Maxwell's equation in vacuum

Gauss's laws

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

Faraday's law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Ampere's law (modified)

$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

The wave equation

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\nabla \times \left(\frac{\partial \mathbf{B}}{\partial t} \right)$$

$$= -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) = -\mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla^2 \mathbf{E} = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} \qquad c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\nabla\times(\nabla\times\mathbf{B}) = \nabla(\nabla\cdot\mathbf{B}) - \nabla^2\mathbf{B} = \nabla\times\left(\mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)$$

$$= \mu_0\epsilon_0\frac{\partial}{\partial t}(\nabla\times\mathbf{E}) = -\mu_0\epsilon_0\frac{\partial^2\mathbf{B}}{\partial t^2}$$

$$\nabla^2\mathbf{B} = \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2}$$

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{4\pi\times10^{-7}\cdot8.85\times10^{-12}}} \quad \text{m/s}$$

3D-Plane waves

$$\psi(\mathbf{r}) = A\sin(\mathbf{k}\cdot\mathbf{r})$$

$$\psi(\mathbf{r}) = B\cos(\mathbf{k}\cdot\mathbf{r})$$

$$\psi(\mathbf{r}) = C\exp(i\mathbf{k}\cdot\mathbf{r})$$

The surface of constant phase:

$$\mathbf{k} \cdot \mathbf{r} = \phi_c$$

$$k_x x + k_y y + k_z z = \phi_c$$

$$\mathbf{r_1}, \quad \mathbf{r_2}$$

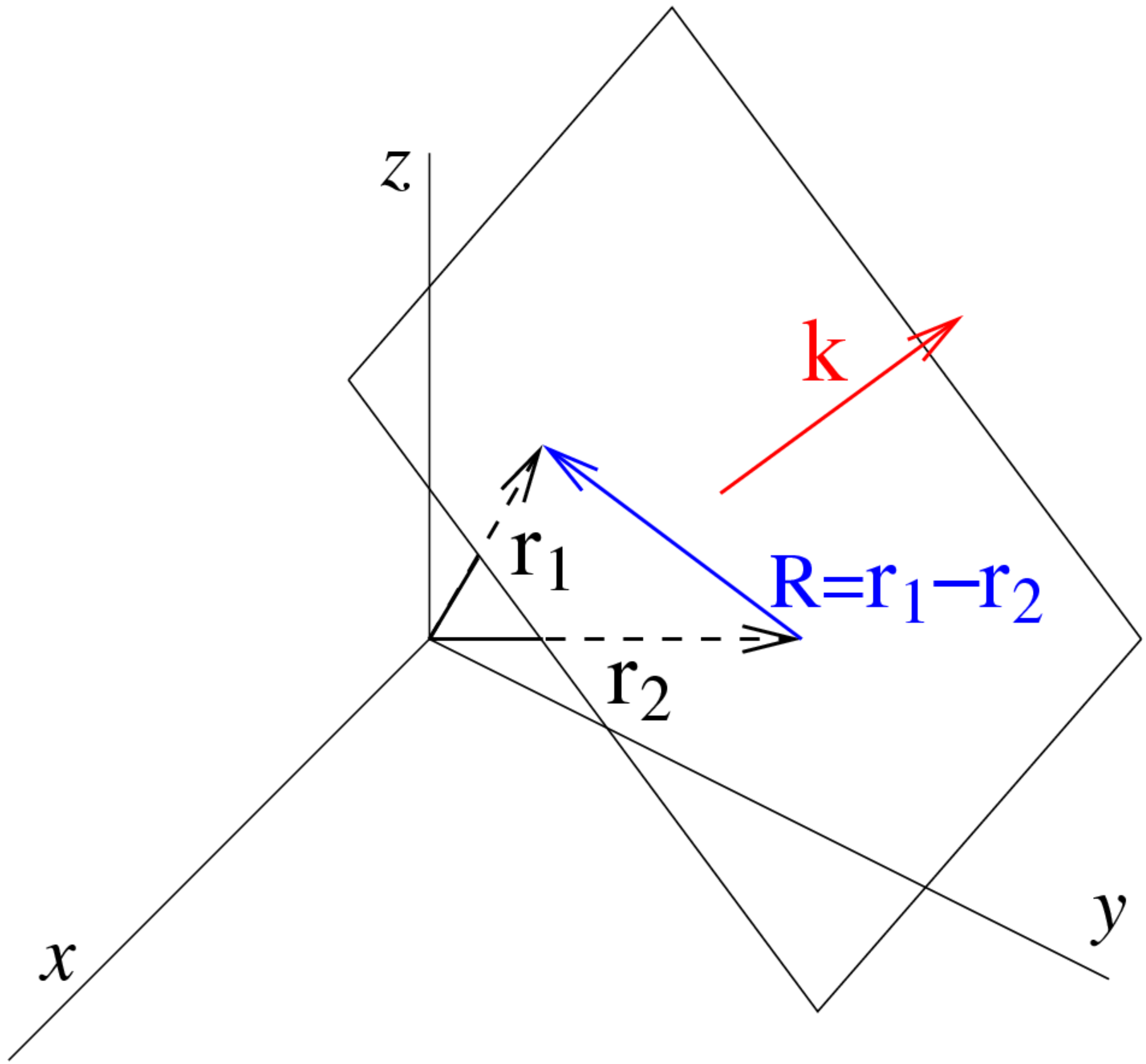
$$\mathbf{k} \cdot \mathbf{r_1} = \mathbf{k} \cdot \mathbf{r_2} = \phi_c$$

