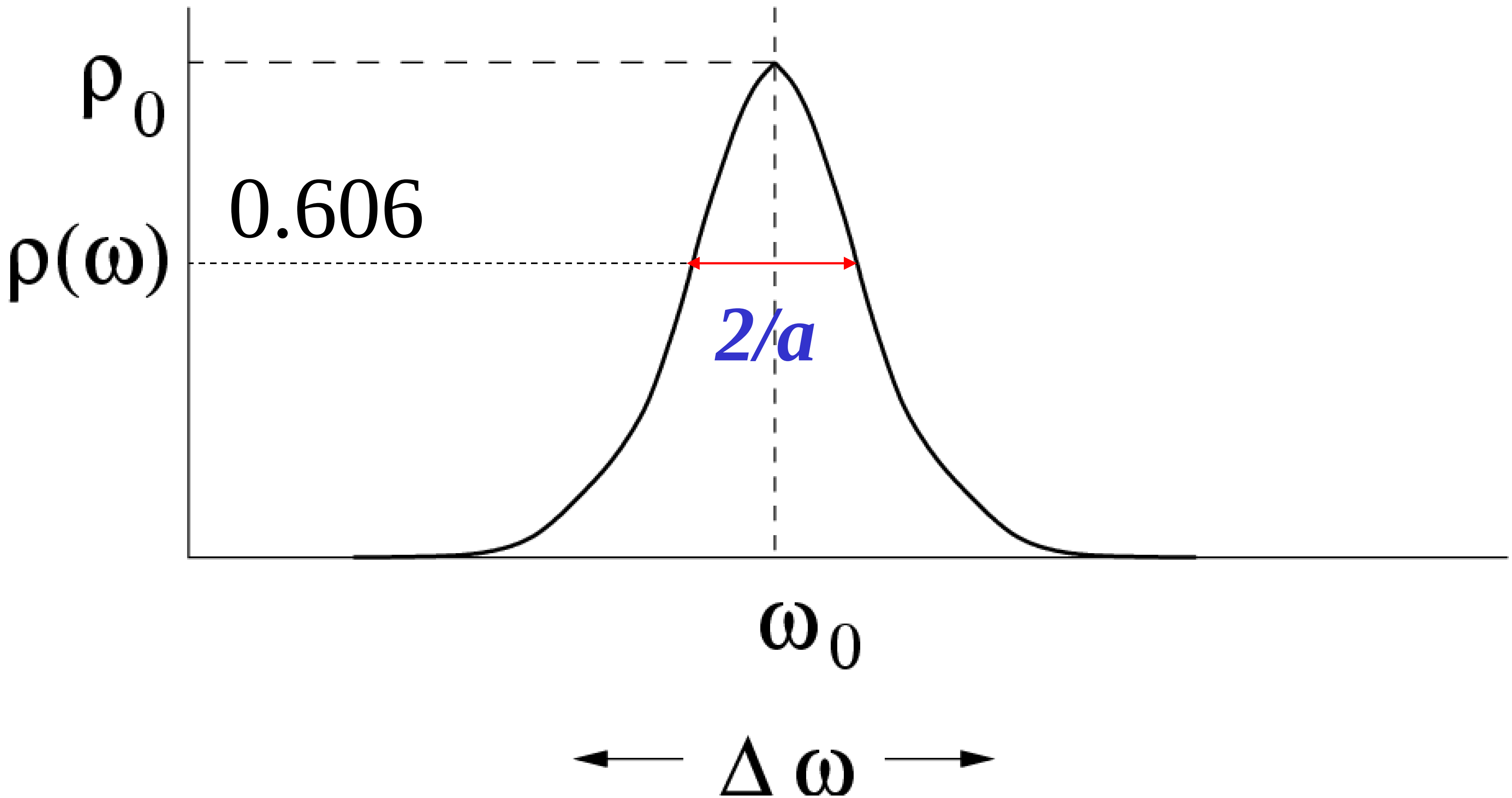
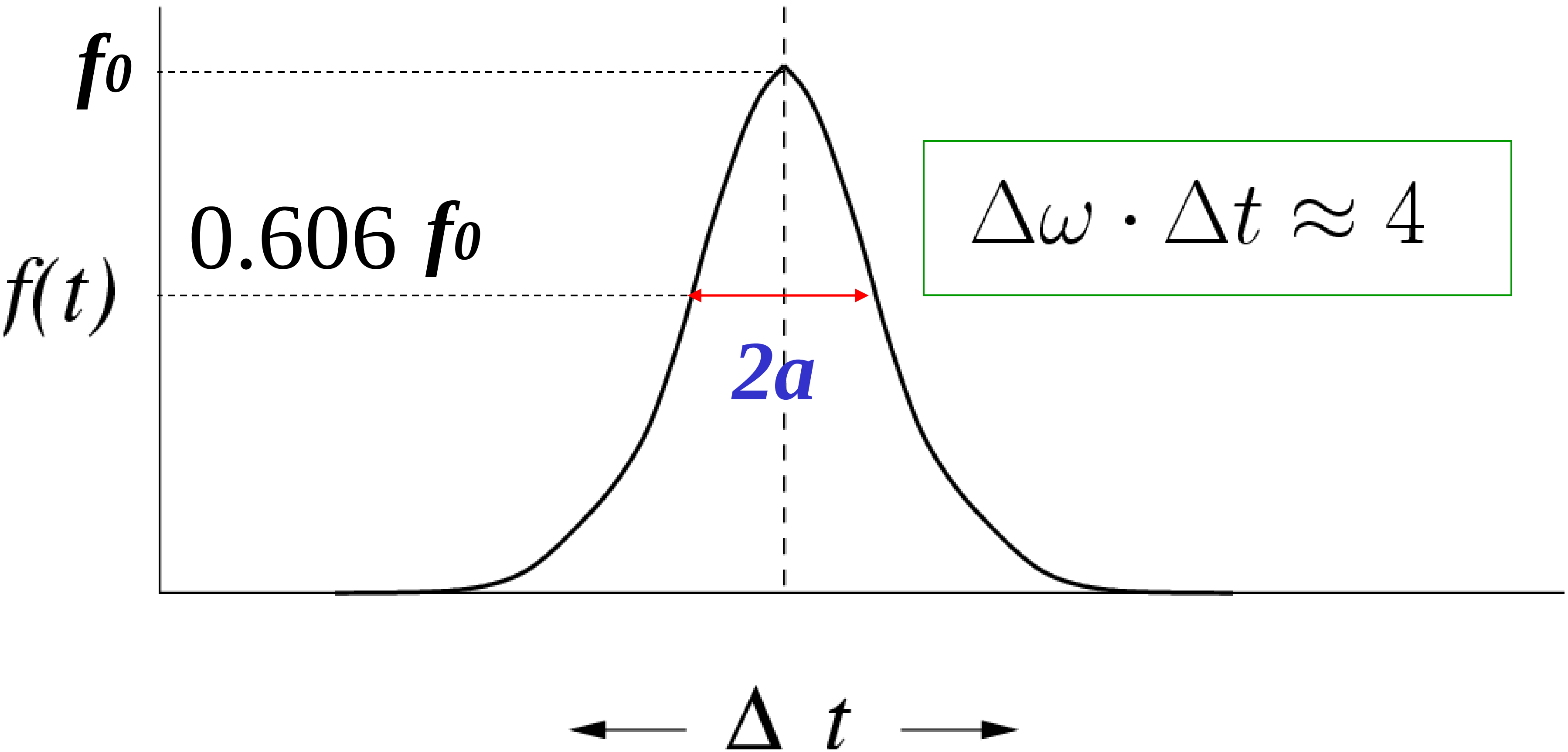
