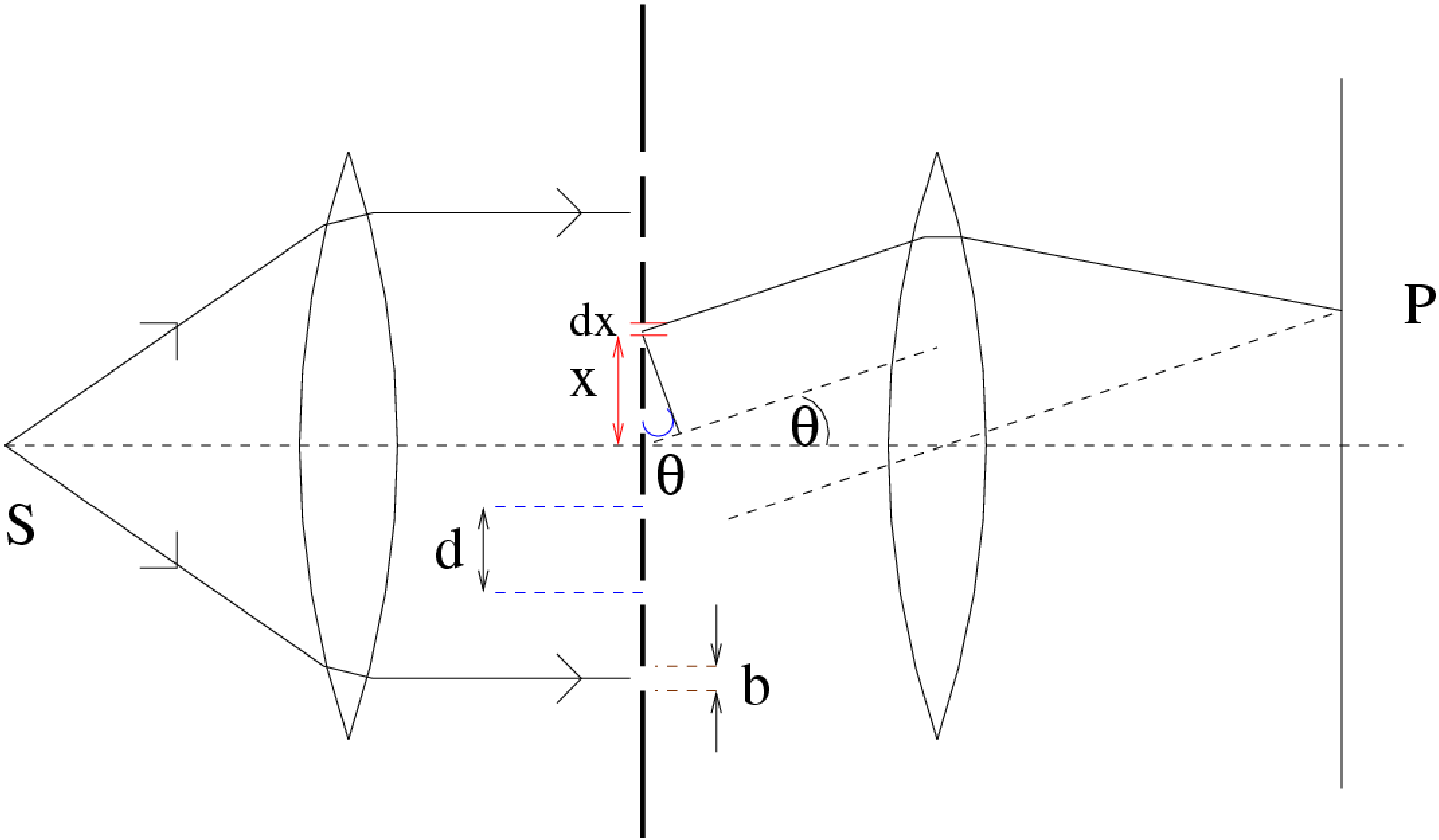
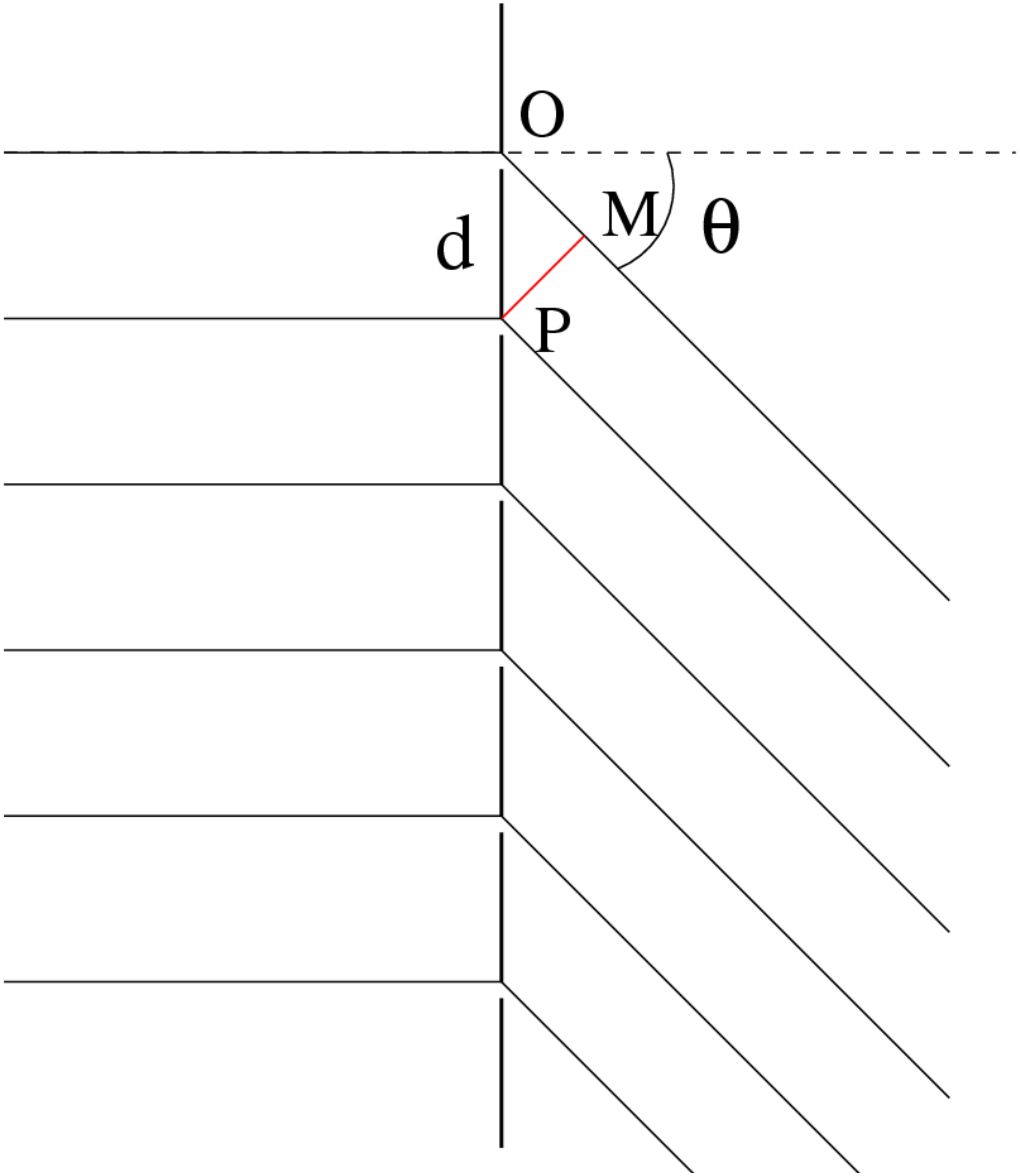
# N slit grating