$$\mathbf{k} \cdot (\mathbf{r_1} - \mathbf{r_2}) = 0$$

$$\mathbf{r_1} - \mathbf{r_2} = \mathbf{R}$$

$$\mathbf{k} \cdot \mathbf{R} = 0$$

Vectors $\mathbf{k}$ and $\mathbf{R}$ are orthogonal to each other. So the surface swapped by a constant phase is a two dimensional plane and the vector $\mathbf{k}$ is normal to that plane.


z
k
r1
R=r1−r2
r2
x
y

Spatial periodic behaviour of $\psi(\mathbf{r})$

$$\psi(\mathbf{r}) = \psi(\mathbf{r} + \lambda\hat{\mathbf{k}})$$

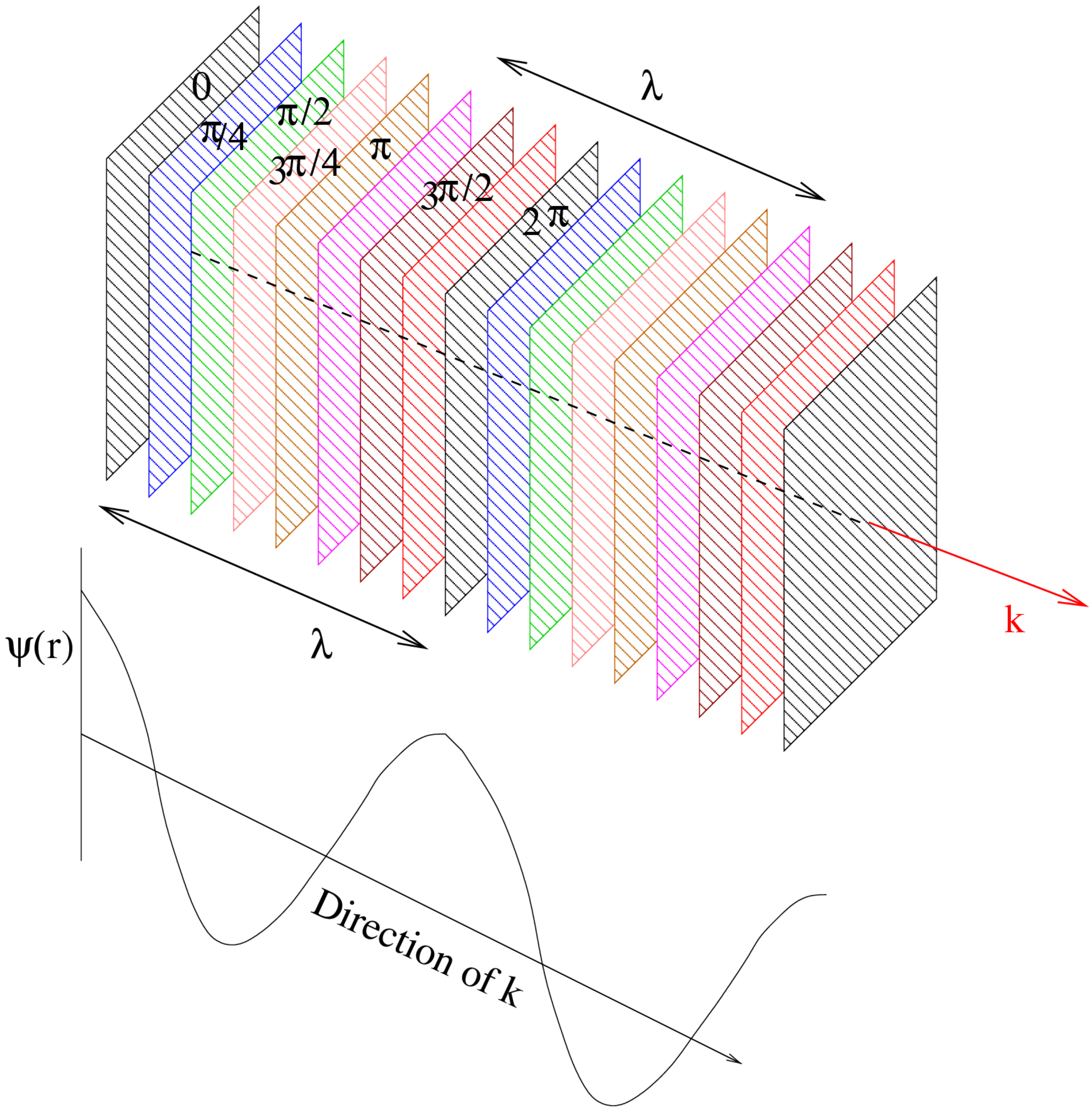
Not to be confused with unit vector along the z direction

$$\hat{\mathbf{k}} = \mathbf{k}/k, \quad k = |\mathbf{k}|$$

$$C \exp(i\mathbf{k}\cdot\mathbf{r}) = C \exp(i\mathbf{k}\cdot(\mathbf{r} + \lambda\hat{\mathbf{k}}))$$

$$= C \exp(i\mathbf{k}\cdot\mathbf{r}) \exp(i\lambda k)$$

$$\Longrightarrow \lambda k = 2\pi$$


0
π/4
π/2
3π/4
π
3π/2
2 π
λ
λ
k
ψ(r)
Direction of k

Traveling 3D wave

$$\psi(\mathbf{r}, t) = C \exp(i(\mathbf{k} \cdot \mathbf{r} \mp \omega t))$$

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$v^2 = \omega^2 / k^2$$

Electromagnetic waves in vacuum

$$\nabla^2\mathbf{E} = \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}$$

$$\nabla^2\mathbf{B} = \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2}$$

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}\exp(i(\mathbf{k}\cdot\mathbf{r} - \omega t))$$

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}\exp(i(k_x x + k_y y + k_z z - \omega t))$$

$$\mathbf{B}(\mathbf{r},t) = \mathbf{B}\exp(i(k_x x + k_y y + k_z z - \omega t))$$

$$\mathbf{k} \cdot \mathbf{k} = k_x^2 + k_y^2 + k_z^2 = k^2 = \omega^2 / c^2$$

$$\nabla \cdot \mathbf{E}(\mathbf{r}, t) = \mathbf{k} \cdot \mathbf{E} = 0$$

Wave vector **k** is perpendicular to **E**

$$\nabla \cdot \mathbf{B}(\mathbf{r}, t) = \mathbf{k} \cdot \mathbf{B} = 0$$

Wave vector **k** is perpendicular to **B**

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t}$$

$$\mathbf{k} \times \mathbf{E} = \omega \mathbf{B}$$

$$\hat{\mathbf{k}} \times \mathbf{E} = \frac{\omega}{k}\mathbf{B} = c\mathbf{B}$$

