Superposition of several sinusoidal waves with same frequency and polarization (oscillations in the same plane) but different amplitudes and phases is again a *sinusoidal* wave with the same frequency. Resultant amplitude and phase are obtained by adding the individual phasors vectorially.

Superposition

$$|A_n| \cos(\omega t + \phi_n)$$

$$= \mathrm{Re}\ \mathbf{A}_n \exp(i\omega t)$$

$$\mathbf{A}_n = |A_n| \exp(i\phi_n)$$

$$\mathbf{A}_1 \exp(i\omega t) + \mathbf{A}_2 \exp(i\omega t) \cdot$$

$$+ \mathbf{A}_3 \exp(i\omega t) + \cdots$$

$$= \sum_n \mathbf{A}_n \exp(i\omega t)$$

$$= \mathbf{A} \exp(i\omega t)$$

Resultant phasor

$$\mathbf{A} = |A| \exp(i\phi)$$

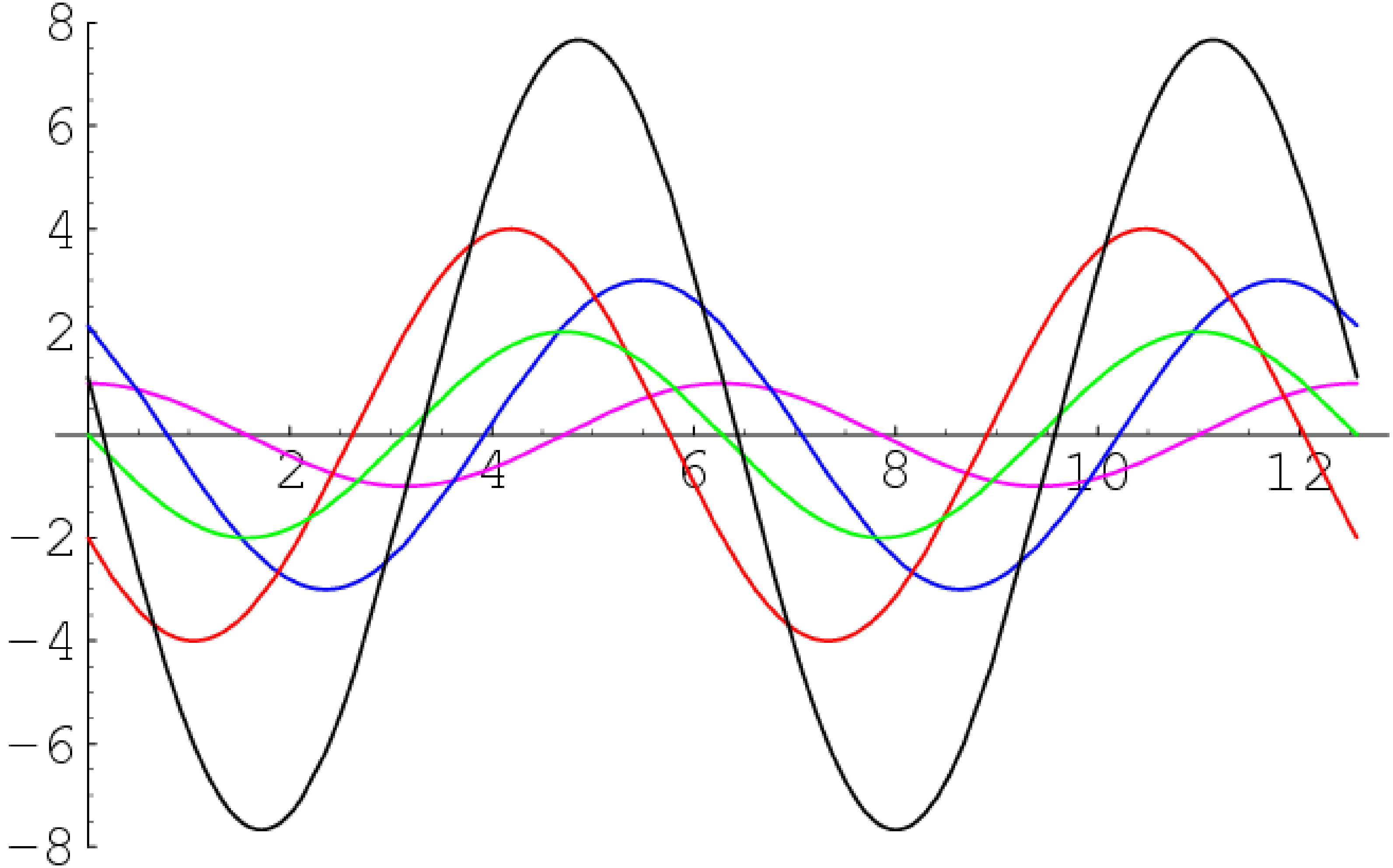
$$= \sum_n \mathbf{A}_n = \sum_n |A_n| \exp(i\phi_n)$$