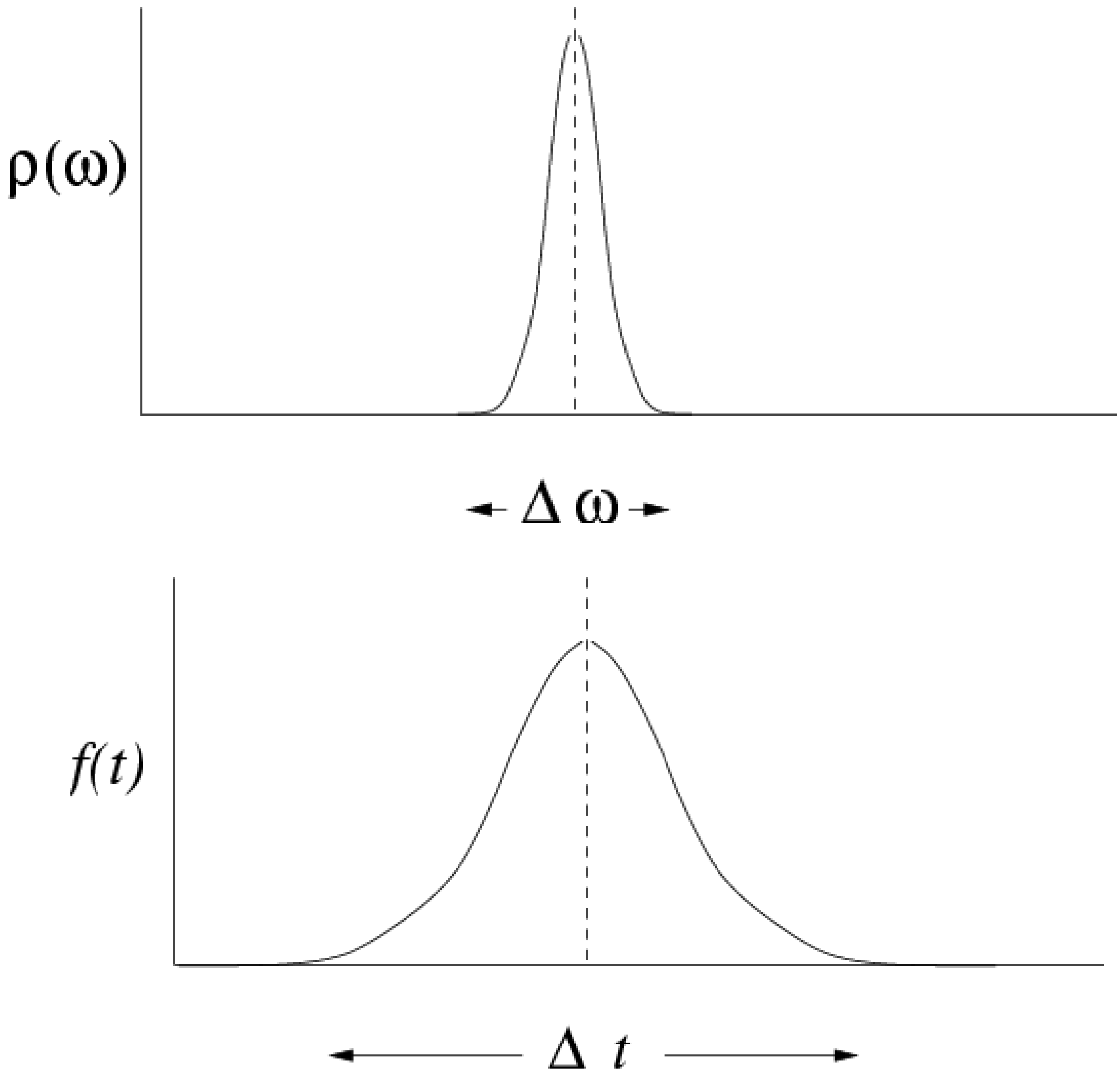
$$\rho(\omega) = \rho_0 \exp(-a^2(\omega - \omega_0)^2/2)$$