B is perpendicular to **E**

$$\nabla \times \mathbf{B}(\mathbf{r},t) = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}(\mathbf{r},t)}{\partial t}$$

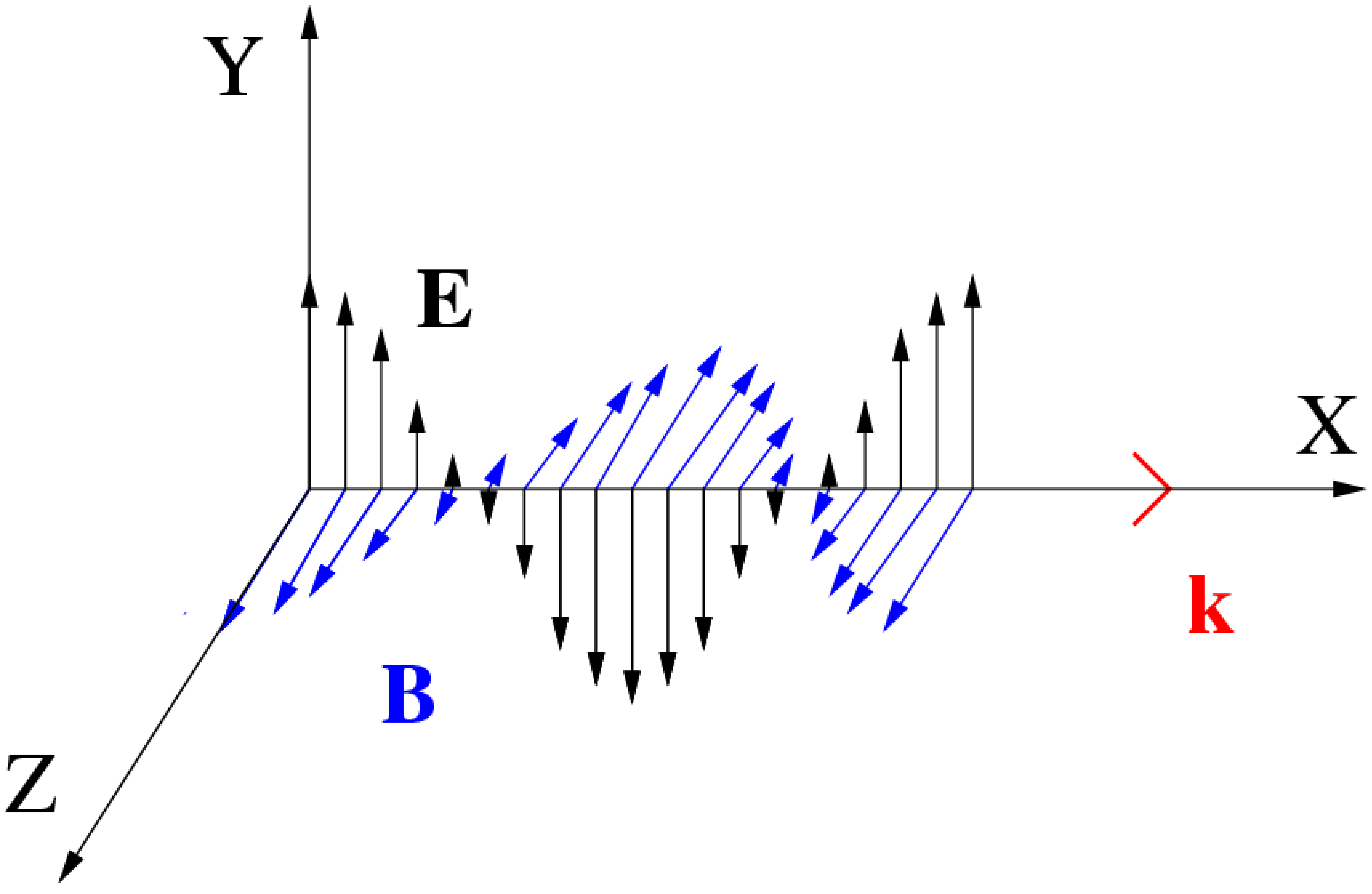
$$\mathbf{k} \times \mathbf{B} = -\frac{\omega}{c^2}\mathbf{E}$$

$$\mathbf{B} \times \hat{\mathbf{k}} = \frac{\omega}{kc^2}\mathbf{E} = \frac{1}{c}\mathbf{E}$$

