Oscillating charges and dipole radiation

For large distances (*r*) the first two terms die as $1/r^2$, whereas the last term (radiation term) survives because it decays as $1/r$.

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0 c^2}\left[\frac{d^2\mathbf{e}_{\mathbf{r}'}}{dt^2}\right]$$

$$E_z(r,t) = \frac{q}{4\pi\epsilon_0 c^2 r} a_z(t - r/c)$$

$$a_z(t - r/c) = \frac{d^2\mathbf{z}(t - r/c)}{dt^2}$$

$$\mathbf{E}(r,t) = -\frac{\mu_0 p_0}{4\pi r}\omega^2 \cos(\omega(t - r/c)) \sin\theta\hat{\theta}$$

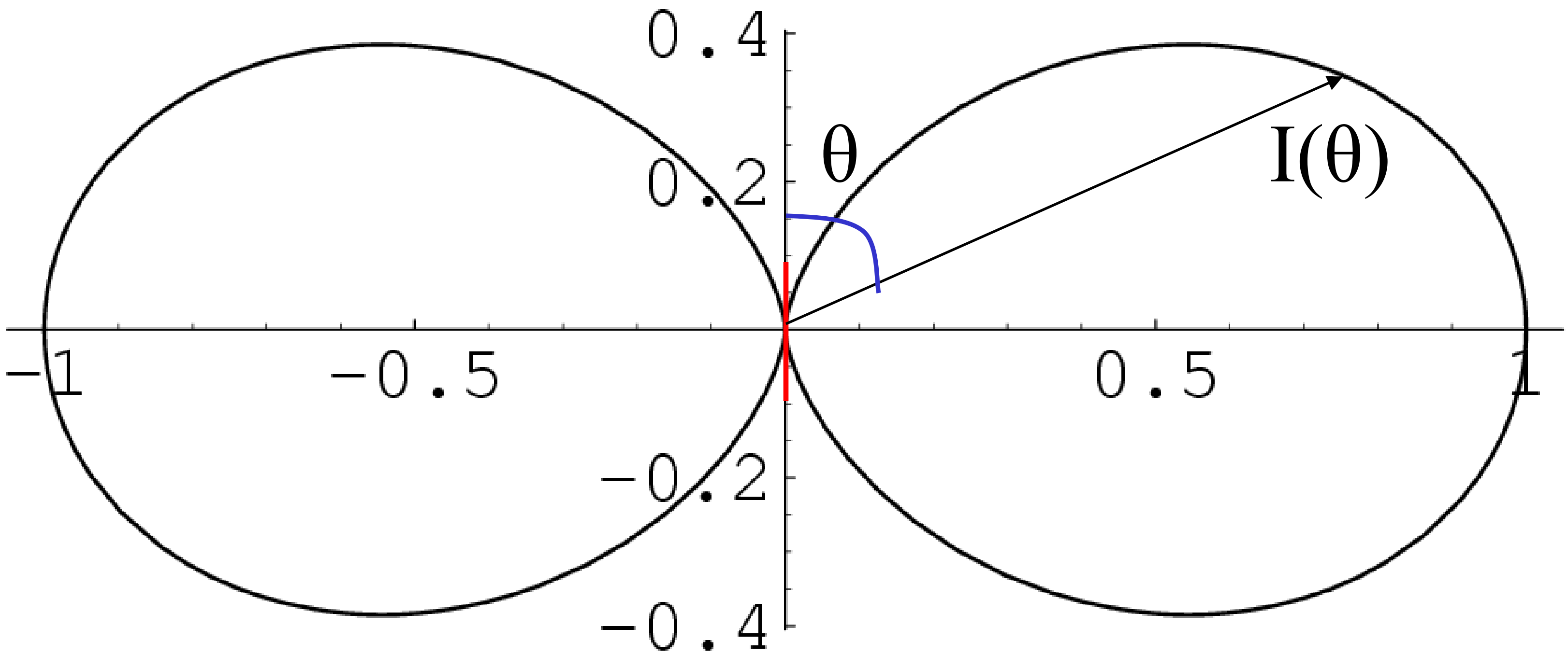
Energy radiated by an electric oscillating dipole
Poynting vector

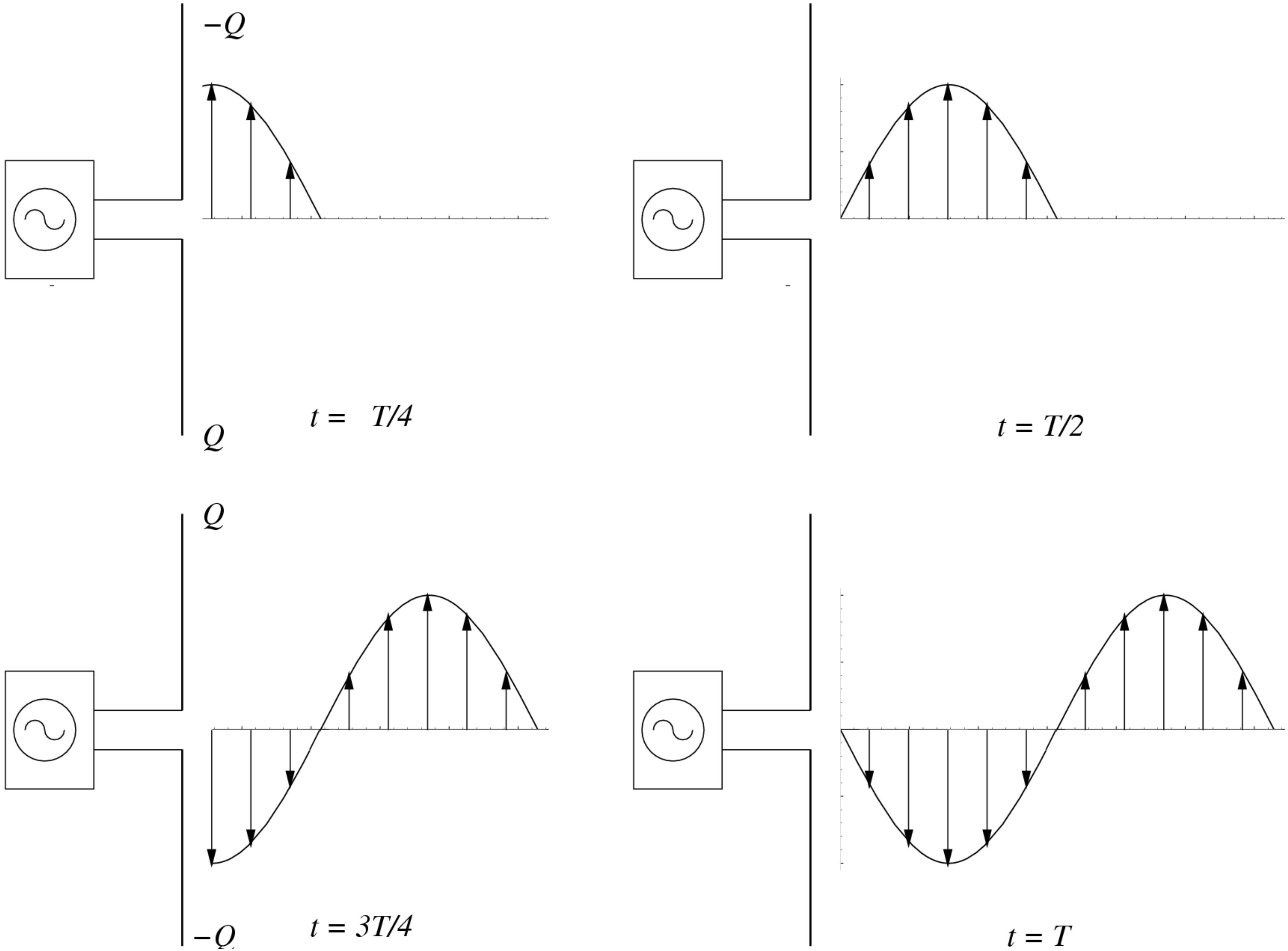
$$\mathbf{S} = (\mathbf{E} \times \mathbf{B})/\mu_0$$