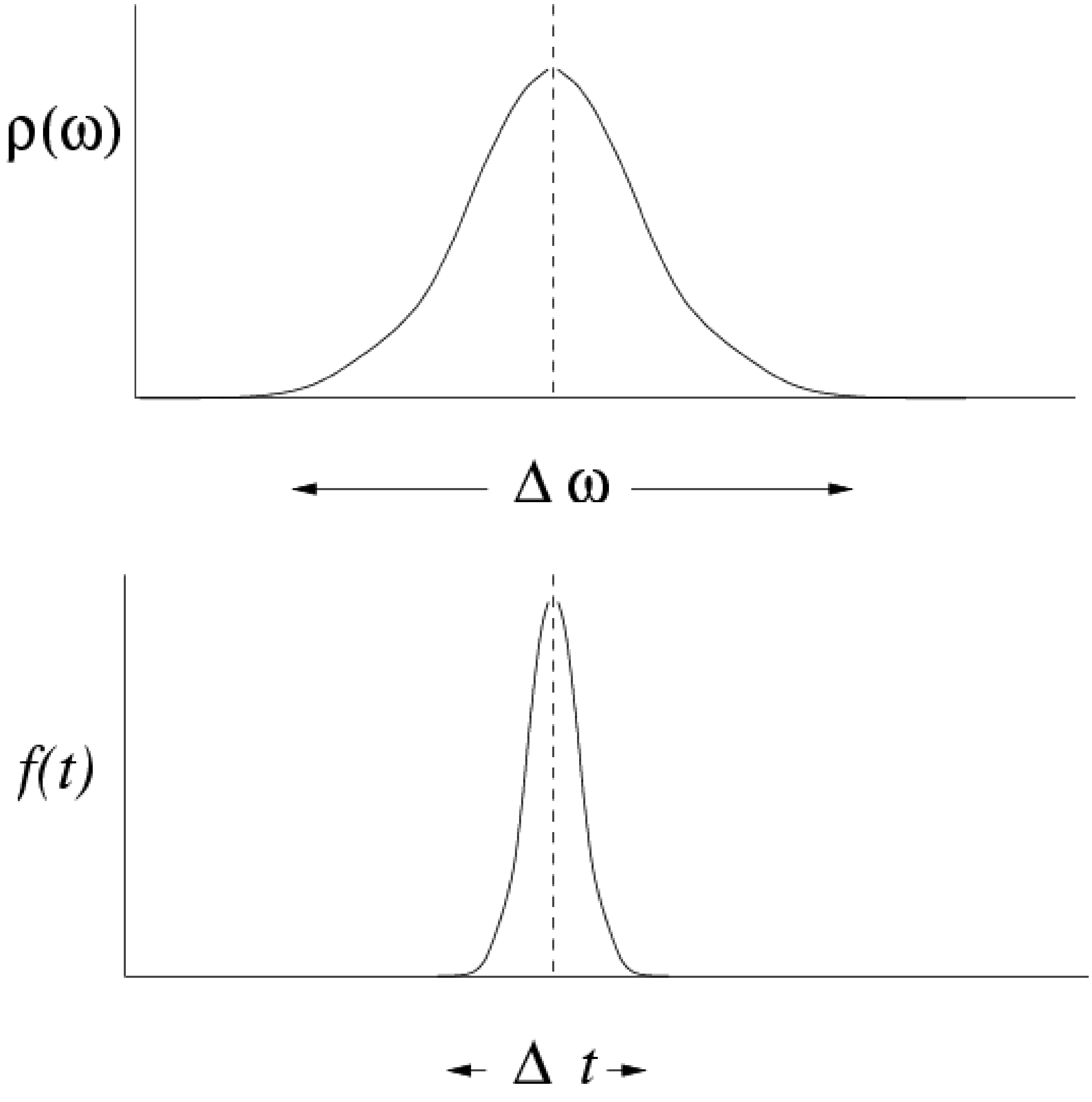
$$\sim \exp(-(t-t_0)^2/2a^2)$$



$$\Delta\omega \cdot \Delta t \approx 4$$



ρ(ω)
Δ ω
f(t)
Δ t


ρ(ω)
Δ ω
f(t)
Δ t

Fourier transform

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \exp(-i\omega t) dt$$

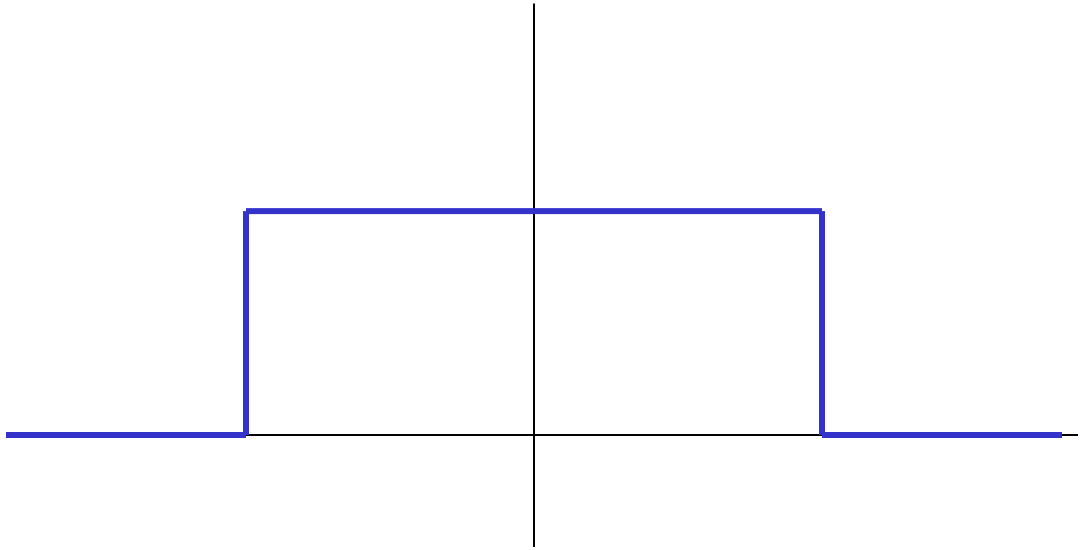
$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) \exp(i\omega t) d\omega$$