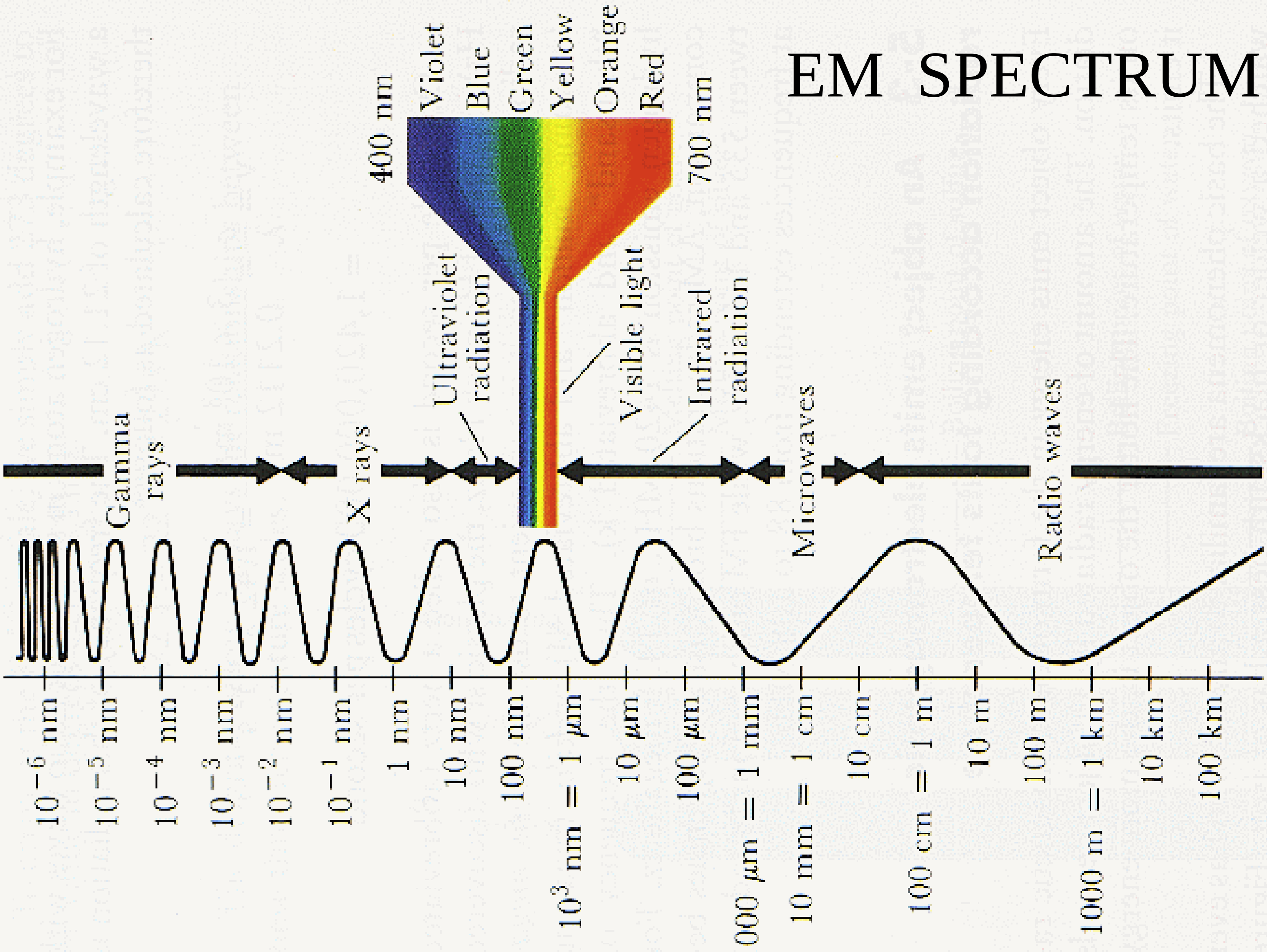
$$c\mathbf{B} \times \hat{\mathbf{k}} = \mathbf{E}$$

