W. Heisenberg

E. Schrödinger

Uncertainty relations &
Matrix mechanics (1932)

Wave mechanics
(1933)

P.A. M. Dirac

Relativistic theory of electron (1933)

W. Pauli

Exclusion principle (1945)

Heisenberg's Uncertainty Relations

$$\Delta x \Delta p_x \sim \hbar, \quad \Delta y \Delta p_y \sim \hbar,$$

$$\Delta z \Delta p_z \sim \hbar, \quad \Delta E \Delta t \sim \hbar$$

$$\hbar = h/2\pi$$

Minimum Uncertainty (Gaussian wave packets)

$$\boxed{\Delta x \Delta p_x = \hbar/2, \quad etc.}$$

Electrons confined in atoms

$$x \sim 10^{-10} m$$

$$x \sim \Delta x$$

$$\Delta p \sim \hbar / \Delta x$$

$$\Delta p \sim p$$

$$E = p^2/2m = \hbar^2/(2m(\Delta x)^2)$$

$$= (10^{-34})^2/(2 \times 10^{-30} \times (10^{-10})^2)$$

$$= 5 \times 10^{-19} \text{ J} \quad \sim 3 \text{ eV}$$

Hydrogen atom ionisation potential : 13.6 eV

α particles confined in nucleus

Size of the nucleus $x \sim 2 \times 10^{-15} m$

$$E = p^2/2m = \hbar^2/(2m(\Delta x)^2)$$

$$= (10^{-34})^2/(2\times 4\times 1.67\times 10^{-27}\times 4\times (10^{-15})^2)$$

$$= 2 \times 10^{-13}\ \mathrm{J} \sim 1\ \mathrm{MeV}$$

Harmonic oscillator: Ground state

$$E = \frac{p_x^2}{2m} + \frac{1}{2}kx^2$$

$$x \sim \Delta x$$

$$\Delta p_x \sim p_x \sim \hbar / 2\Delta x$$

$$E = \frac{\hbar^2}{8m(\Delta x)^2} + \frac{1}{2}k(\Delta x)^2$$

$$\frac{\partial E}{\partial \Delta x} = -\frac{\hbar^2}{4m(\Delta x)^3} + k(\Delta x) = 0$$

$$\Rightarrow (\Delta x)^2 = \frac{\hbar}{2\sqrt{mk}}$$

$$E = \frac{\hbar^2}{8m(\frac{\hbar}{2\sqrt{mk}})} + \frac{1}{2}k(\frac{\hbar}{2\sqrt{mk}})$$

$$E = \frac{\hbar}{2}\sqrt{\frac{k}{m}} = \frac{\hbar}{2}\omega = \frac{h\nu}{2}$$

Zero point energy

Hydrogen atom: Ground state

$$E = \frac{p^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r}$$

$$r \sim \Delta r$$

$$\Delta p \sim p \sim \hbar/\Delta r$$

$$E = \frac{\hbar^2}{2m(\Delta r)^2} - \frac{e^2}{4\pi\epsilon_0 \Delta r}$$

$$\frac{\partial E}{\partial \Delta r} = -\frac{\hbar^2}{m(\Delta r)^3} + \frac{e^2}{4\pi\epsilon_0 (\Delta r)^2} = 0$$

$$\Rightarrow \Delta r = \frac{4\pi\epsilon_0}{me^2}\,\hbar^2$$

$$\Delta r = \frac{(1.05 \times 10^{-34})^2}{9 \times 10^9 \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2}$$