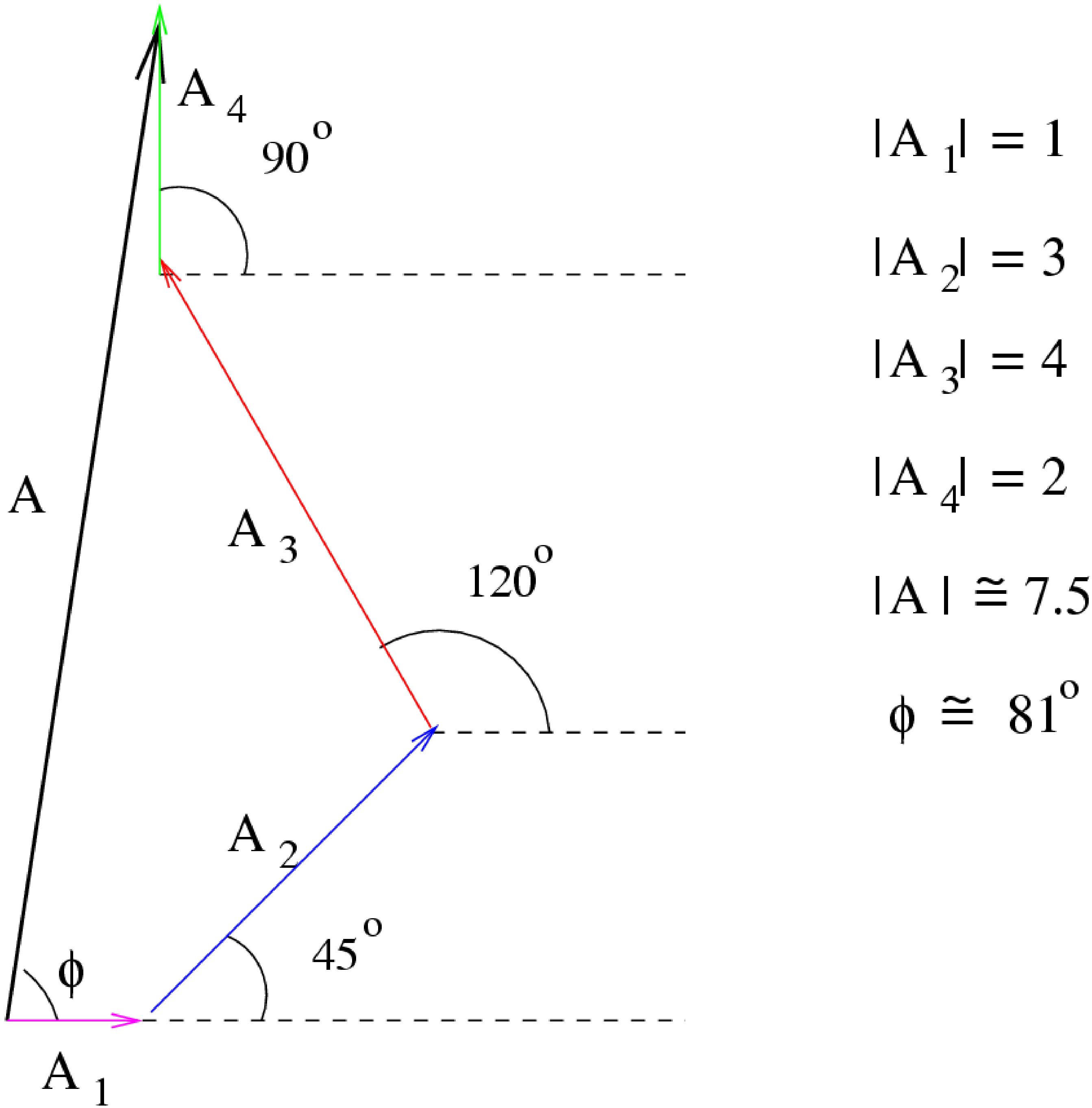
Energy (Intensity):