# Normal incidence

# Transmission grating



$$A = |A|\exp(i\phi) = a + a\exp(i\delta) + a\exp(i2\delta)$$
$$+ a\exp(i3\delta) + \cdots a\exp(i(n-1)\delta)$$

$$A = a[1 - \exp(in\delta)]/[1 - \exp(i\delta)]$$
$$= a\frac{\sin(n\delta/2)}{\sin(\delta/2)}\exp(i(n-1)\delta/2)$$

$$\boxed{|A| = |a|\frac{\sin(n\delta/2)}{\sin(\delta/2)}}$$

$$n = N$$ = Number of slits

Amplitude due to each single slit

$$|a| = \sin\beta/\beta$$

$$\delta = 2\pi d \sin\theta/\lambda$$

$$\gamma = \pi d \sin\theta/\lambda = \delta/2$$

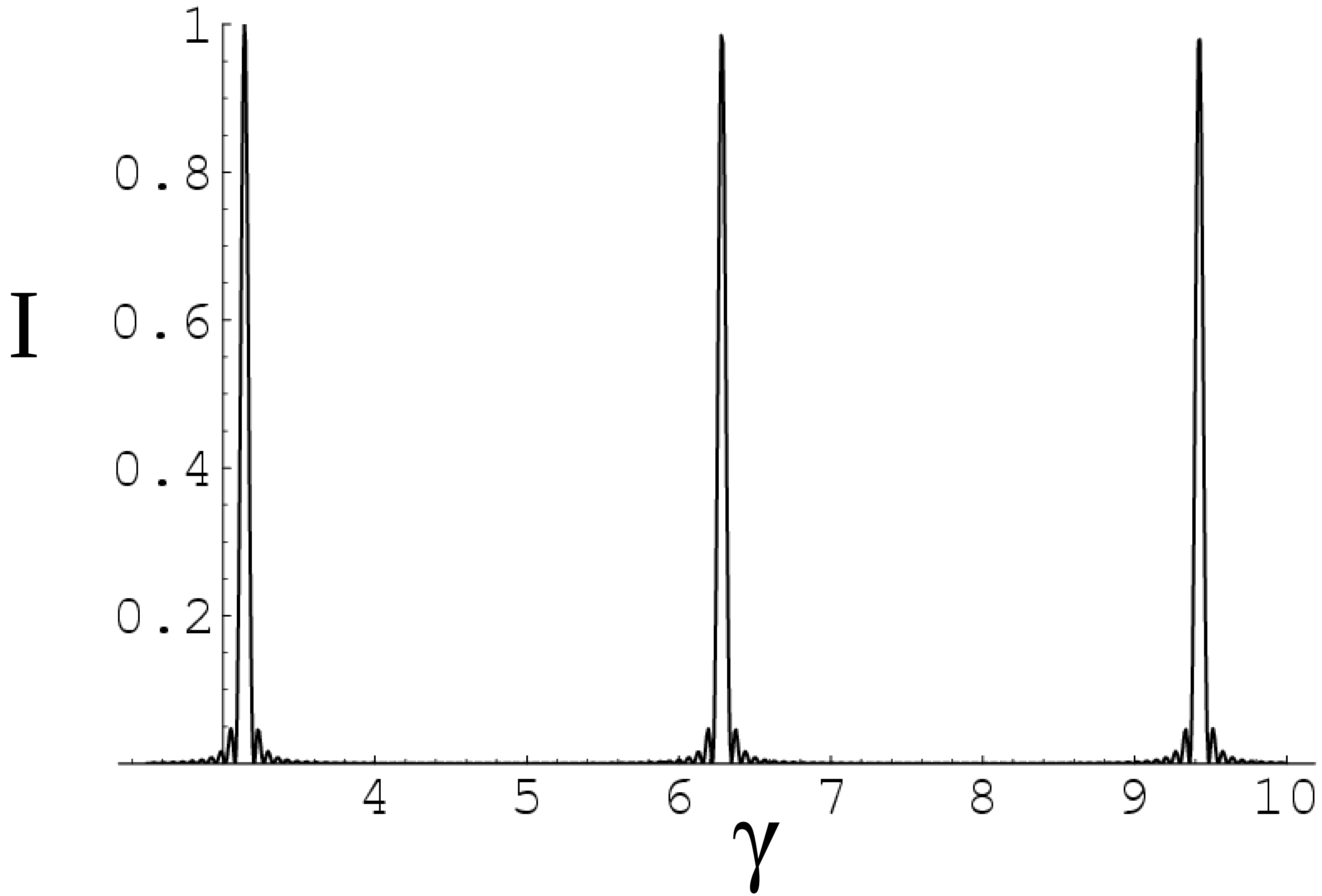
$$\beta = \pi b \sin\theta/\lambda$$

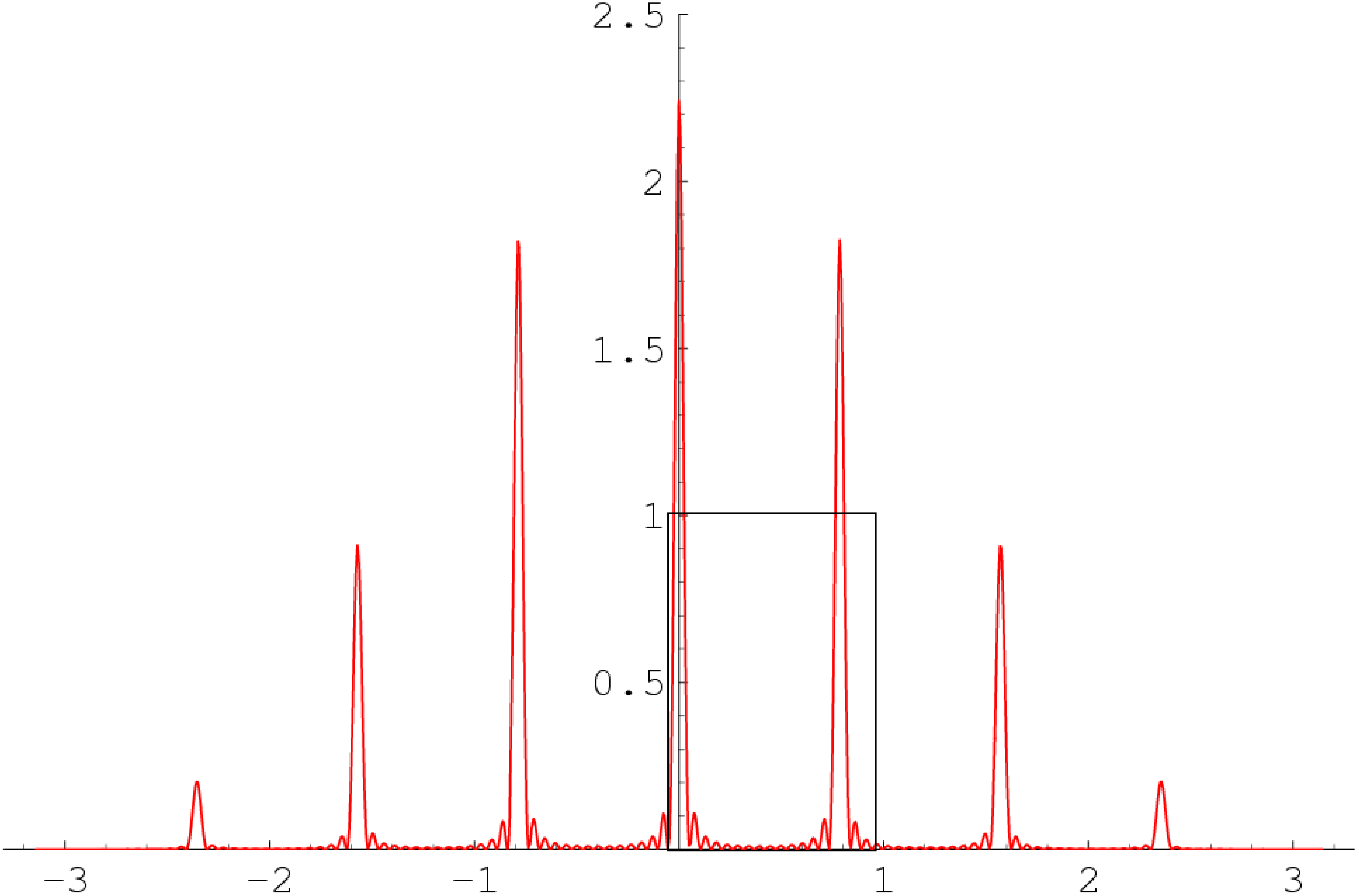
# Intensity pattern

$$I(\theta) = I_0 \frac{\sin^2 \beta}{\beta^2} \cdot \frac{\sin^2 N\gamma}{\sin^2 \gamma}$$

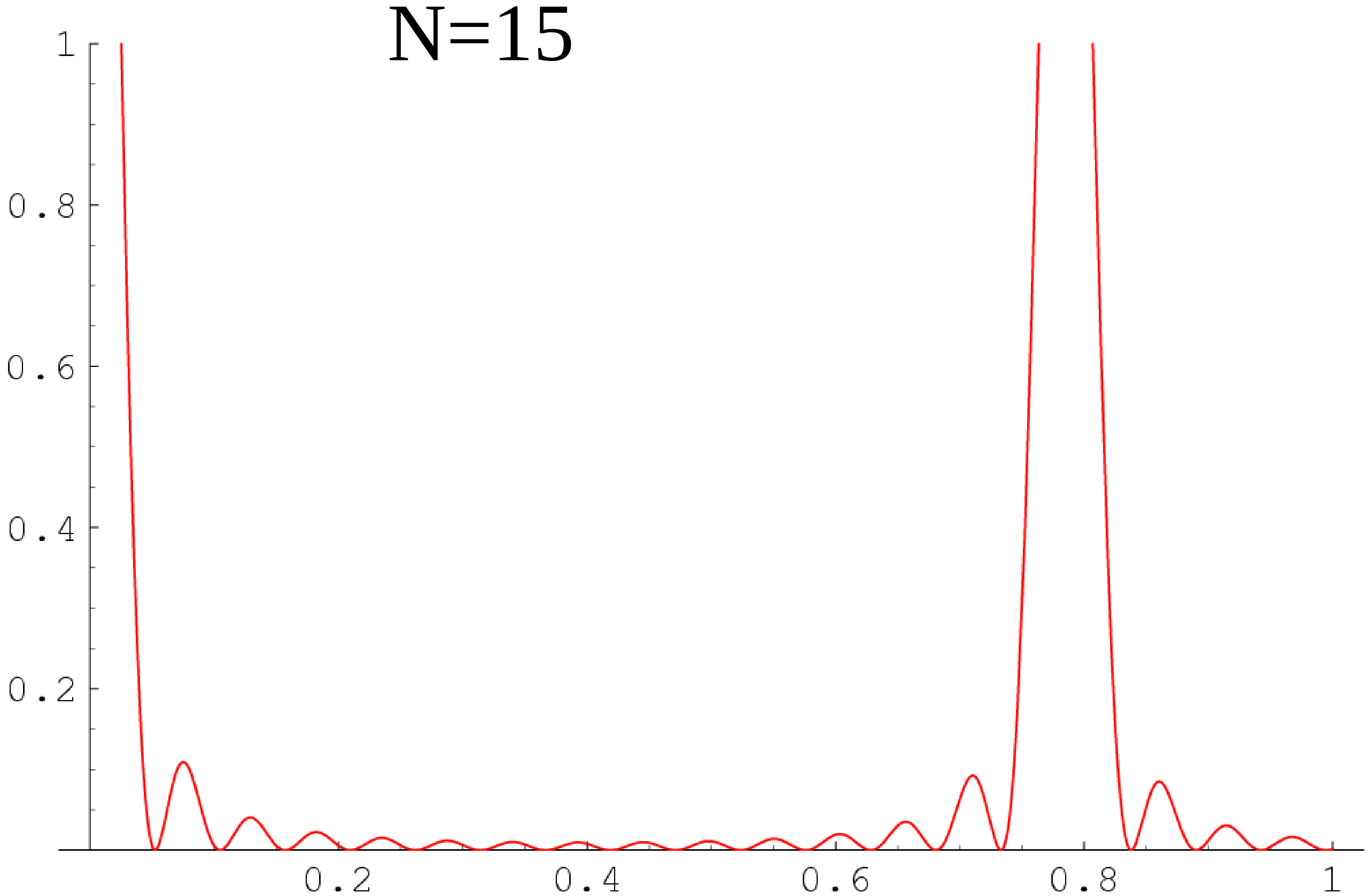
$$I(\theta) = I_0 \frac{\sin^2 N\gamma}{\sin^2 \gamma}$$ for small b