Intensity averaged over one complete cycle

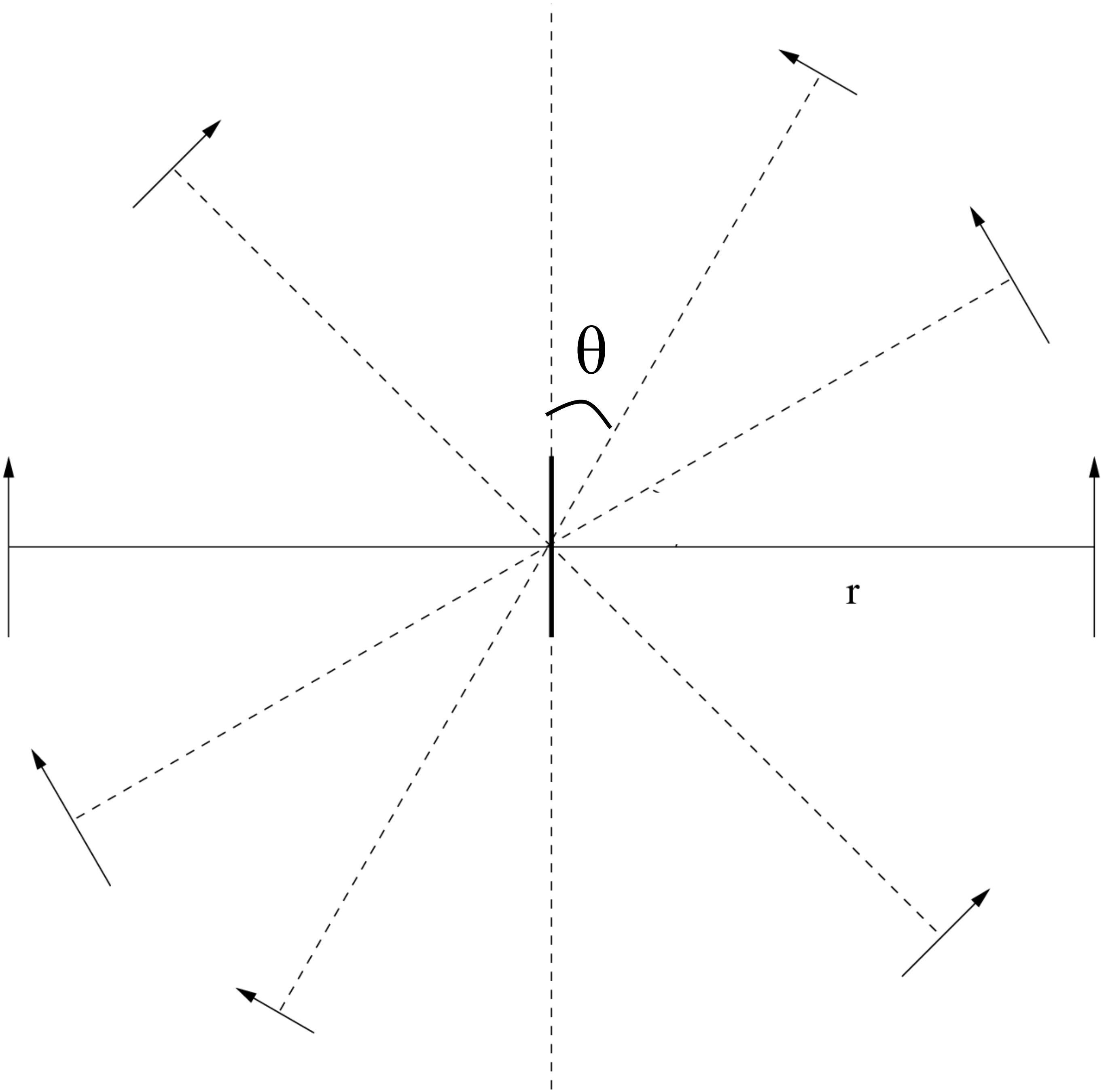
$$<\mathbf{S}> = \left(\frac{\mu_o p_o^2 \omega^4}{32\pi^2 c}\right) \frac{\sin^2\theta}{r^2} \hat{\mathbf{r}}$$

Radiation intensity –Polar Plot
(for a fixed distance, I(0)=1)