k-x space

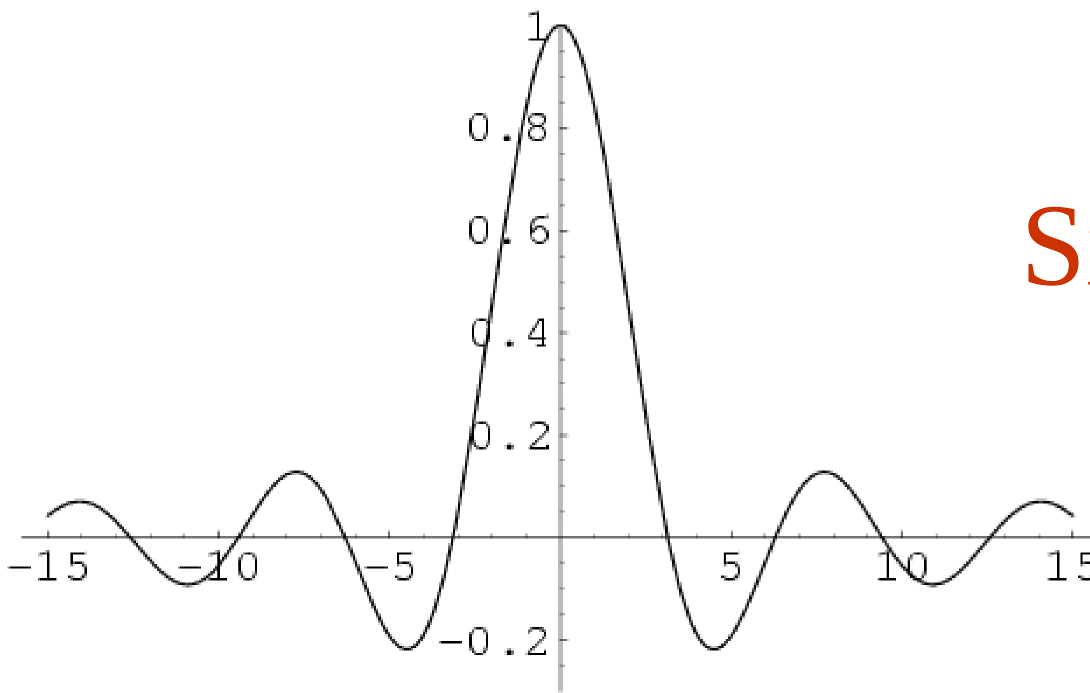
$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \exp(-ikx) dx$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(k) \exp(ikx) dk$$

F(k)