$$I = \mathbf{A}\mathbf{A}^* = |A|^2$$

$$\cos t + 3\cos(\pi/4 + t) + 4\cos(2\pi/3 + t)$$

$$+2\cos(\pi/2 + t)$$





A
A 1
A 2
A 3
A 4
φ
45°
120°
90°
|A 1| = 1
|A 2| = 3
|A 3| = 4
|A 4| = 2
|A| ≅ 7.5
φ ≅ 81°

Traveling waves

$$y(x,t) = |A_n| \cos(kx - \omega t + \phi_n)$$

$$y(x,t) = \mathrm{Re}\, \mathbf{A}_n \exp(i(kx - \omega t))$$

$$e_n(x,t) = \mathbf{E}_n \exp(i(kx - \omega t))$$

$$\mathbf{E}_n = |E_n| \exp(i\phi_n)$$

Wave vector k

(direction of propagation)

$$k = 2\pi/\lambda$$

Phase velocity v_{ph}

$$v_{ph} = \omega/k$$

Problem

Superposition of large number of phasors of equal amplitude *a* and equal successive phase difference θ. Find the resultant phasor.

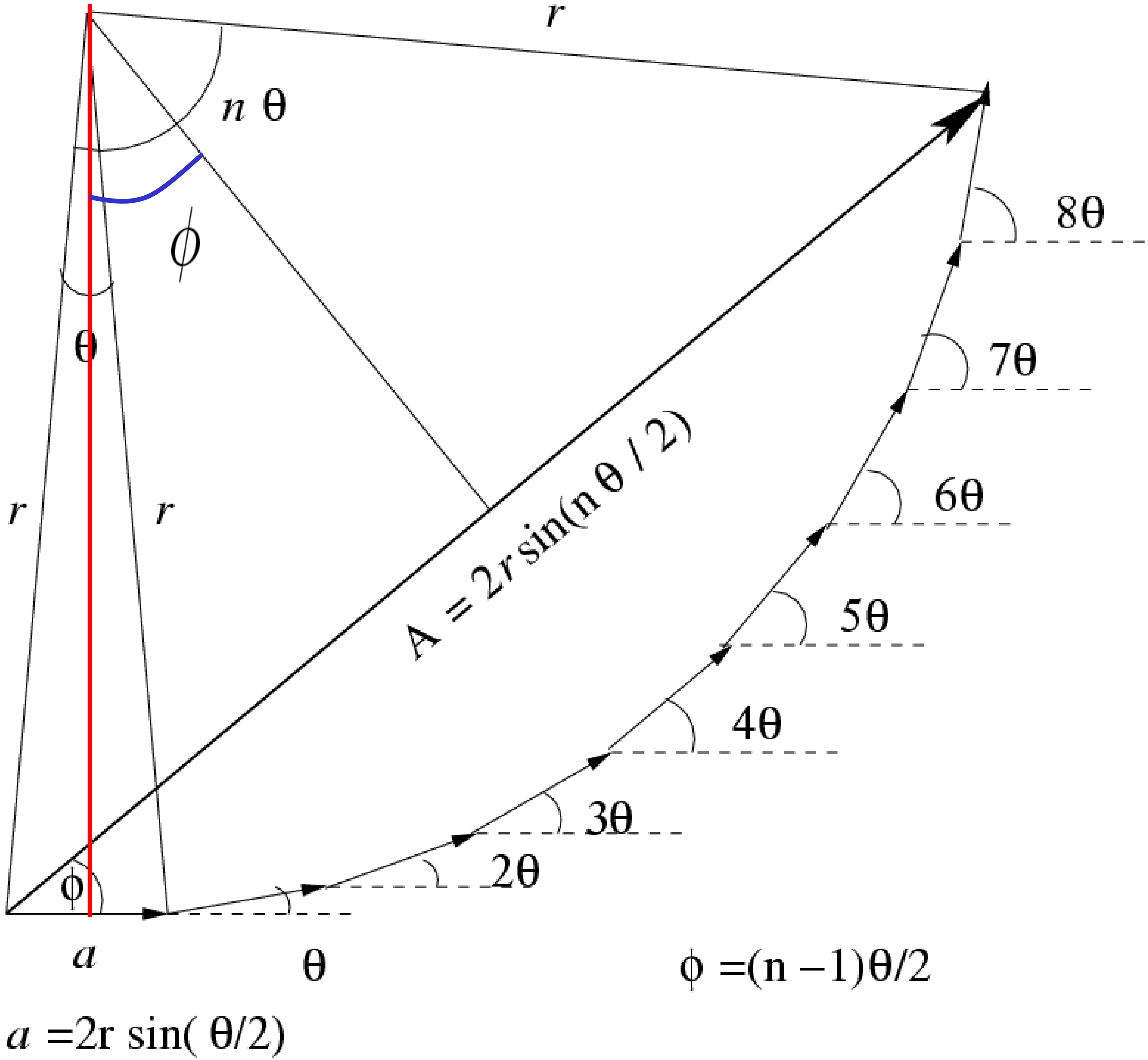
$$A = |A|\exp(i\phi) = a + a\exp(i\theta) + a\exp(i2\theta)$$
$$+a\exp(i3\theta) + \cdots a\exp(i(n-1)\theta)$$