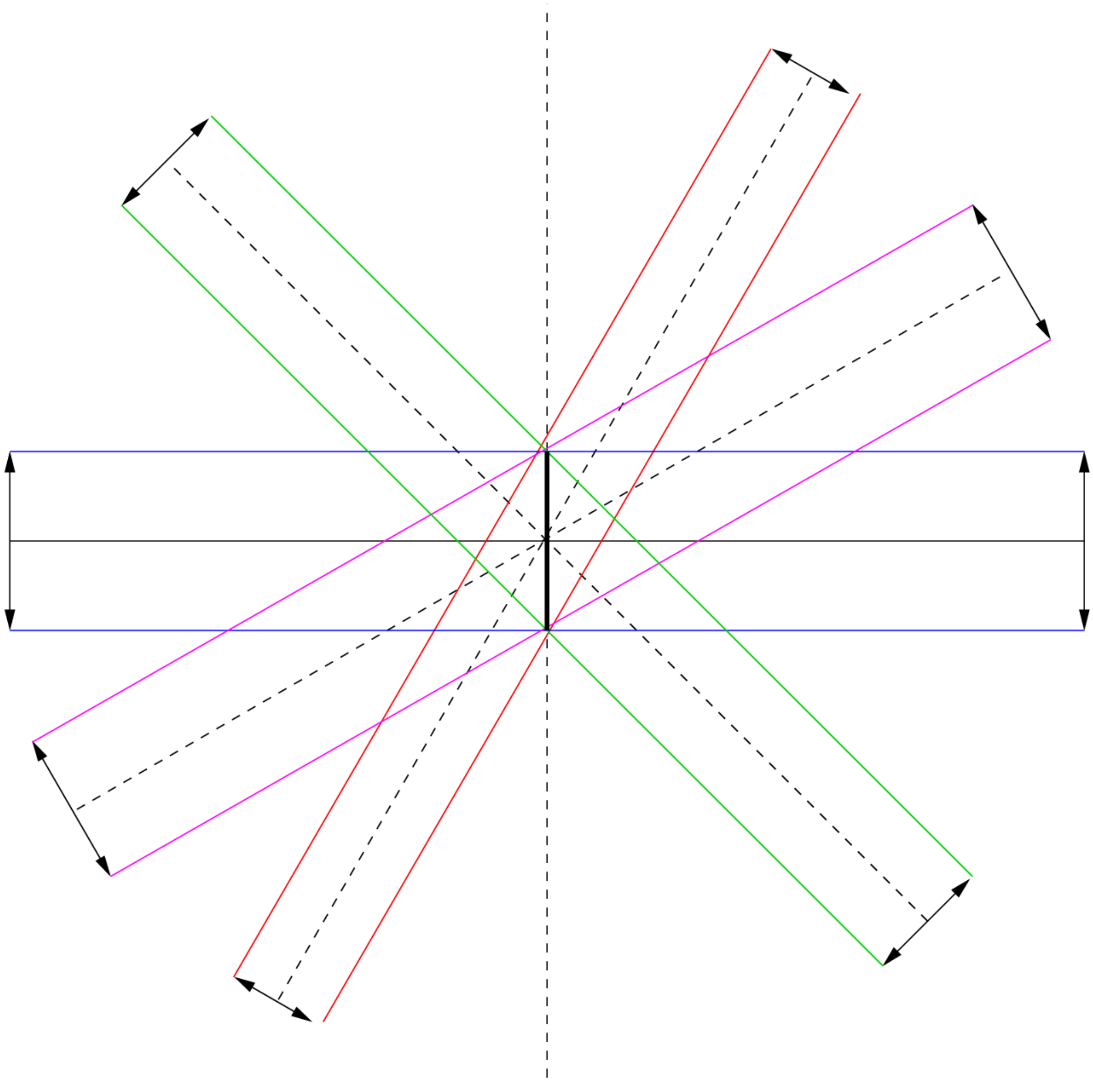




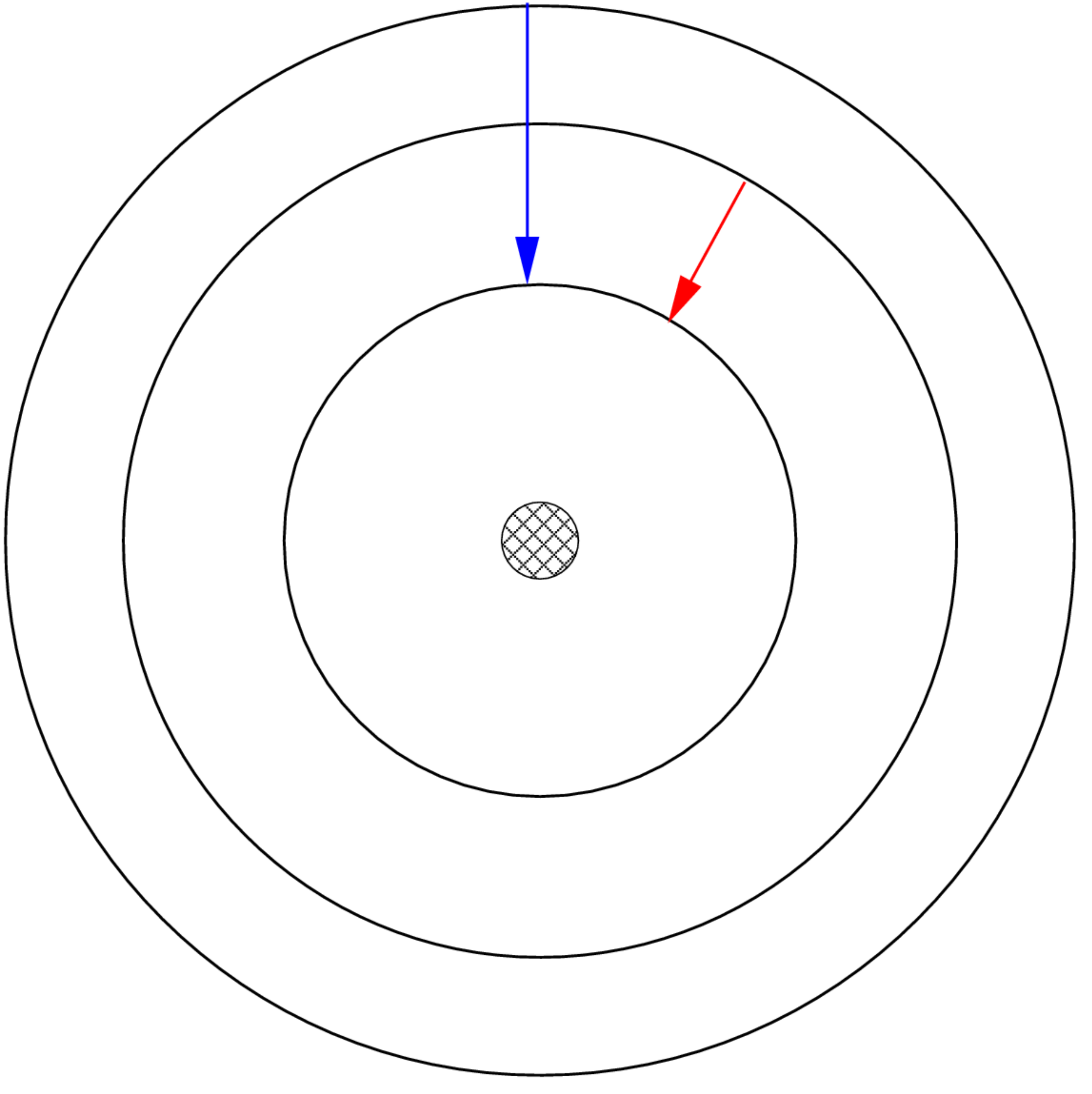
−Q
Q
t = T/4
t = T/2
Q
−Q
t = 3T/4
t = T