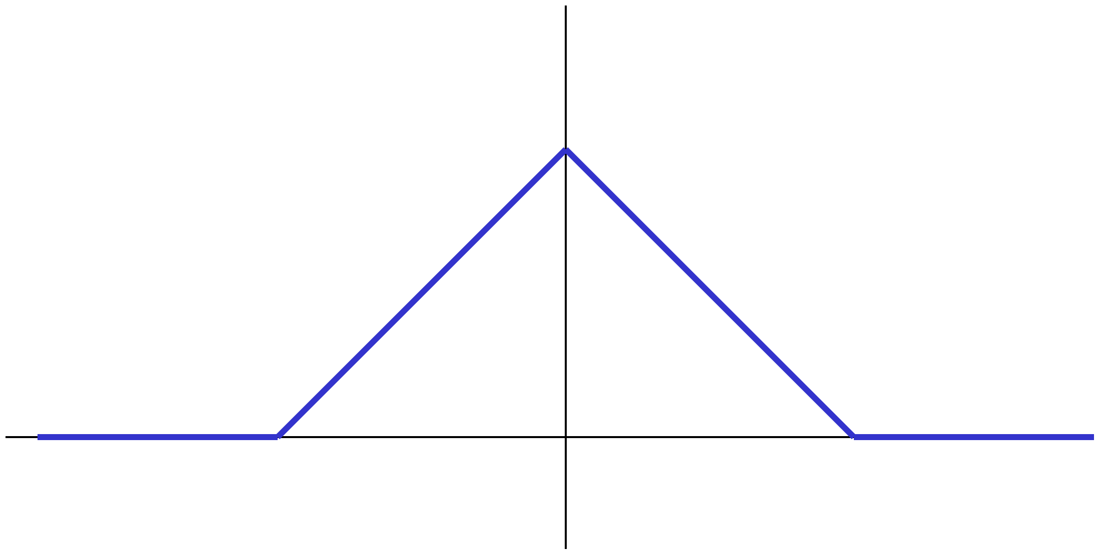


$f(x)$

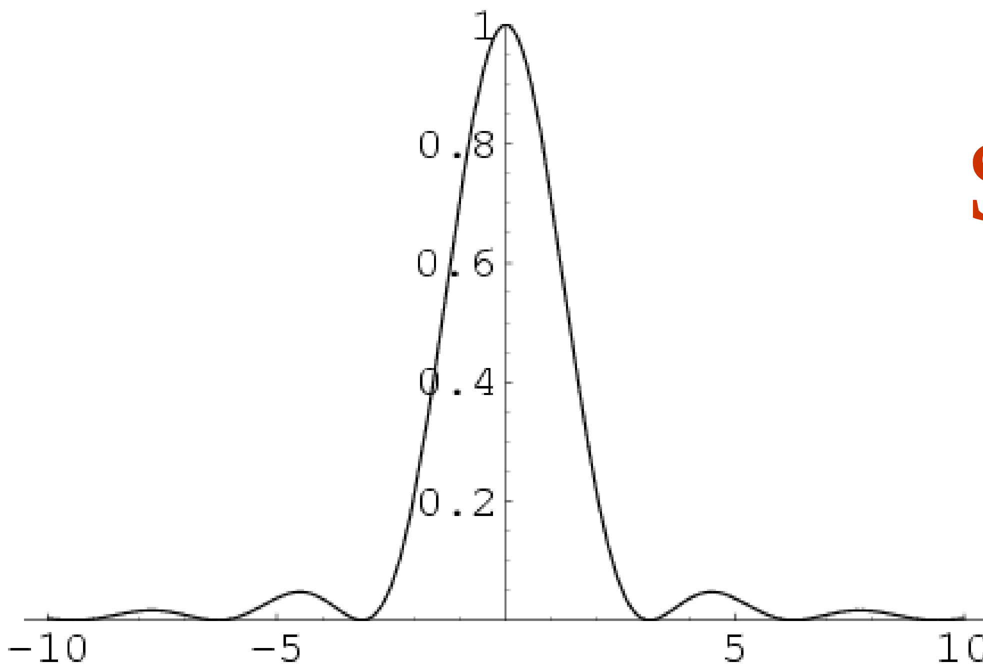


Sin x / x

F(k)

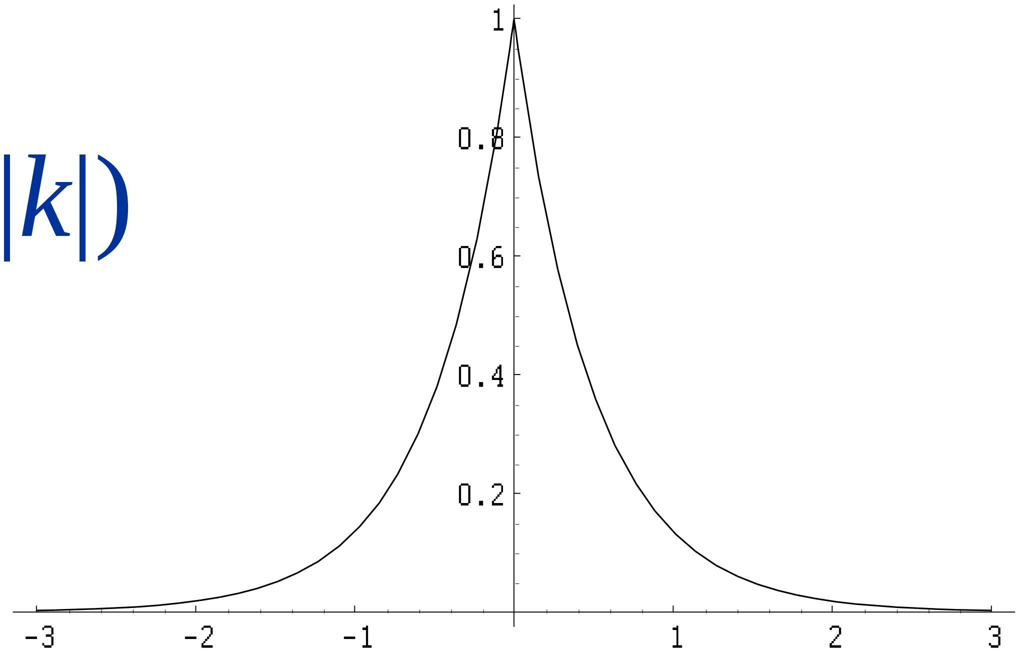


f(*x*)