$$A = a[1 - \exp(in\theta)]/[1 - \exp(i\theta)]$$

$$= a\frac{\sin(n\theta/2)}{\sin(\theta/2)}\exp(i(n-1)\theta/2)$$

$$|A| = a\frac{\sin(n\theta/2)}{\sin(\theta/2)}$$

$$\phi = (n-1)\theta/2$$


r
n θ
ϕ
θ
r
r
A = 2r sin(n θ / 2)
8θ
7θ
6θ
5θ
4θ
3θ
2θ
ϕ
a
θ
ϕ =(n −1)θ/2
a =2r sin(θ/2)

When n is large and θ and a are small such that

$$n\theta/2 = \alpha$$

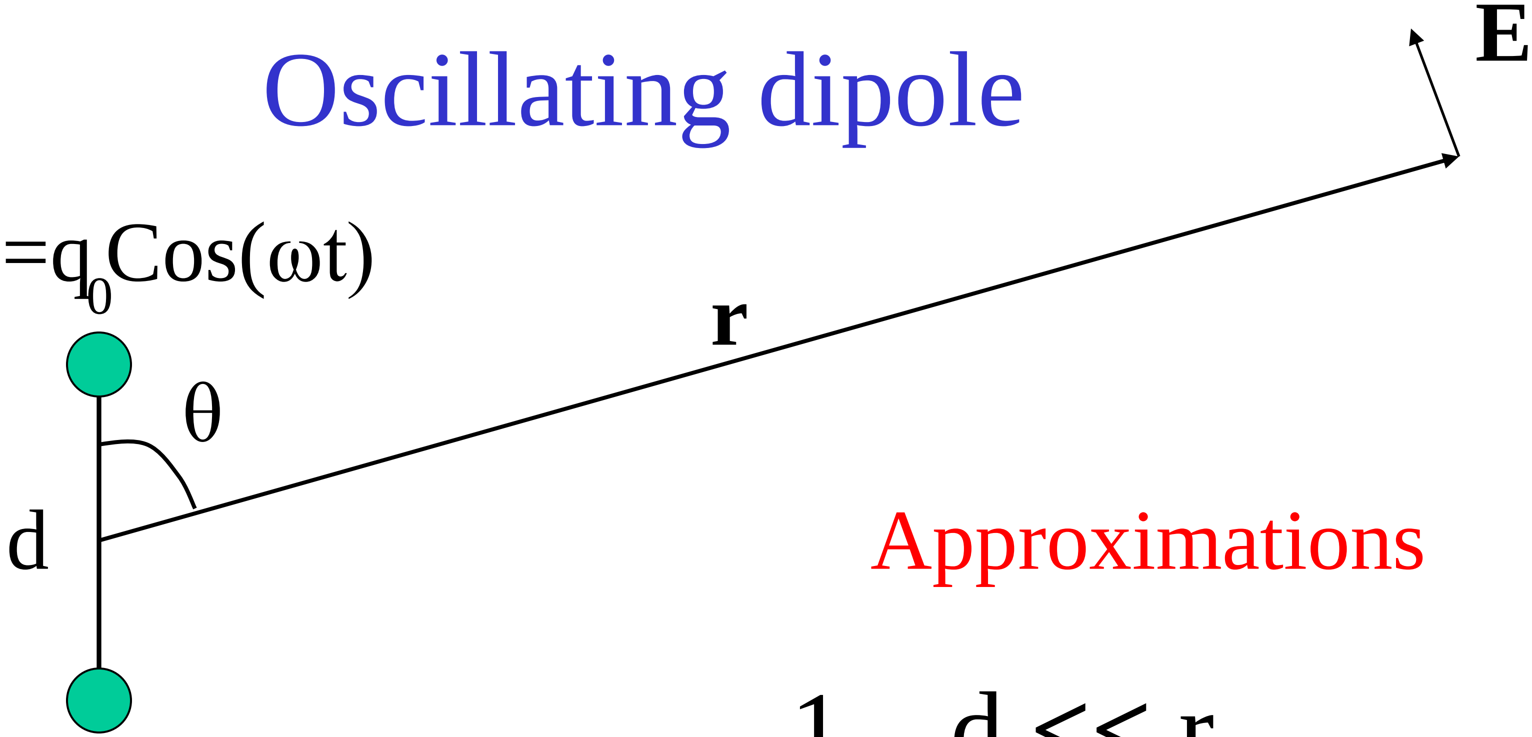
$$na = A_0$$

$$A = (A_0 \sin\alpha/\alpha)\exp(i\alpha)$$

$$I = AA^* = A_0^2 \sin^2\alpha/\alpha^2$$

Oscillating dipole

$q(t) = q_0 \text{Cos}(\omega t)$



$q(t) = -\, q_0 \text{Cos}(\omega t)$

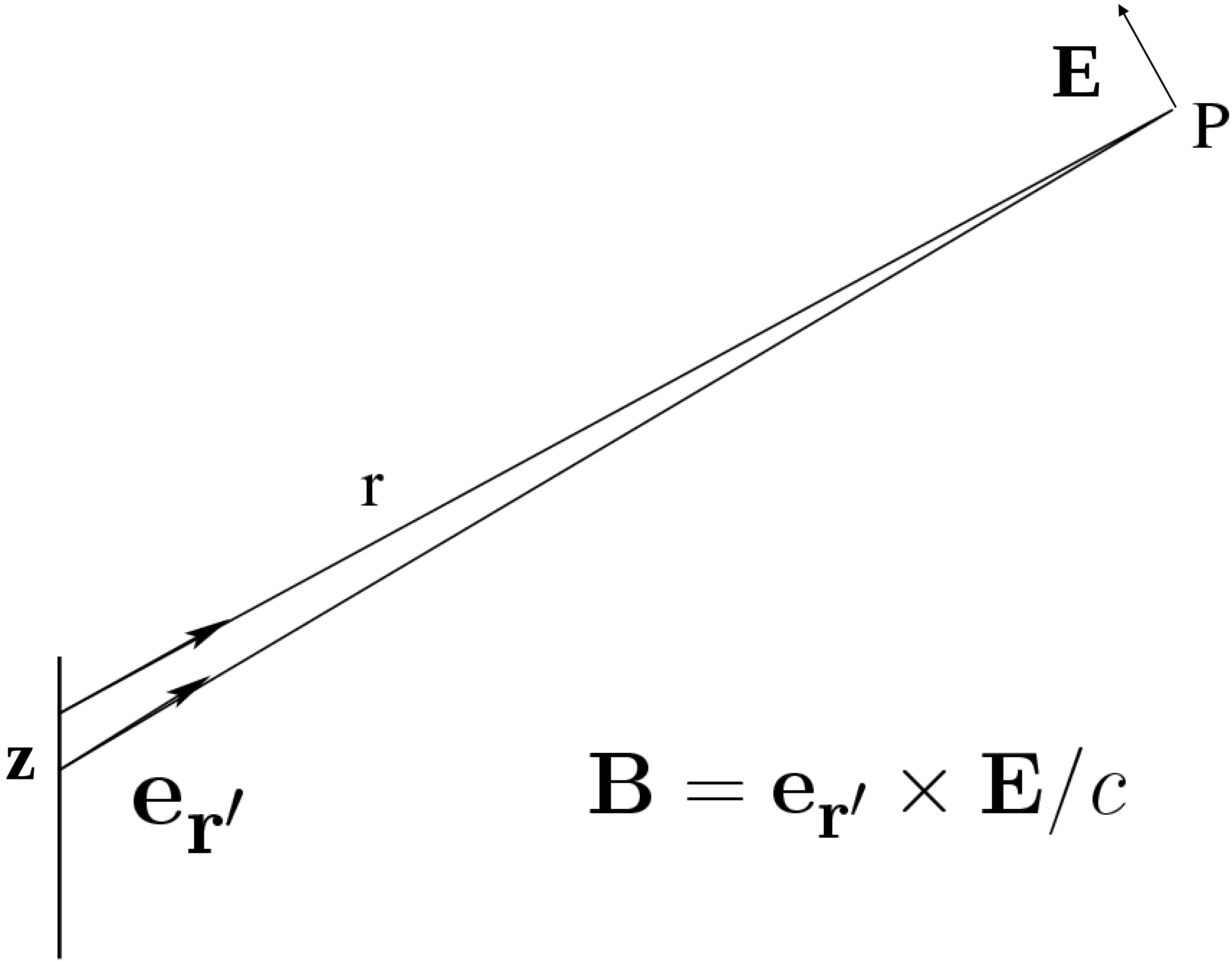
$\mathbf{p}(t) = p_0 \text{Cos}(\omega t)\, \mathbf{z}$