B, k and **E** make a right handed Cartesian co-ordinate system

Electromagnetic waves in vacuum



EM SPECTRUM



400 nm
Violet
Blue
Green
Yellow
Orange
Red
700 nm
Gamma rays
X rays
Ultraviolet radiation
Visible light
Infrared radiation
Microwaves
Radio waves
10^{-6} nm
10^{-5} nm
10^{-4} nm
10^{-3} nm
10^{-2} nm
10^{-1} nm
1 nm
10 nm
100 nm
10^3 nm = 1 μm
10 μm
100 μm
1000 μm = 1 mm
10 mm = 1 cm
10 cm
100 cm = 1 m
10 m
100 m
1000 m = 1 km
10 km
100 km

Gamma rays	$< 10^{-2}$ nm	$> 3x10^{19}$ Hz
X-rays	$10^{-2} \sim 10$ nm	$3x10^{19} \sim 3x10^{16}$ Hz
Ultraviolet	$10 \sim 400$ nm	$3x10^{16} \sim 8x10^{14}$ Hz
Visible/Light	$400 \sim 700$ nm	$8x10^{14} \sim 4x10^{14}$ Hz
Infrared	700 nm $\sim$ 1 mm	$4x10^{14} \sim 3x10^{11}$ Hz
Microwave	$1 \sim 300$ mm	$3x10^{11} \sim 10^{9}$ Hz
Radio	> 300 mm	$< 10^{9}$ Hz

Gamma rays

High energetic beams: materials are transparent to it

Typical energy range: in MeV or more

Rest mass of electron= $m_e c^2 \cong 0.5$ MeV

1MeV= 1 Million electron Volts= 1.6×10^{-13} Joules

$$h\nu \cong 10^{-33} \times 10^{20} = 10^{-13} \text{ Joules } \sim \text{MeV}$$