# Intensity



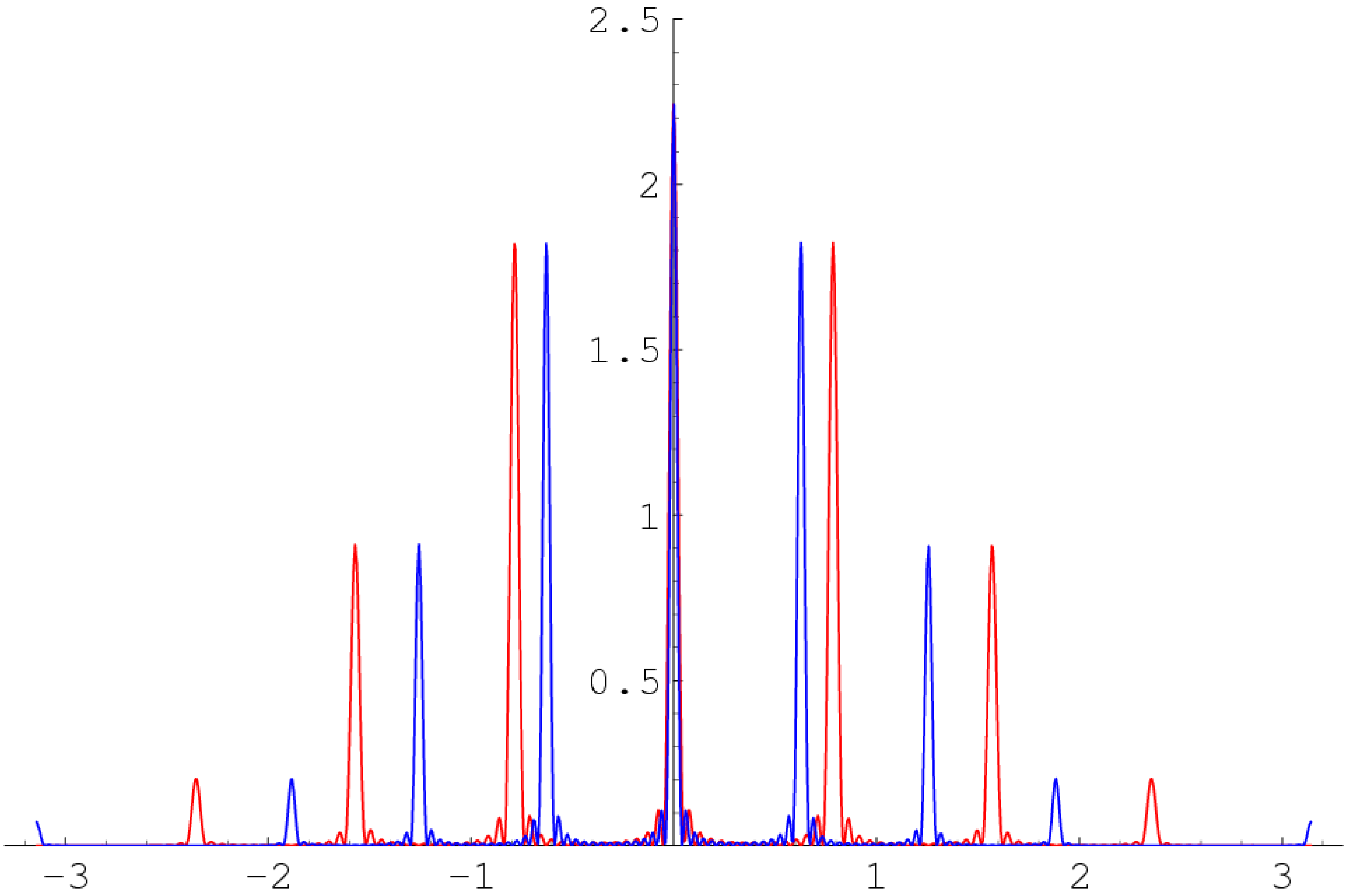


2.5
2
1.5
1
0.5
−3
−2
−1
1
2
3

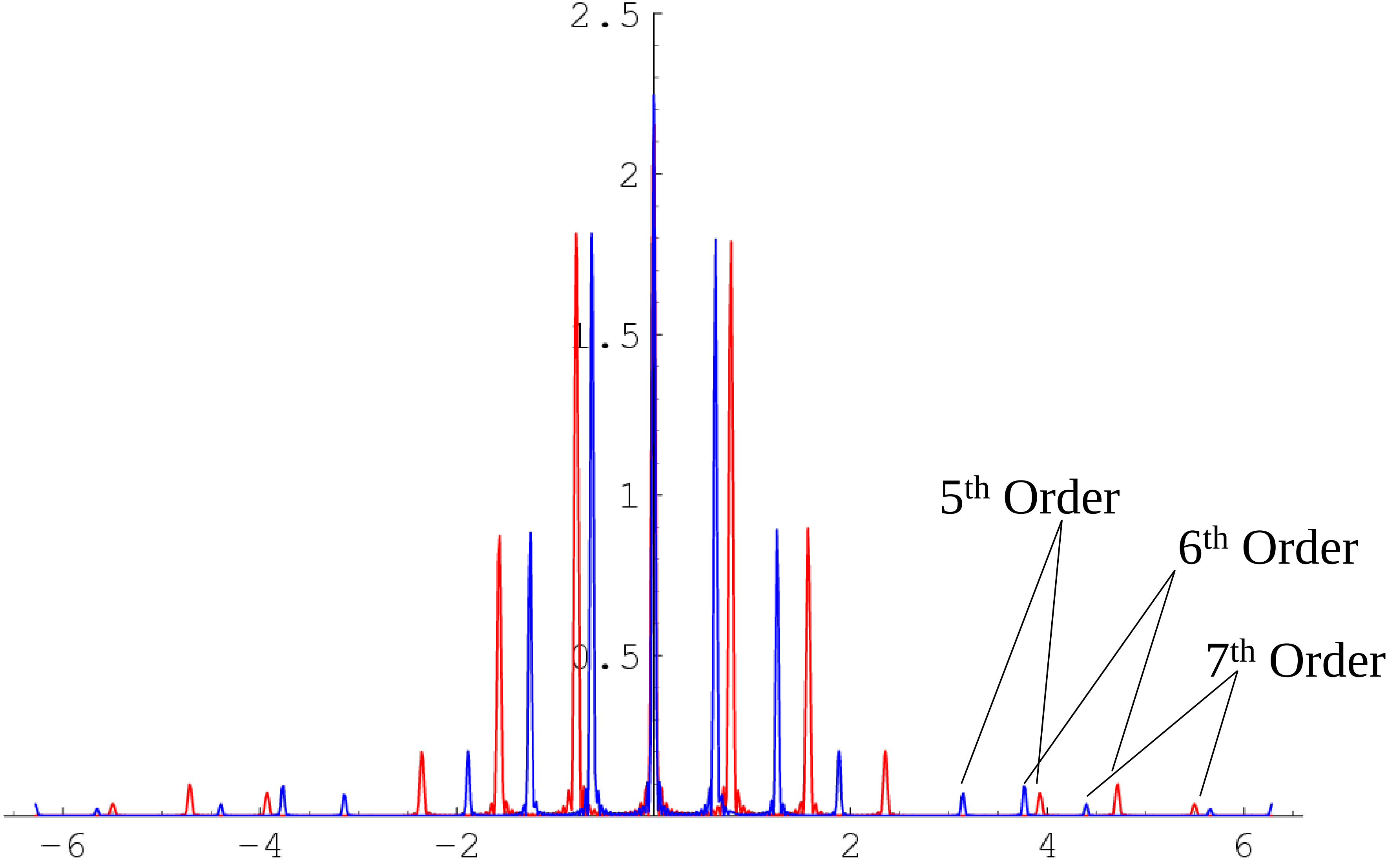


N=15
1
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1

# For two wavelengths



# d/b = 4



## Principal maxima

$$N\gamma = mN\pi \Leftrightarrow d\sin\theta_m = m\lambda$$

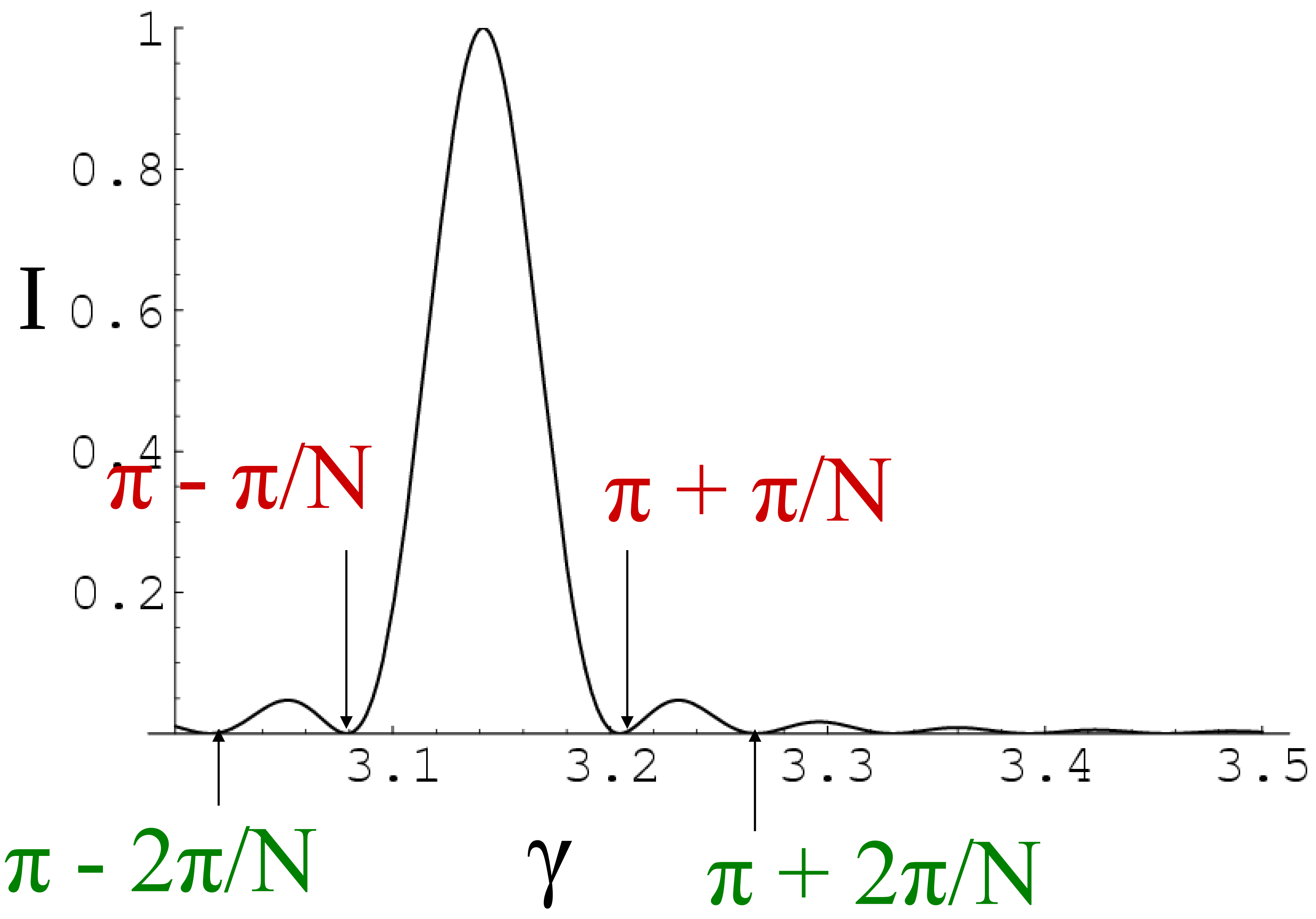