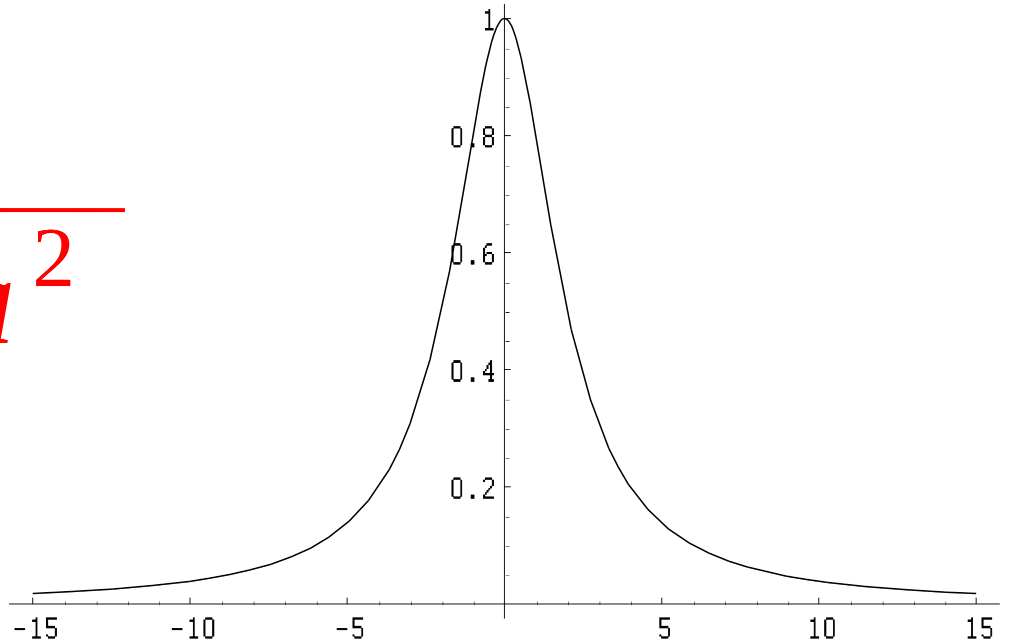
$\mathrm{Sin}^2 x \, / \, x^2$

$$F(k) = \mathrm{Exp}(-a|k|)$$

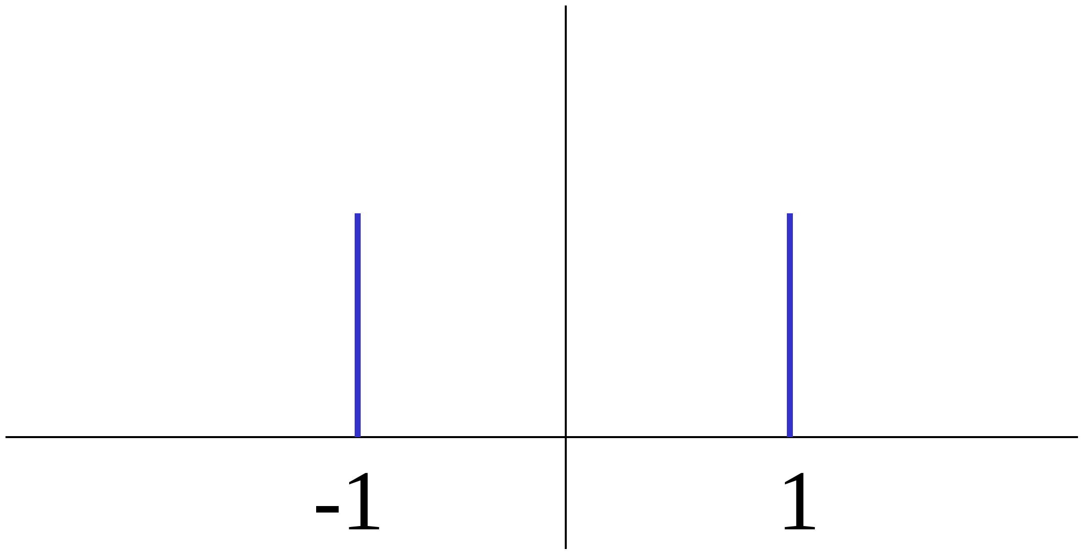