$$= 0.528 \times 10^{-10} \text{ m} = 0.528 \text{ A}^\circ$$

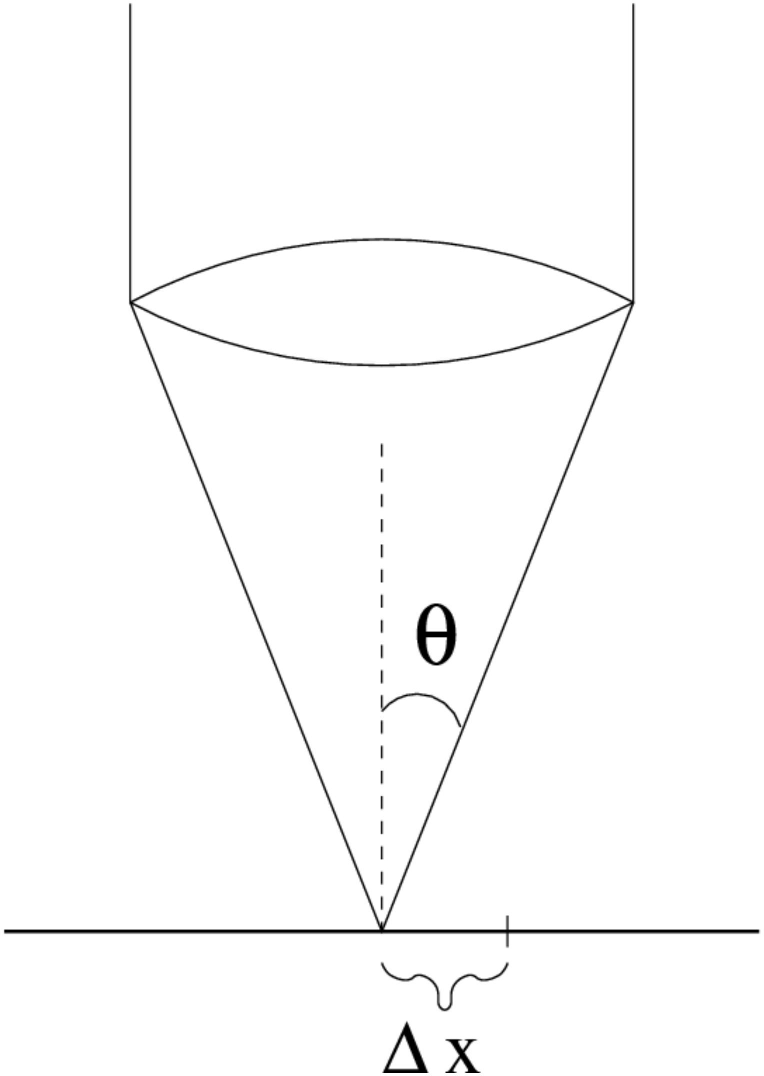
Bohr radius

Minimum energy

$$E = -\frac{me^4}{2 \times (4\pi\epsilon_0)^2\hbar^2} = -13.6\text{eV}$$

Position measurement in a microscope

Abbe Sine condition for resolution



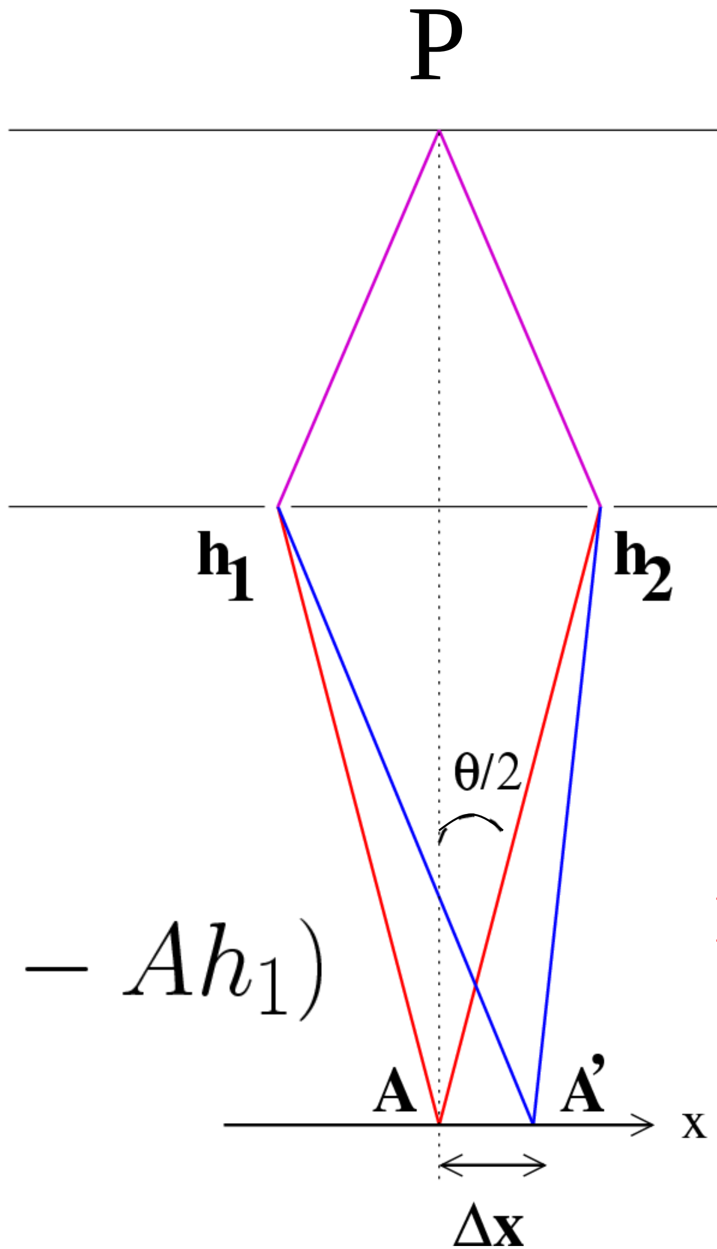
$$\Delta x \sim \frac{\lambda}{sin\theta}$$

$$|p| = \frac{h\nu}{c} = \frac{h}{\lambda}$$

$$\Delta p_x \sim \frac{h}{\lambda} sin\theta$$

$$\Delta p_x . \Delta x \sim h$$

Double slit experiment



Condition for a maximum at P

$$\phi = \frac{2\pi}{\lambda}(Ah_2 - Ah_1)$$

$$= 0$$

You want to know through which hole photon has traveled

$$\Delta p_x < \frac{2h}{\lambda}\sin(\theta/2)$$

This implies uncertainty in position

$$\Delta x > \frac{h}{2}\frac{1}{\Delta p_x}$$

$$\Delta x > \frac{h}{2}\left[\frac{\lambda}{2h\sin(\theta/2)}\right]$$

$$\Delta x > \frac{\lambda}{4\sin(\theta/2)}$$

the atom could just as well be at postion A’

The phase difference is

$$\phi' = \frac{2\pi}{\lambda}(A'h_2 - A'h_1)$$

$$\phi' = \frac{2\pi}{\lambda} 2\sin(\theta/2)\Delta x$$

$$> \pi$$

A condition for
a minimum at P

Interference washed out !