$$m = 0, \pm 1, \pm 2, \cdots$$

## Minima

$$\sin N\gamma = 0 \Leftrightarrow N\gamma = \pm n\pi, n \neq 0, N, 2N, \cdots$$

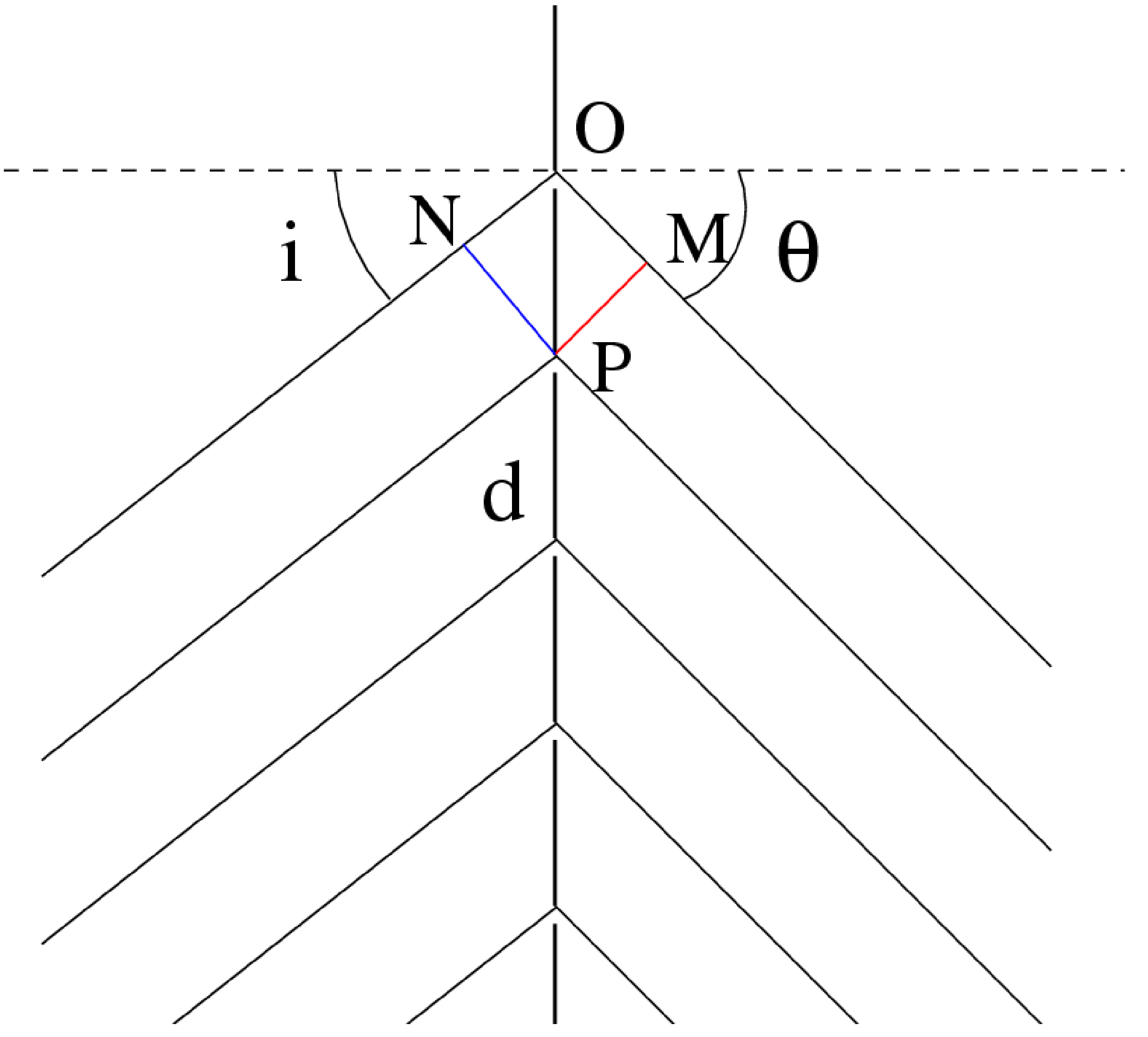
$$N\gamma = (mN+1)\pi \Leftrightarrow d\sin(\theta_m + (\Delta\theta)_w)$$

$$= m\lambda + \frac{\lambda}{N}$$



I
1
0.8
0.6
0.4
0.2
3.1
3.2
3.3
3.4
3.5
γ
π - π/N
π + π/N
π - 2π/N
π + 2π/N

