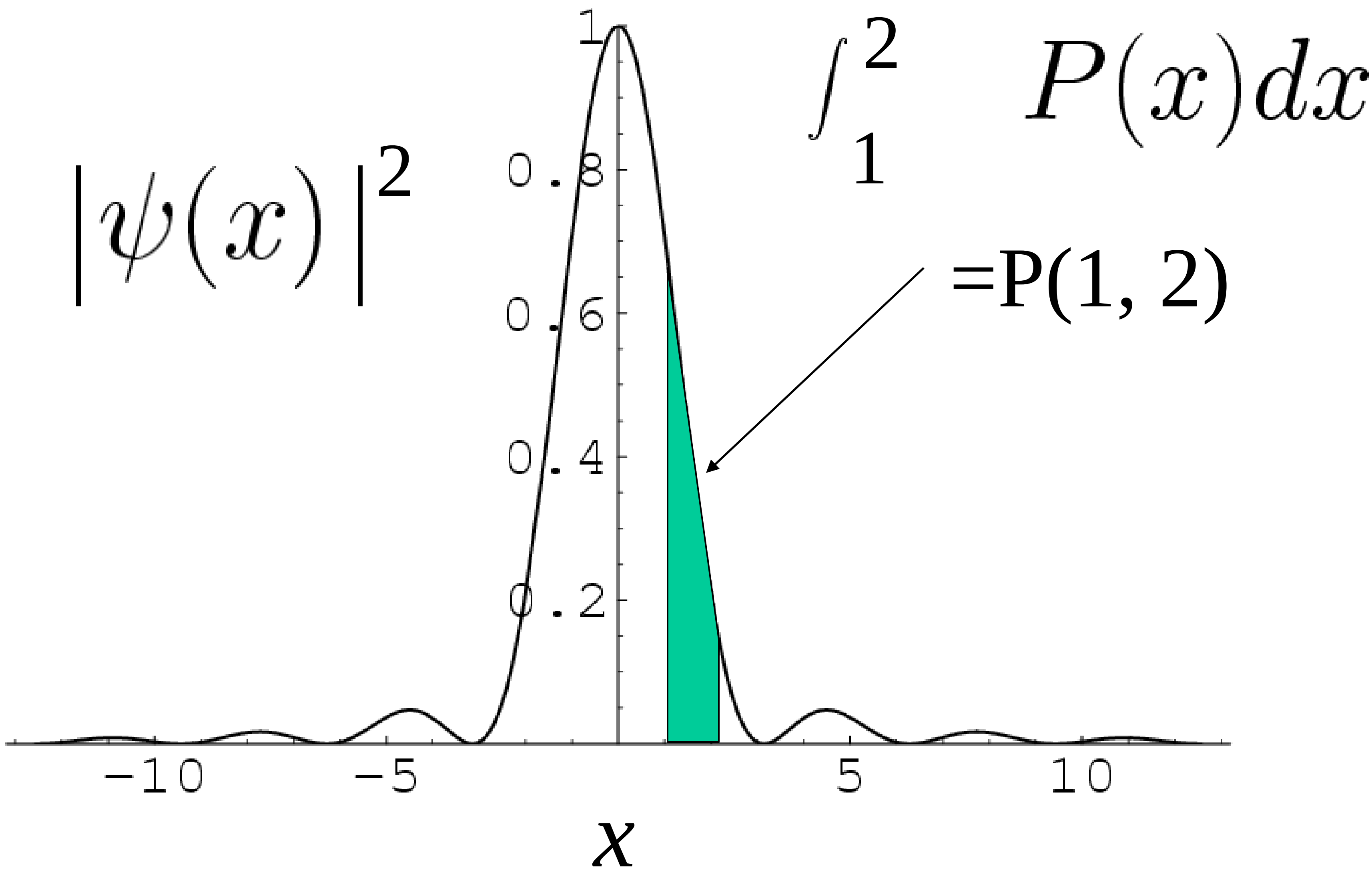
$$P(x)dx = \psi^*(x)\psi(x)dx$$

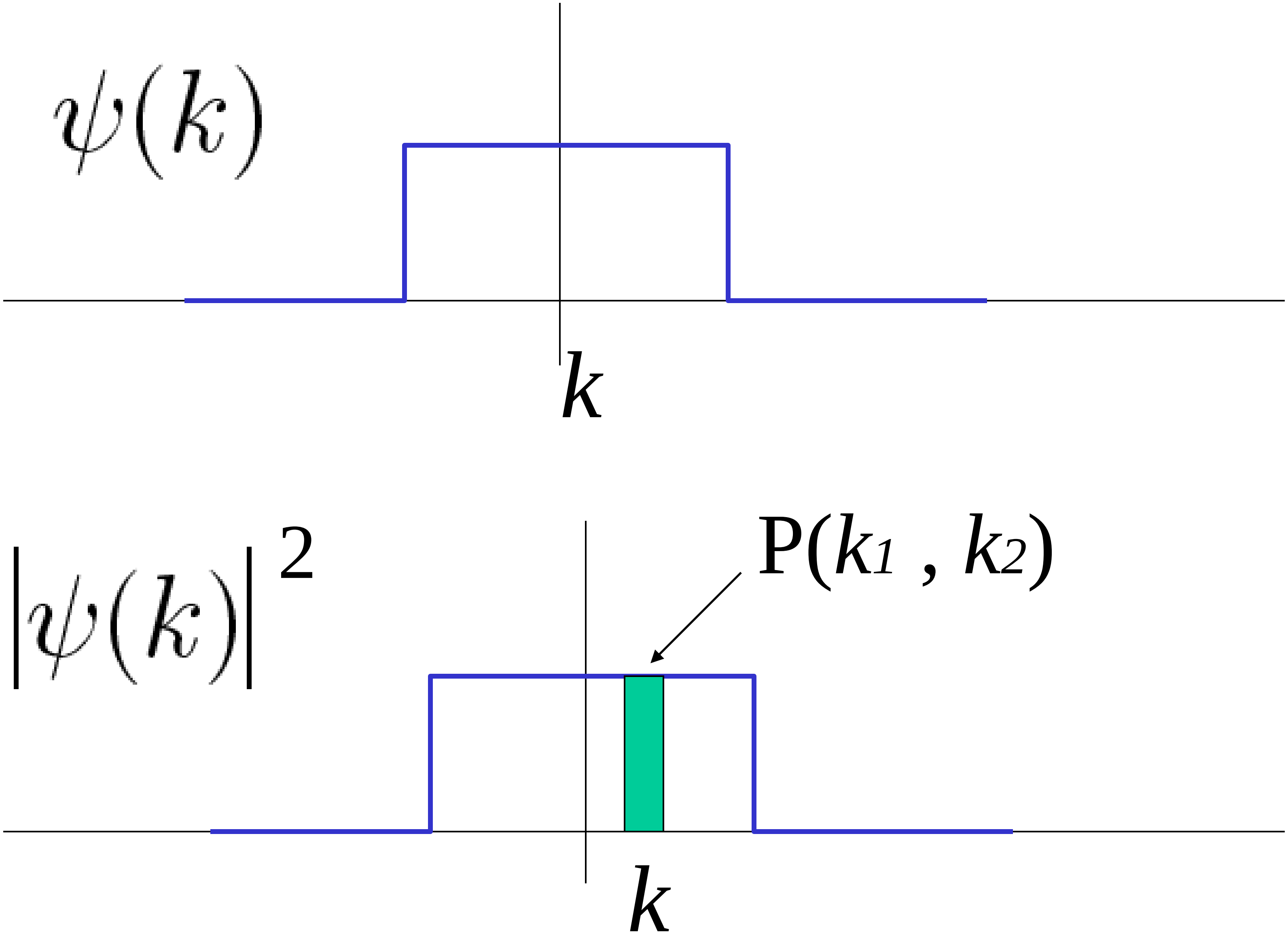
$$\int_{-\infty}^{\infty} P(x)dx$$

$$= \int_{-\infty}^{\infty} \psi^*(x)\psi(x)dx = 1$$



$\psi(x)$
1
0.8
0.6
0.4
0.2
-0.2
-15
-10
-5
5
10
15
x


$|\psi(x)|^2$
$\int_1^2 P(x)dx$
=P(1, 2)
1
0.8
0.6
0.4
0.2
−10
−5
5
10
x



$\psi(k)$
k
$|\psi(k)|^2$
P(k_1 , k_2)
k

$$\psi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x) \exp(-ikx) dx$$

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(k) \exp(ikx) dk$$

$$p = \hbar k$$

k and $k + dk$

$$\hbar = h/2\pi$$

$$P(k)dk = \psi^*(k)\psi(k)dk$$

Superposition

$$\psi = a\psi_1 + b\psi_2$$

Physical observables are replaced with Hermitian operators

$$\hat{p}_x = -i\hbar\frac{\partial}{\partial x}\ , \quad E = i\hbar\frac{\partial}{\partial t}$$

$$E = p^2/2m + V$$

Operator algebra

Linear operators

1) $$\hat{O}\psi = \phi$$

2) $$\hat{O}(\psi_1 + \psi_2) = \hat{O}\psi_1 + \hat{O}\psi_2$$