More particle like behaviour

Sources: Nuclear transitions

Pair annihilation-Electron-positron annihilation

X -rays

Typical energy range: in 0.1-100 KeV

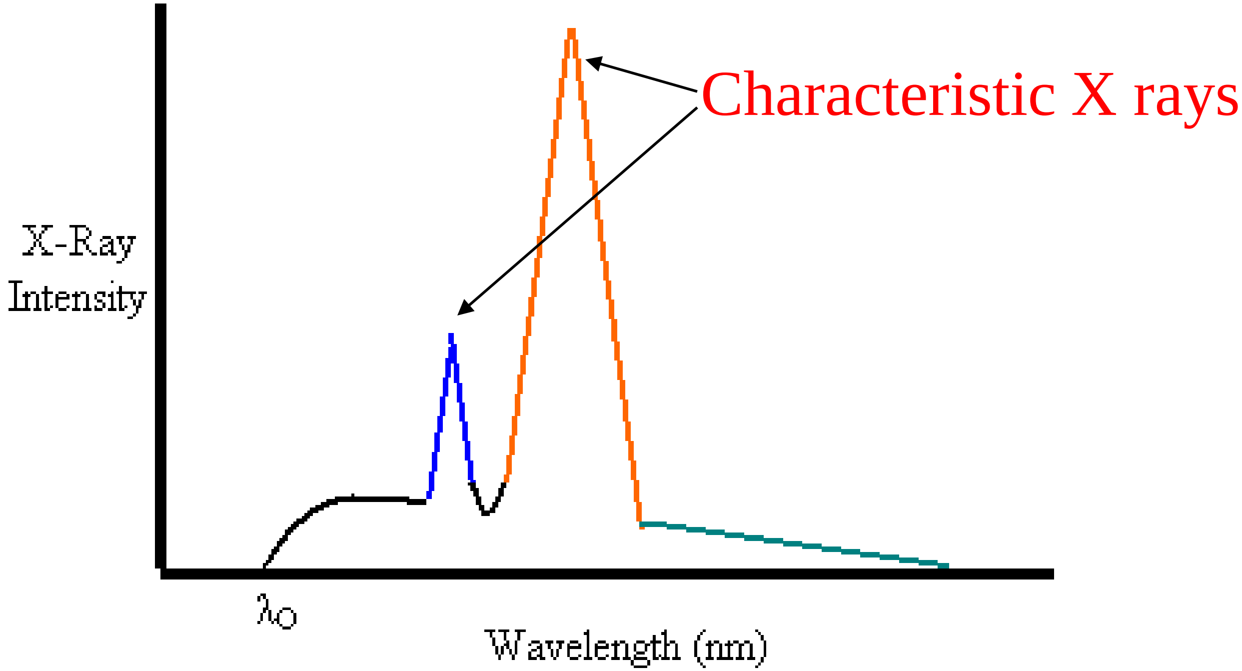
Particle like behaviour

Sources: Atomic transitions (inner electrons)

Characteristic X-rays

Energetic electrons fired on a material target

Bremsstrahlung



Wavelengths of atomic dimensions or less

Good probe for finding structures of substances

Hard X-rays :10-100 KeV

Human body is transparent : Diagnostic X-rays

Ultraviolet

Typical energy range: in 3-100 eV

Sources: Sun

Atomic transitions (when electrons make long jumps)

Ozone layer in atmosphere absorbs UV from the Sun and create Ionosphere

Wavelengths < 300 nm is germicidal

Aquaguard

Visible/Light

Sources: Atomic transitions (outer electrons)

Hot glowing metal filament:Thermal
Continuous radiation : White light

Discharge through gas filled tubes:
Characteristic lines: Line spectra

Wave like behaviour
Interference is easily shown

Infrared

Sources: Molecules

Rotational and vibrational transitions

CO_2 and H_2O vibrational levels ~ 0.2-0.8 eV

Thermal radiation from human body peaks around 0.01mm:

many snakes sense these wavelengths

Incandescent lamps radiate 50% of energy in IR region

Microwave

Communication band extends in microwave

Global System for Mobile (GSM) operates in 900/1800/1900 MHz bands

Sources:Electron spin/Nuclear spin

21cm(1.420 GHz) line of Hydrogen

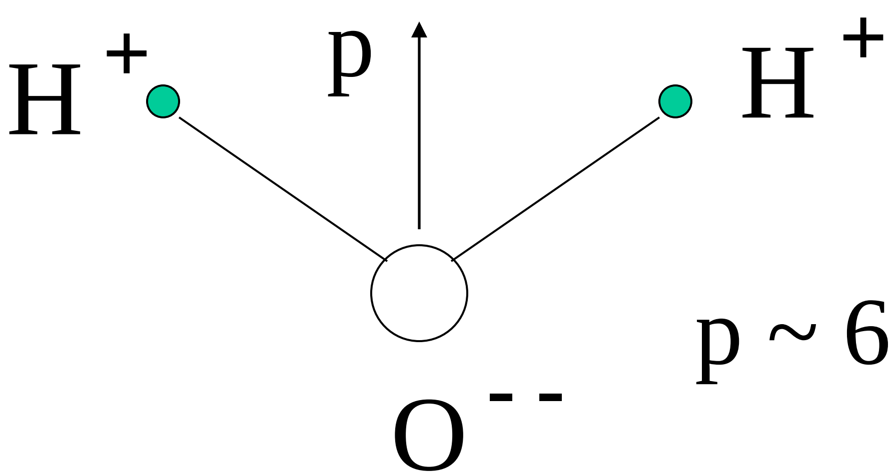
Useful in locating hydrogen in space

CMBR: Cosmic Microwave Background Radiation

2.73 K radiation from all direction

Remnant of Hot Big Bang of the past

Polar molecules like water absorb
EM radiations in microwave
Try to align their dipole moment with the external field, setting the them in rotation.
Usual Microwave ovens use
12.2 cm or 2.45 GHz



$p \sim 6.2 \times 10^{-30}$ C-m

Radio

Radio and TV communication

UHF and VHF ~ 1GHz is used for TV and FM

Medium waves ~ 0.5-1.5 MHz
and Short waves ~ 15 MHz
for radio transmission

Sources: Electronic Circuits
In space: Radio sources