# Oblique incidence




$$d(\sin i + \sin \theta_m) = m\lambda$$

# Width of principal maxima

$$(\Delta\theta)_w \approx \frac{\lambda}{Nd\cos\theta_m}$$

**Dispersive power of grating** $D \equiv \frac{(\Delta\theta)_s}{\Delta\lambda}$

$$d\cos\theta_m(\Delta\theta)_s = m\Delta\lambda$$

$$D = \frac{m}{d\cos\theta_m}$$

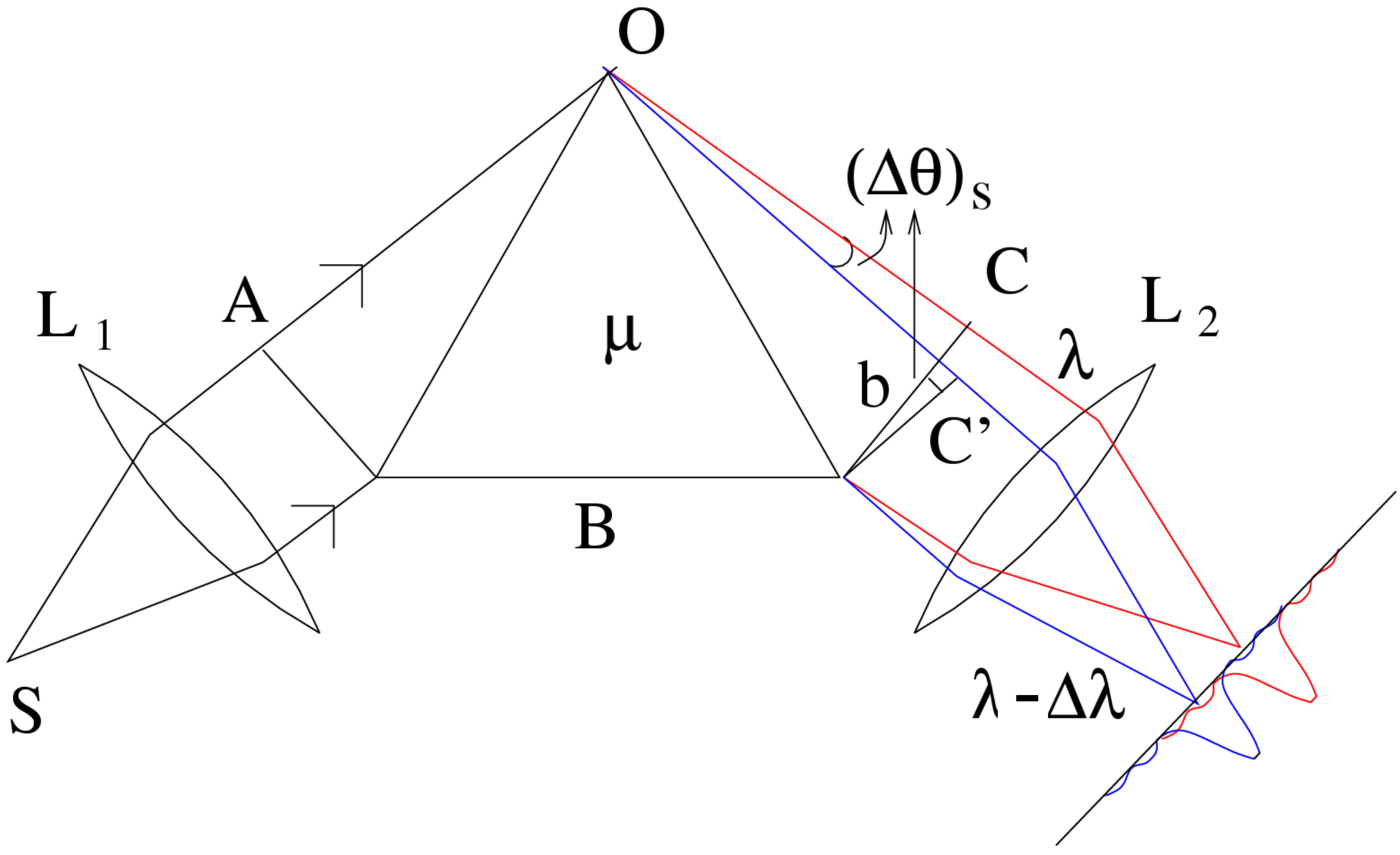
# Chromatic resolving power of a grating