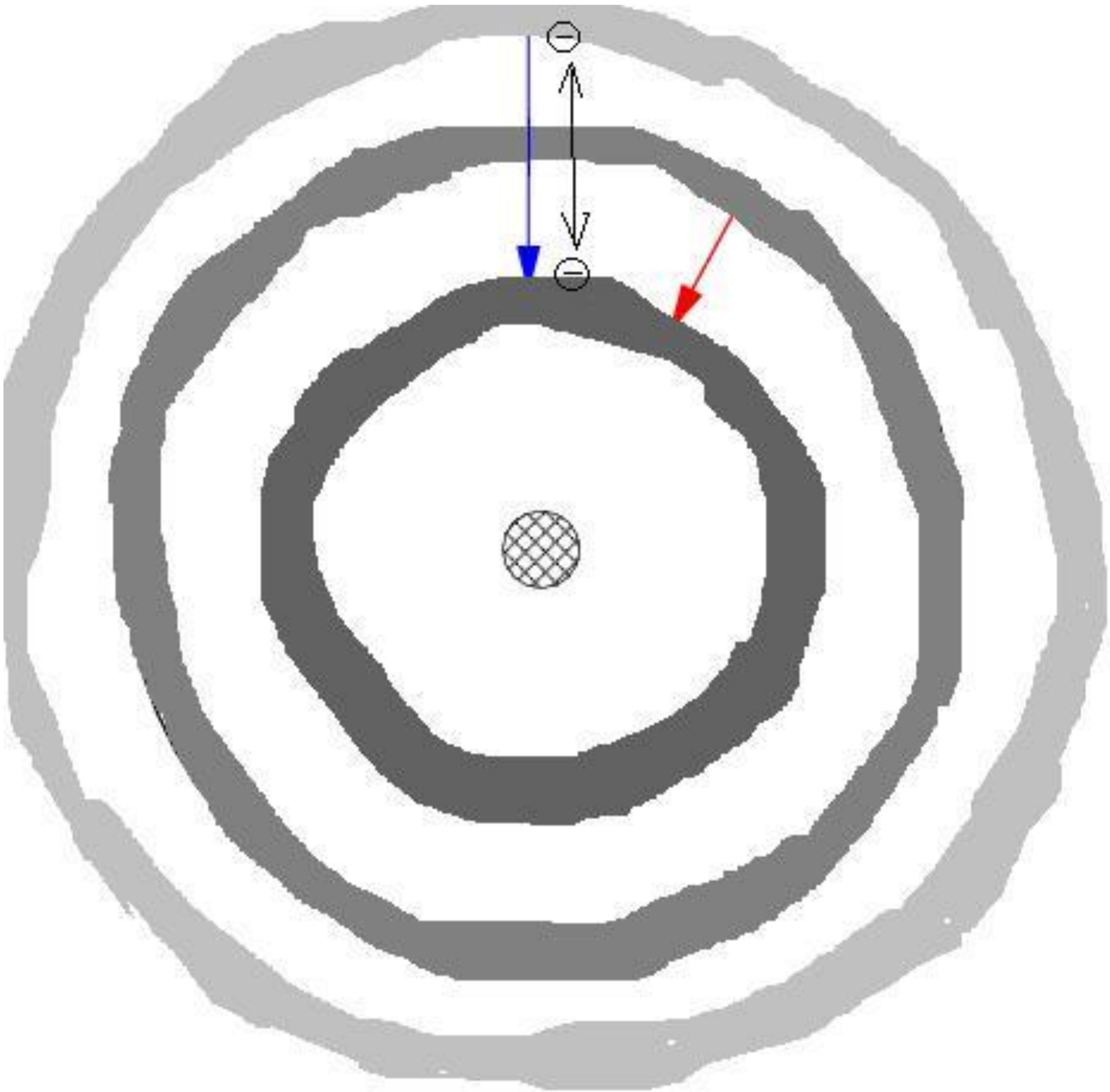


Dipole Radiation



Dipole Radiation





Two-beam interference

$$e_1(t) = E\exp(i\omega t)$$
$$e_2(t) = E\exp(i(\omega t + \phi))$$

Resultant phasor:

$$P = E(1 + \exp(i\phi))$$

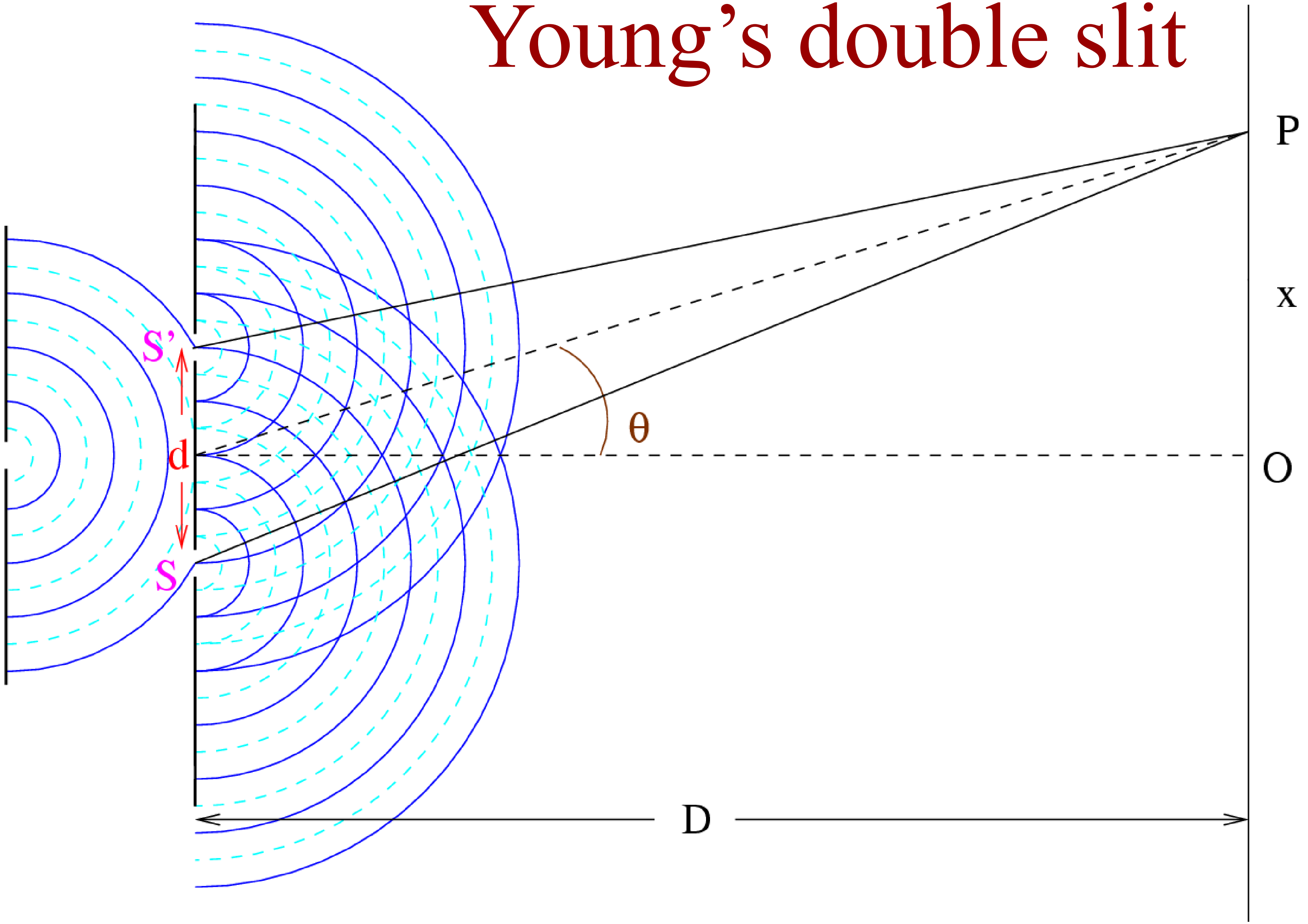
Intensity: $$I = PP^*$$

$$= E^2(1+\exp(i\phi))(1+\exp(-i\phi))$$

$$= 2E^2(1 + \cos\phi) = 4E^2\cos^2(\phi/2)$$

$$I = I_m \cos^2(\phi/2)$$

Young's double slit



Path difference: $SP - S'P$

$$= \sqrt{D^2 + (x + d/2)^2}$$

$$- \sqrt{D^2 + (x - d/2)^2}$$

$$D >> x, d$$

$$= D[1 + (x + d/2)^2/D^2]^{1/2}$$

$$- D[1 + (x - d/2)^2/D^2]^{1/2}$$

$$(1 + y)^n = 1 + nx$$

$$1 >> y$$

$$= [(x + d/2)^2 - (x - d/2)^2]/2D$$

$$= [(2x)d]/2D = xd/D$$

For a bright fringe,

$$SP - S'P = m\lambda$$

For a dark fringe,

$$SP - S'P = (2n + 1)\lambda/2$$

Position of m[th] maximum from centre

$$x_m \approx m\lambda D/d$$

Fringe width

$$\Delta x = \lambda D/d$$

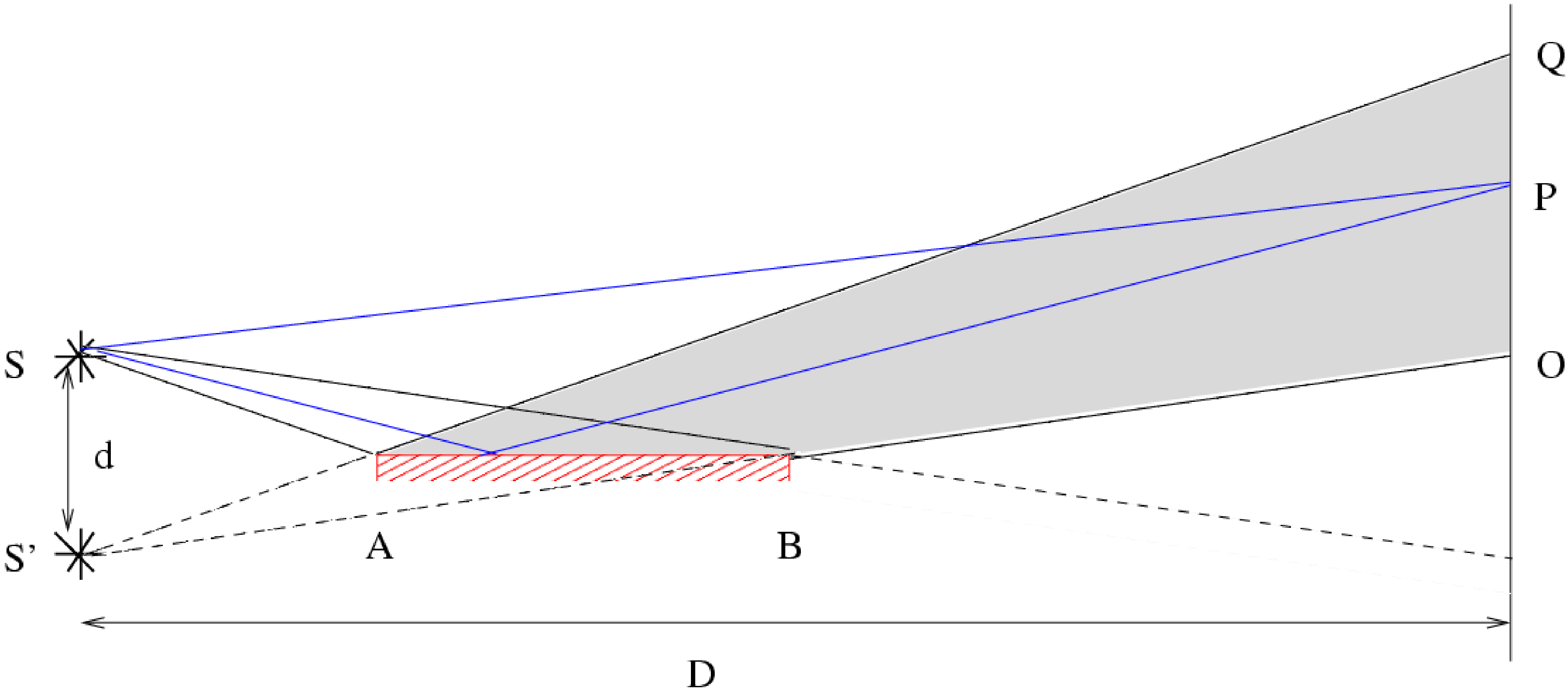
Phase difference:

$$\phi = 2\pi(SP - S'P)/\lambda$$

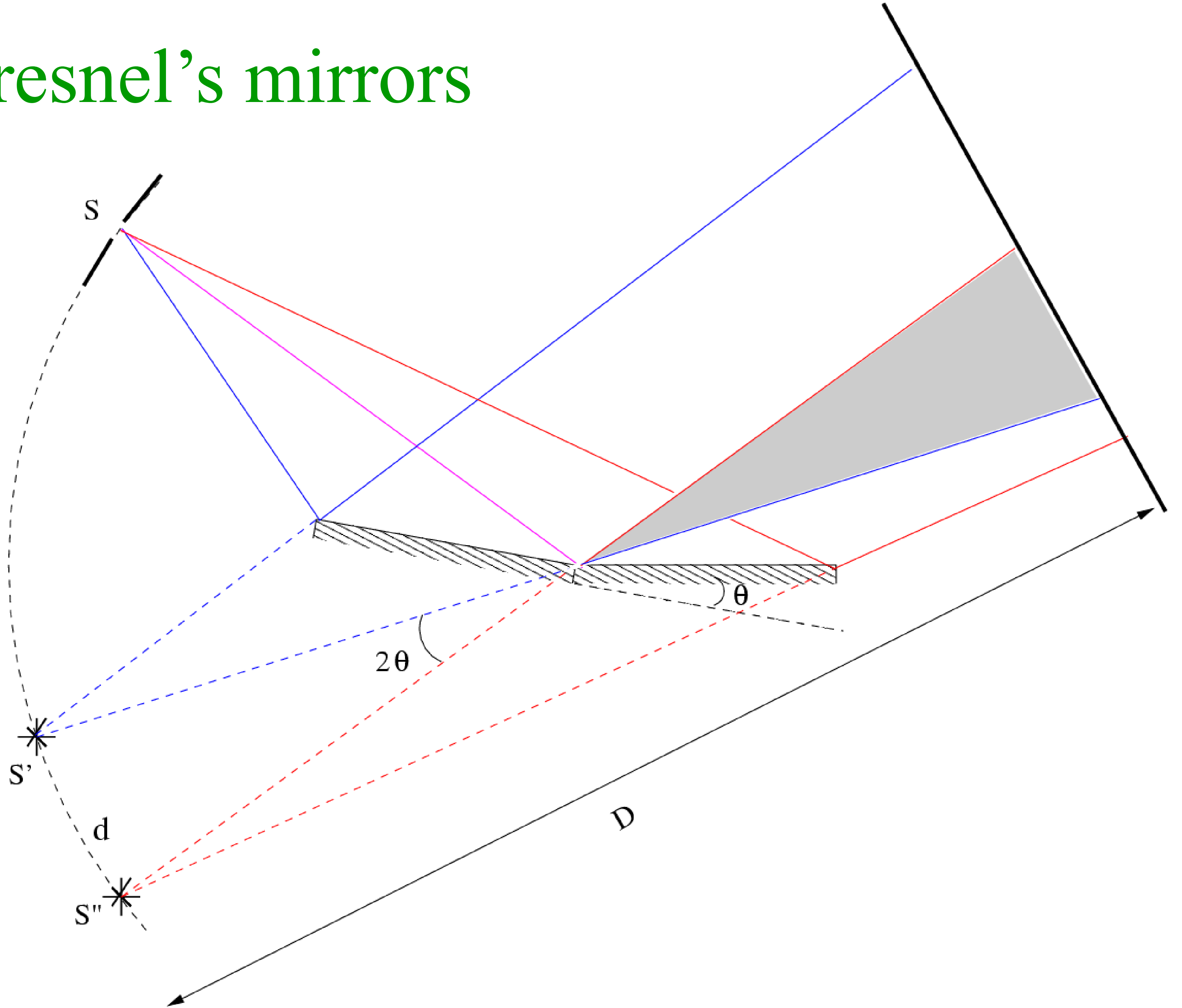
Realisations of Young's double slit interference