Maximun dipole moment

$p_0 = q_0 d$

Approximations

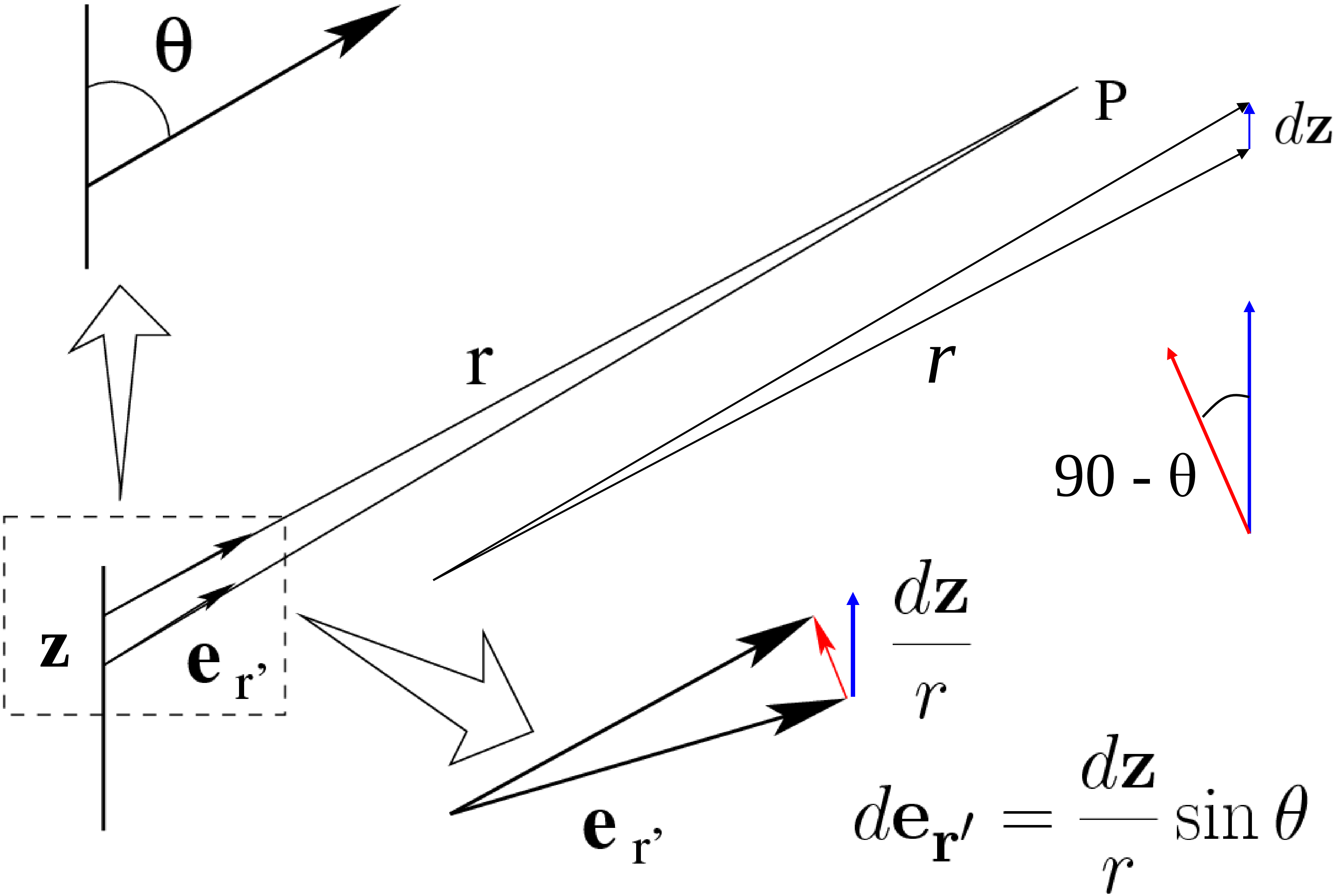
1. $d \ll r$
2. $d \ll c/\omega \sim \lambda$
3. $c/\omega \sim \lambda \ll r$


E
P
r
z
$\mathbf{e}_{\mathbf{r}'}$
$\mathbf{B} = \mathbf{e}_{\mathbf{r}'} \times \mathbf{E}/c$

Oscillating charges and dipole radiation

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0}\left[\frac{\mathbf{e_{r'}}}{r'^2} + \frac{r'}{c}\frac{d}{dt}\left(\frac{\mathbf{e_{r'}}}{r'^2}\right) + \frac{1}{c^2}\frac{d^2\mathbf{e_{r'}}}{dt^2}\right]$$

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0 c^2}\left[\frac{d^2\mathbf{e_{r'}}}{dt^2}\right]$$


θ
P
$d\mathbf{z}$
r
r
90 - θ
z
$\mathbf{e}_{r'}$
$\frac{d\mathbf{z}}{r}$
$\mathbf{e}_{r'}$
$d\mathbf{e}_{\mathbf{r}'} = \frac{d\mathbf{z}}{r}\sin\theta$

Oscillating charges and dipole radiation

For large distances (r) the first two terms die as $1/r^2$, whereas the last term (radiation term) survives because it decays as $1/r$.

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0 c^2}\left[\frac{d^2\mathbf{e}_{\mathbf{r}'}}{dt^2}\right]$$

$$E_z(r,t) = \frac{q}{4\pi\epsilon_0 c^2 r} a_z(t - r/c)$$

$$a_z(t - r/c) = \frac{d^2\mathbf{z}(t - r/c)}{dt^2}$$

$$\mathbf{E}(r,t) = -\frac{\mu_0 p_0}{4\pi r}\omega^2 \cos(\omega(t - r/c)) \sin\theta \hat{\theta}$$