$$| \ (\Delta\theta)_s \ | = | \ (\Delta\theta)_w \ |$$

$$d\cos\theta_m \ \frac{\lambda}{Nd\cos\theta_m} = m\Delta\lambda$$

$$C.R.P. \equiv \left(\frac{\lambda}{\Delta\lambda}\right)_{bare} = m\,\mathrm{N}$$

# Chromatic resolving power of a prism



$$(\Delta\theta)_s = (\Delta\theta)_w = \lambda/b$$

$$AO + OC = B\mu(\lambda)$$

$$AO + OC' = B\mu(\lambda - \Delta\lambda)$$

$$OC' = OC + b(\Delta\theta)_s$$

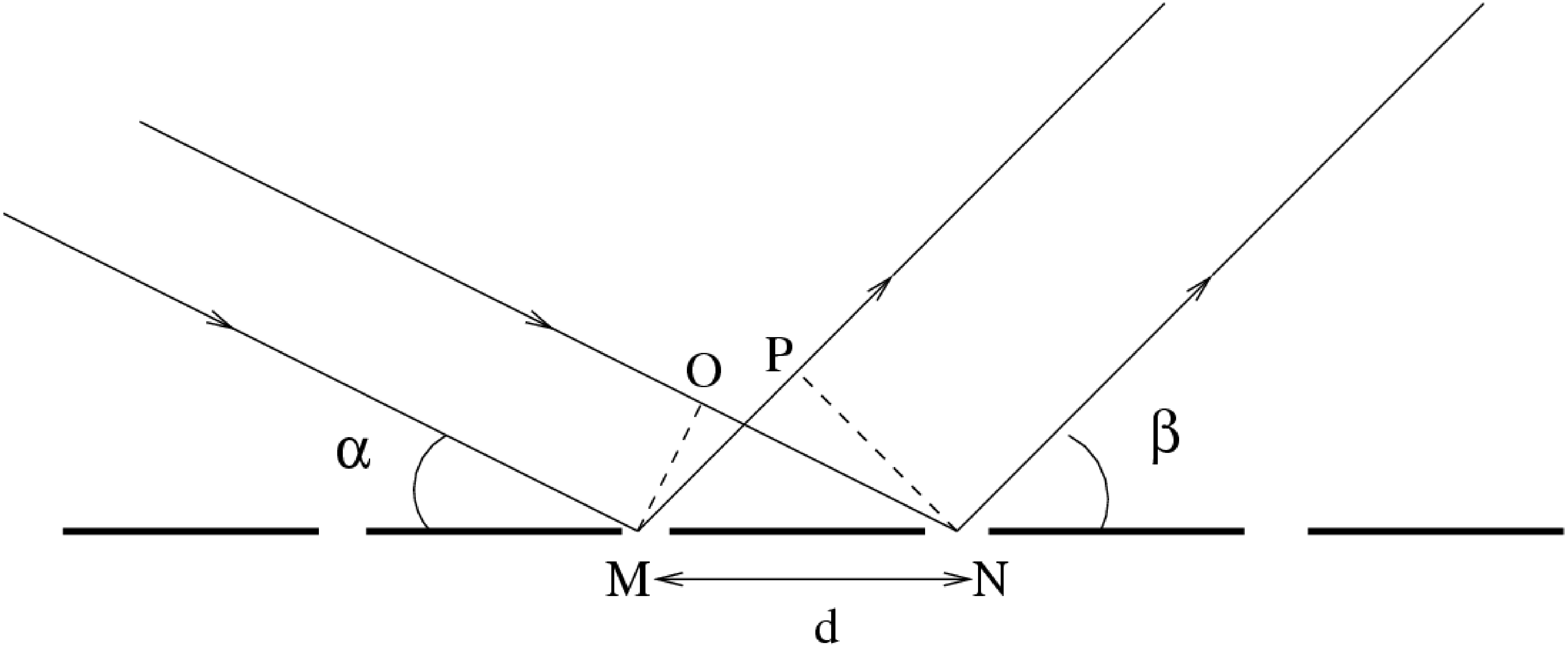
$$b(\Delta\theta)_s = B[\mu(\lambda - \Delta\lambda) - \mu(\lambda)]$$

$$= -B\left(\frac{d\mu}{d\lambda}\right)\Delta\lambda$$

$$C.R.P. \equiv \left(\frac{\lambda}{\Delta\lambda}\right)_{bare} = B\left|\frac{d\mu}{d\lambda}\right|$$