3) $$\hat{O}(C\psi) = C\hat{O}\psi$$

where C is a constant

Examples: Momentum and Energy Operators

$$\hat{p_x} = -i\hbar\frac{\partial}{\partial x} \quad , \qquad E = i\hbar\frac{\partial}{\partial t}$$

Position Operator $\quad \hat{x}\psi(x) = x\psi(x)$

$$\hat{O} = f(\hat{x}) \text{ or } g(\hat{p})$$

Examples: Not linear

$$\hat{O}\psi = \psi^2$$

$$\hat{O}\psi = \psi^*$$

Commutator of two operators

$$[A, B] = AB - BA$$

$$[x, p_x]\psi = [x, -i\hbar\frac{\partial}{\partial x}]\psi$$

$$= \; - i\hbar x\frac{\partial\psi}{\partial x} + i\hbar\frac{\partial(x\psi)}{\partial x}$$

$$= -i\hbar x\frac{\partial\psi}{\partial x} + i\hbar\frac{\partial x}{\partial x}\psi + i\hbar x\frac{\partial\psi}{\partial x}$$

$$= i\hbar\psi$$

$$\boxed{[x, p_x] = i\hbar}$$

$$\langle(\Delta A)^2\rangle\langle(\Delta B)^2\rangle \geq \frac{1}{4} \mid \langle[A, B]\rangle \mid^2$$

Schwarz Inequality

$$A \equiv x, \quad B \equiv p_x$$

$$\langle(\Delta x)^2\rangle\langle(\Delta p_x)^2\rangle \geq \frac{1}{4} \mid \langle[x, p_x]\rangle \mid^2$$