1. Lloyd's mirror

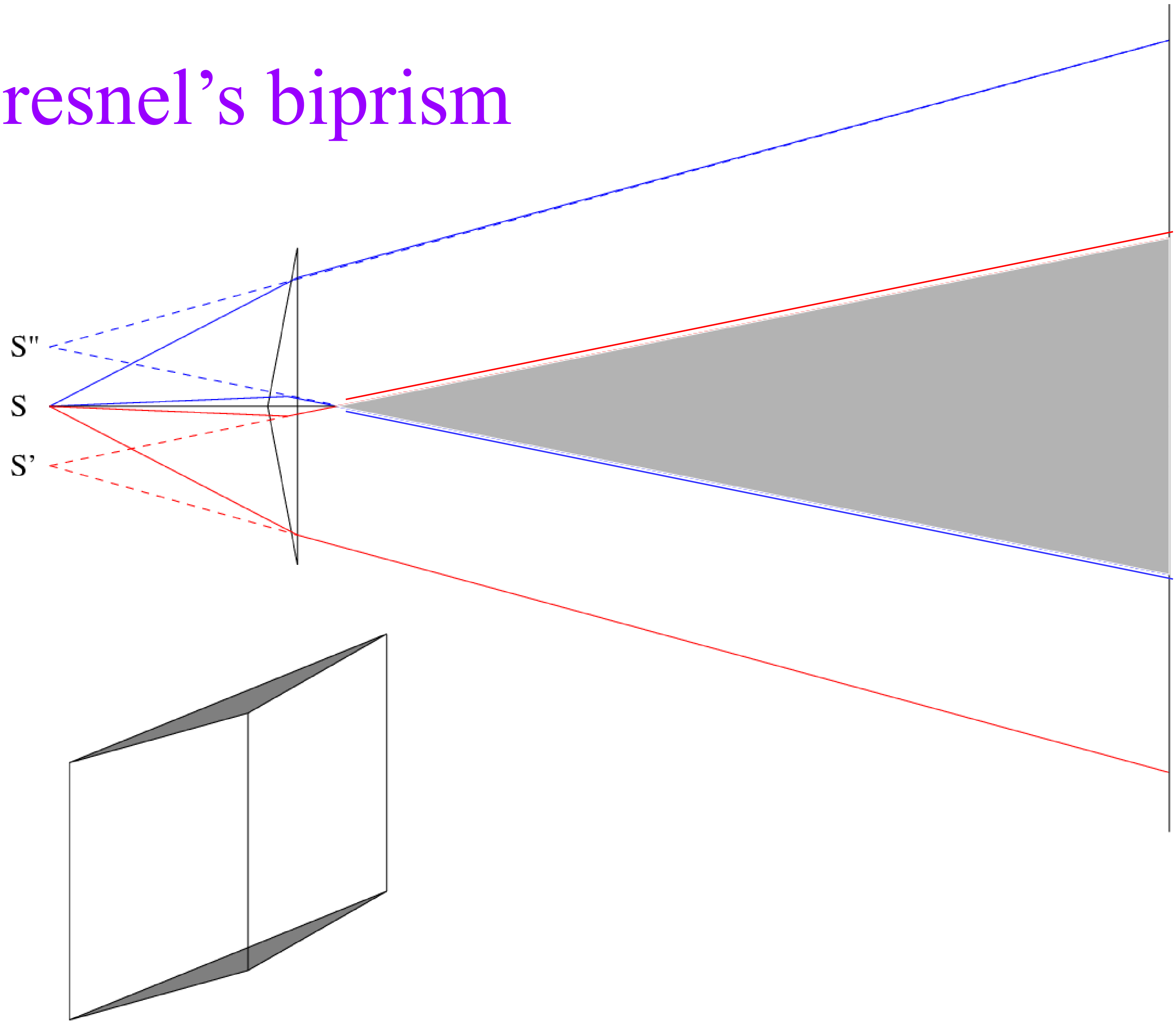
Transverse Section



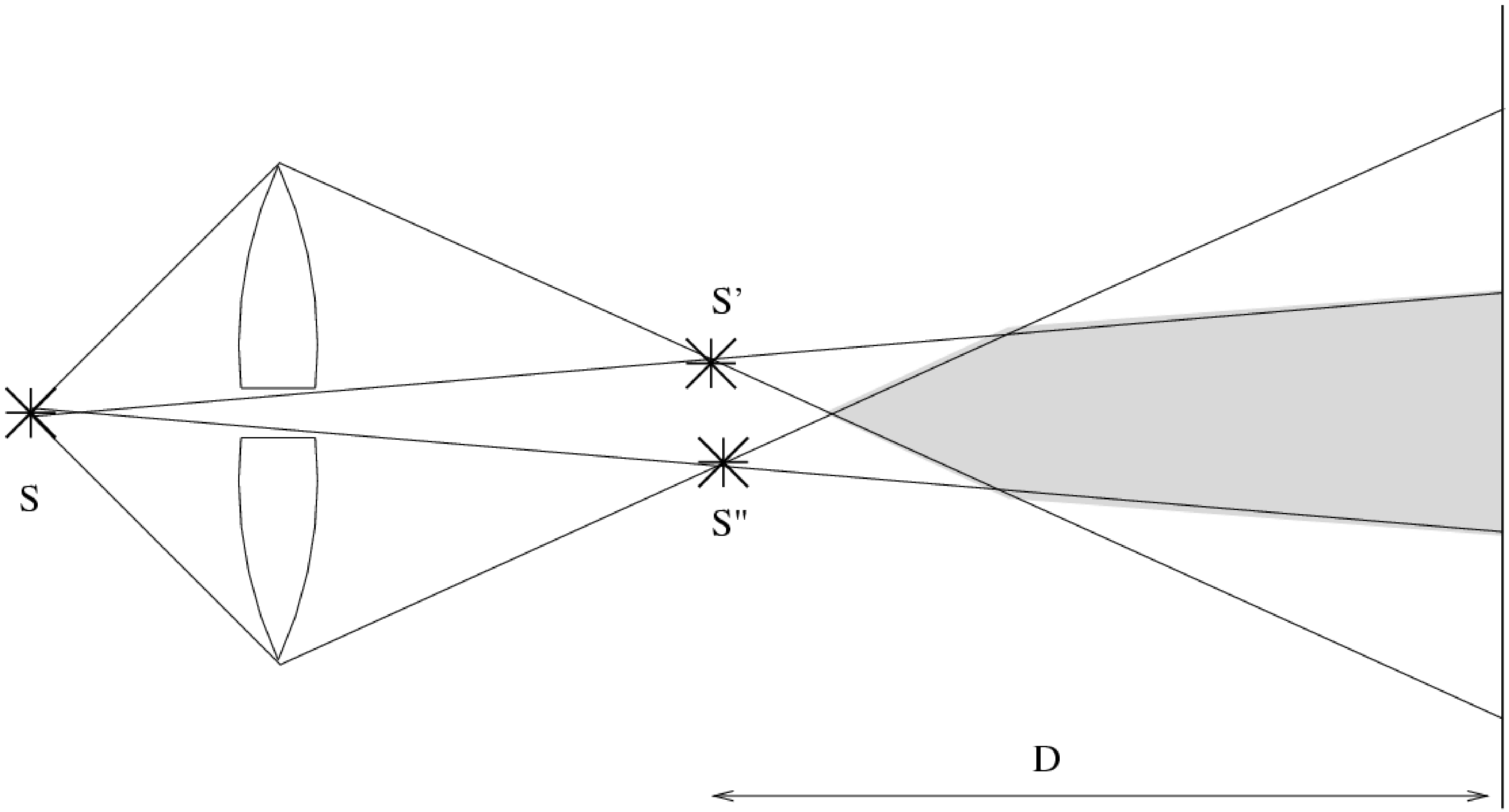
2. Fresnel's mirrors



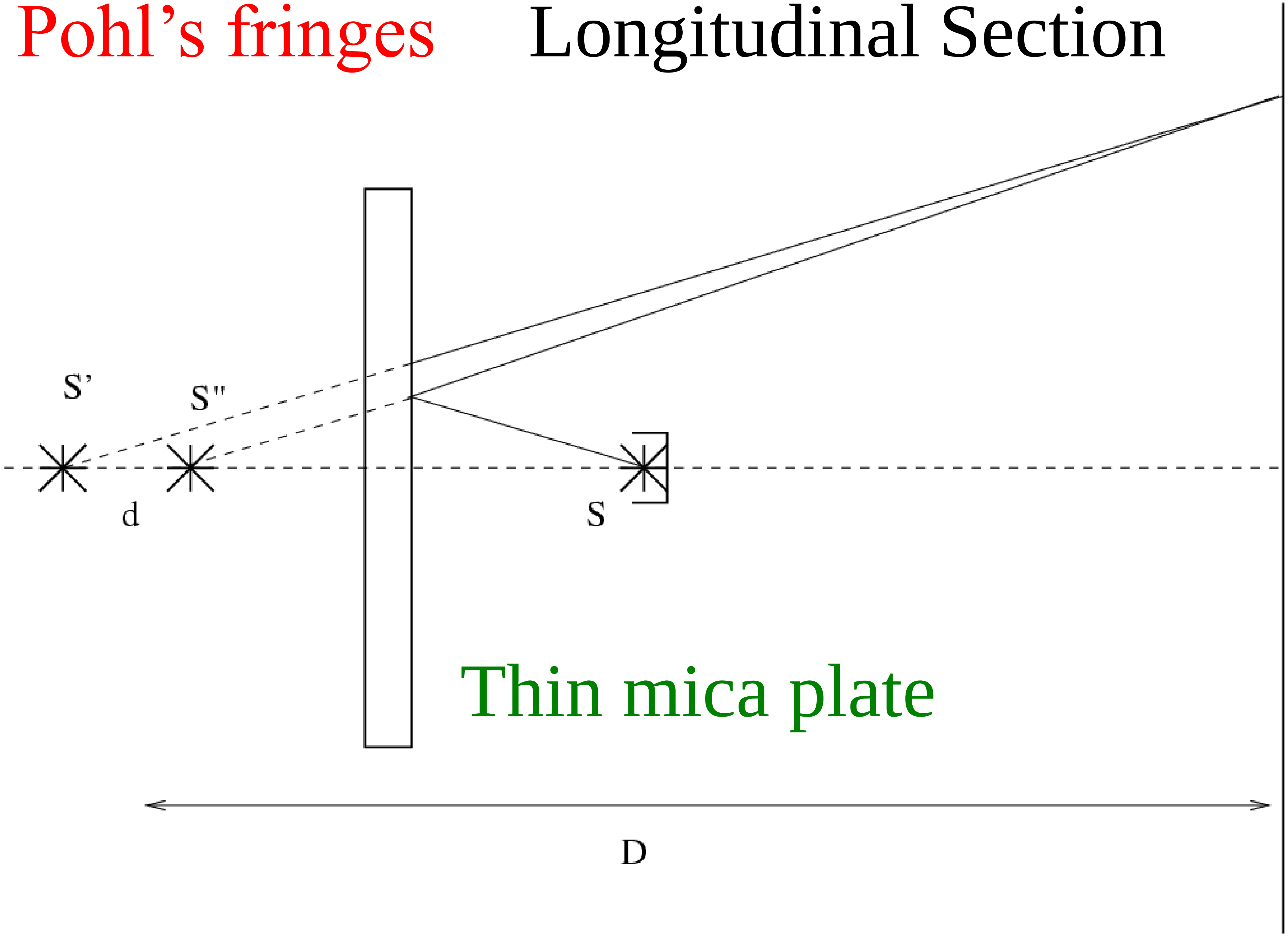
3. Fresnel’s biprism



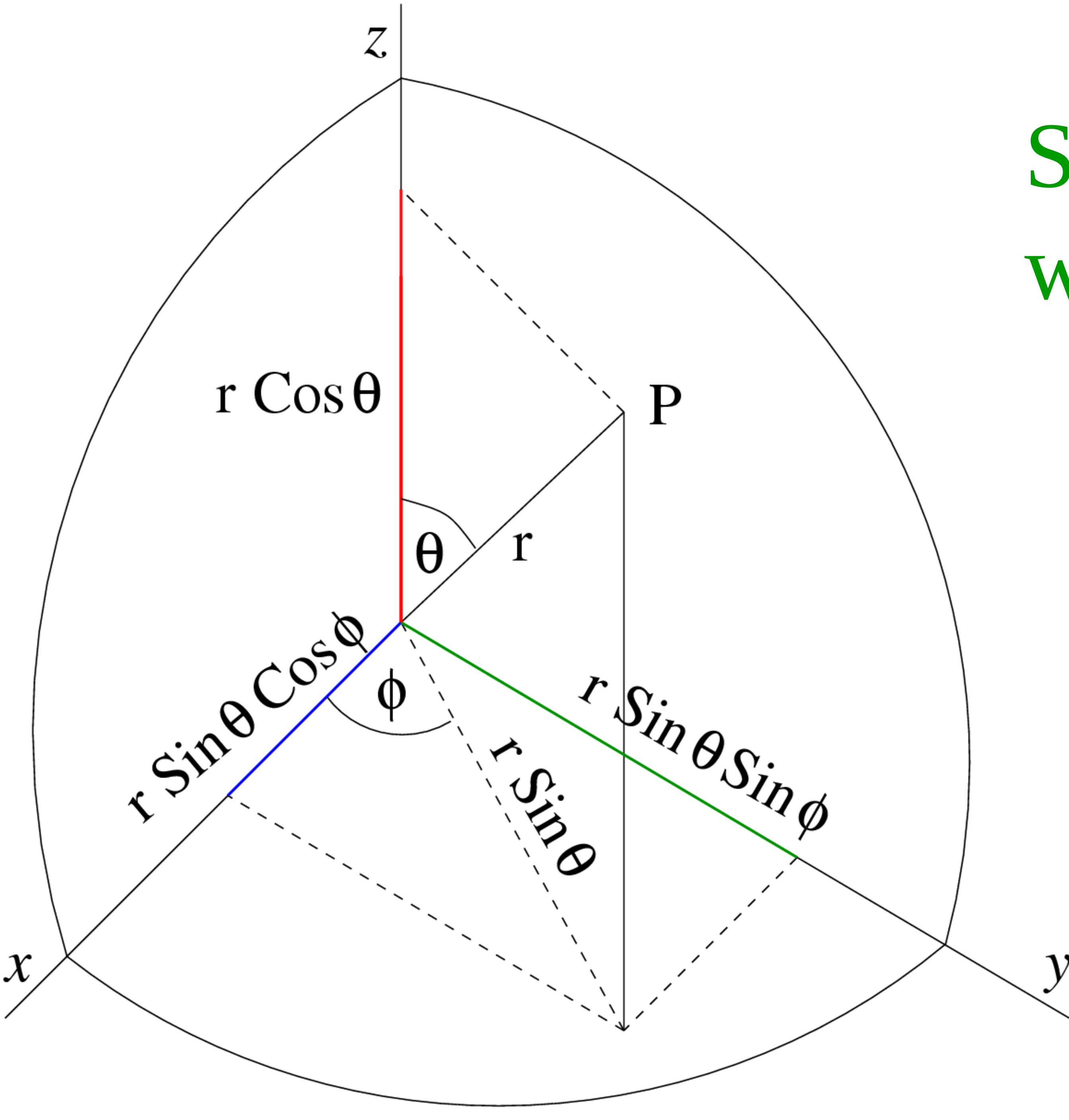
4. Billet's split lens



Pohl's fringes Longitudinal Section



Spherical waves



$$\psi(\mathbf{r}) = \psi(r, \theta, \phi) = \psi(r)$$

$$\nabla^2 \psi(r) = \frac{\partial^2 \psi}{\partial r^2} + \frac{2}{r}\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi)$$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

$$\frac{\partial^2}{\partial r^2}(r\psi) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}(r\psi)$$

$$r\psi(r) = A\exp(ik(r - vt))$$

$$\psi(r) = \frac{A}{r}\exp(ik(r - vt))$$