# Transmission grating

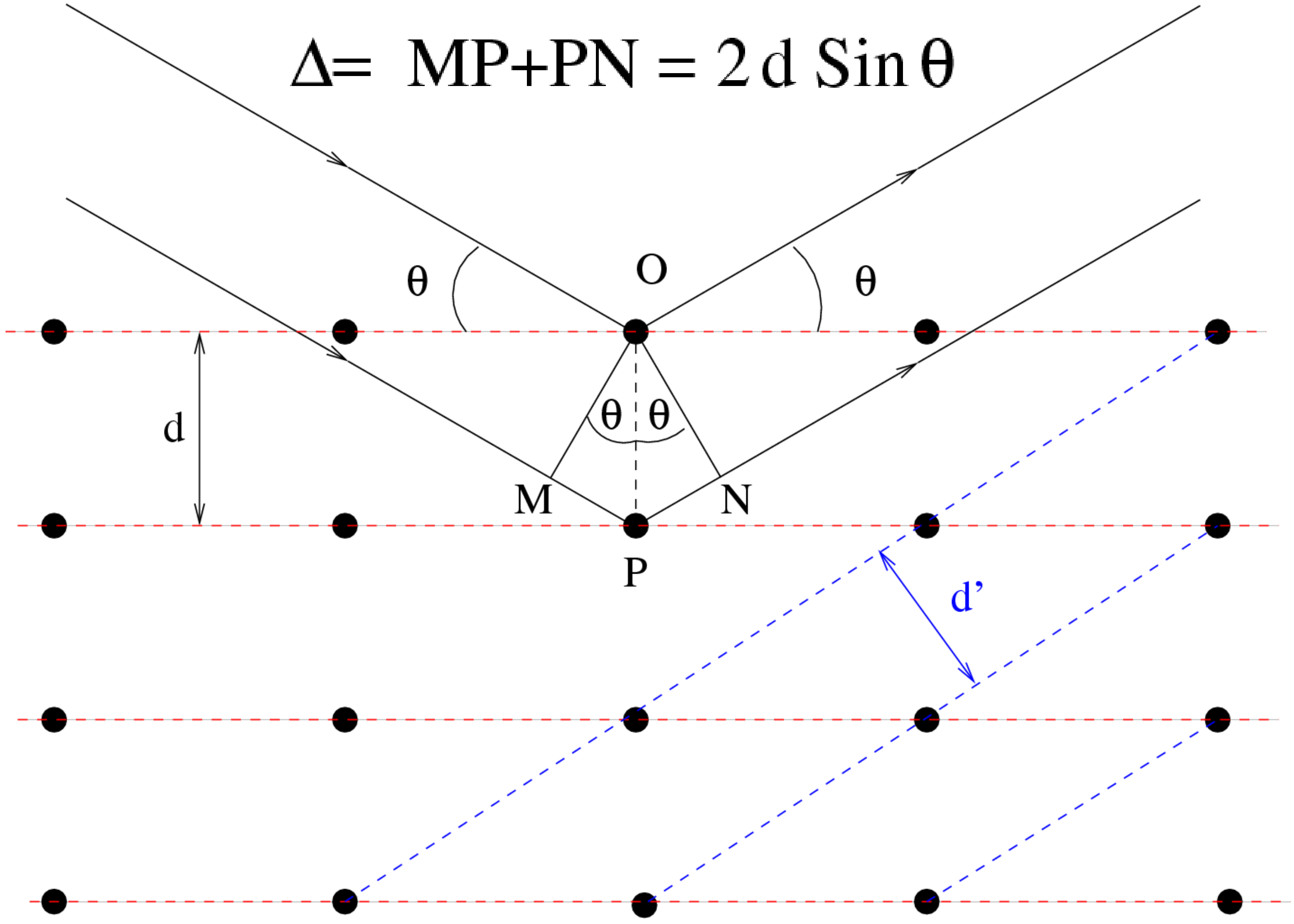
# Reflection grating

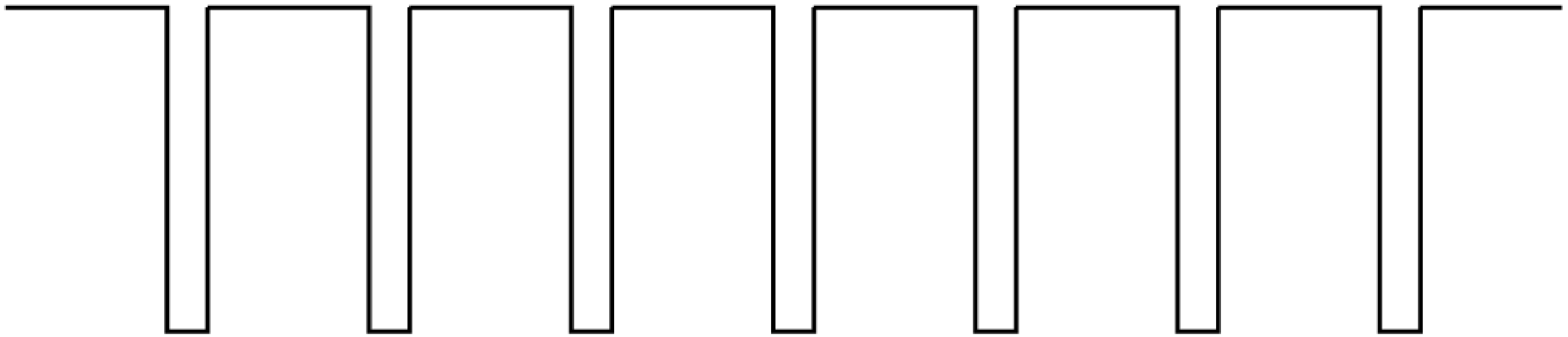



$$\Delta = ON - PM = d(\cos\alpha - \cos\beta)$$

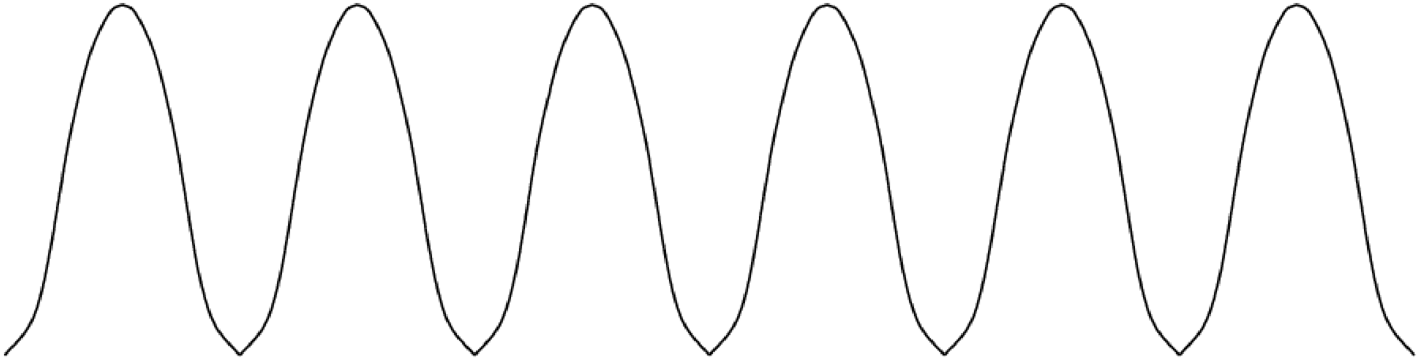
# X-ray diffraction from crystals: Bragg's law

$$2d \sin\theta = n\lambda$$





Phase grating



Ultrasonics