$$\langle(\Delta x)\rangle\langle(\Delta p_x)\rangle \geq \hbar/2$$

Minimal Uncertainty

$$[y, p_y] = i\hbar, \qquad [z, p_z] = i\hbar$$

$$[x, y] = [x, z] = [y, z] = 0$$

$$[p_x, p_y] = [p_x, p_z] = [p_y, p_z] = 0$$

$$[x, p_z] = [x, p_y] = 0$$

$$[y, p_x] = [y, p_z] = 0 = [z, p_x] = [z, p_y]$$

Algebra of commutators

$$[A, A] = 0$$

$$[A, B] = -[B, A]$$

$$[A, \alpha] = 0$$

α constant

$$[A+B,C]=[A,C]+[B,C]$$

$$\begin{aligned}[A,BC] &= ABC - BCA \\ &= ABC - BAC + BAC - BCA \\ &= (AB - BA)C + B(AC - CA) \\ &= [A,B]C + B[A,C]\end{aligned}$$

$$[AB, C] = [A, C]B + A[B, C]$$

$$[AB, CD] = [A, CD]B + A[B, CD]$$

$$= C[A, D]B + [A, C]DB + A[B, C]D + AC[B, D]$$

$$[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0$$

Jacobi identity

Problem $[l_z, x] = ?\,,\ l_z$ = z component of angular momentum

$$= [xp_y - yp_x, x]$$

$$= [xp_y, x] - [yp_x, x]$$

$$= x[p_y, x] + [x, x]p_y$$

$$-y[p_x, x] - [y, x]p_x \quad = i\hbar y$$