$$f(x) = \frac{a}{x^2 + a^2}$$

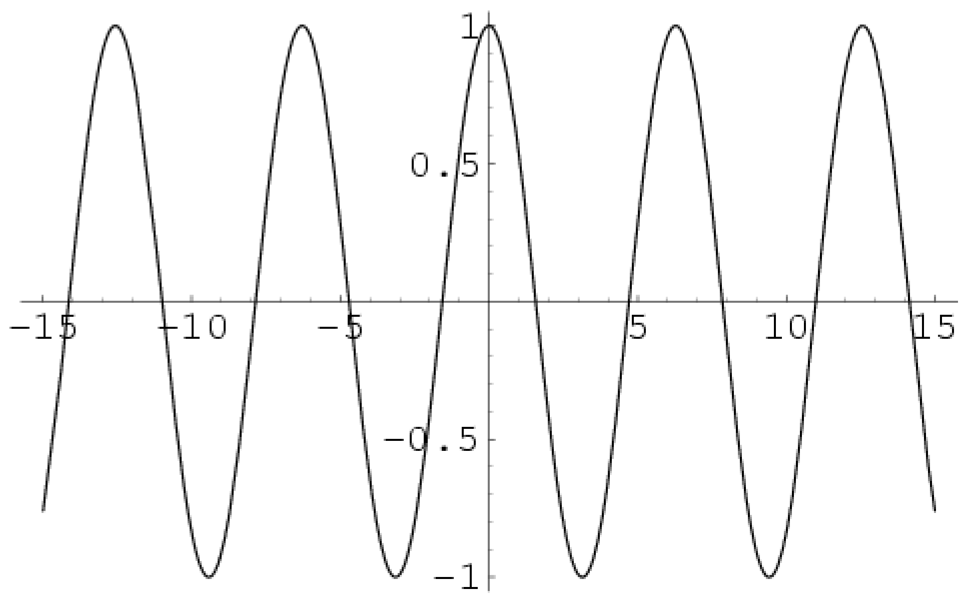
Lorentzian



F(k)

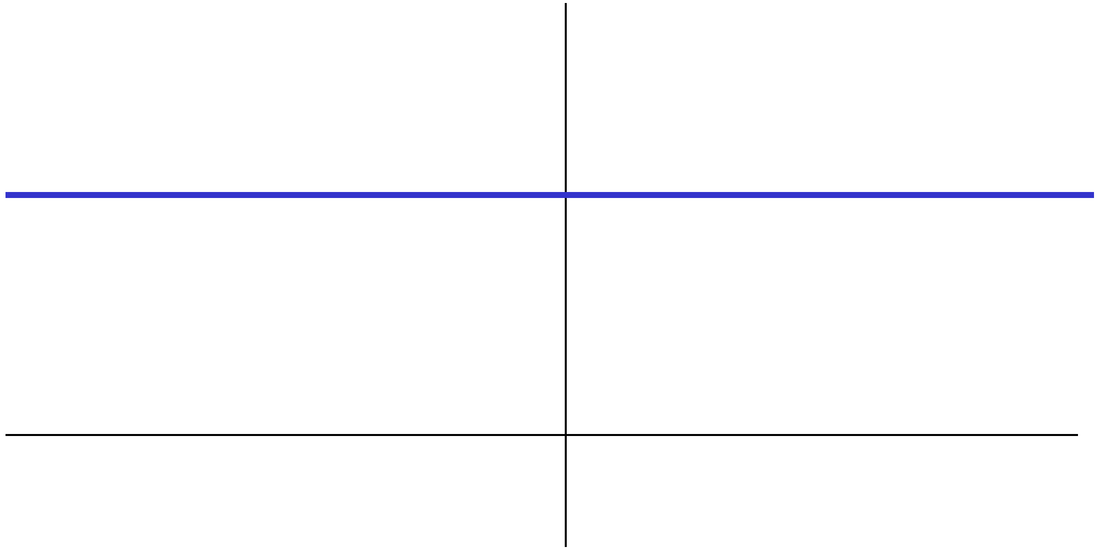


f(x)



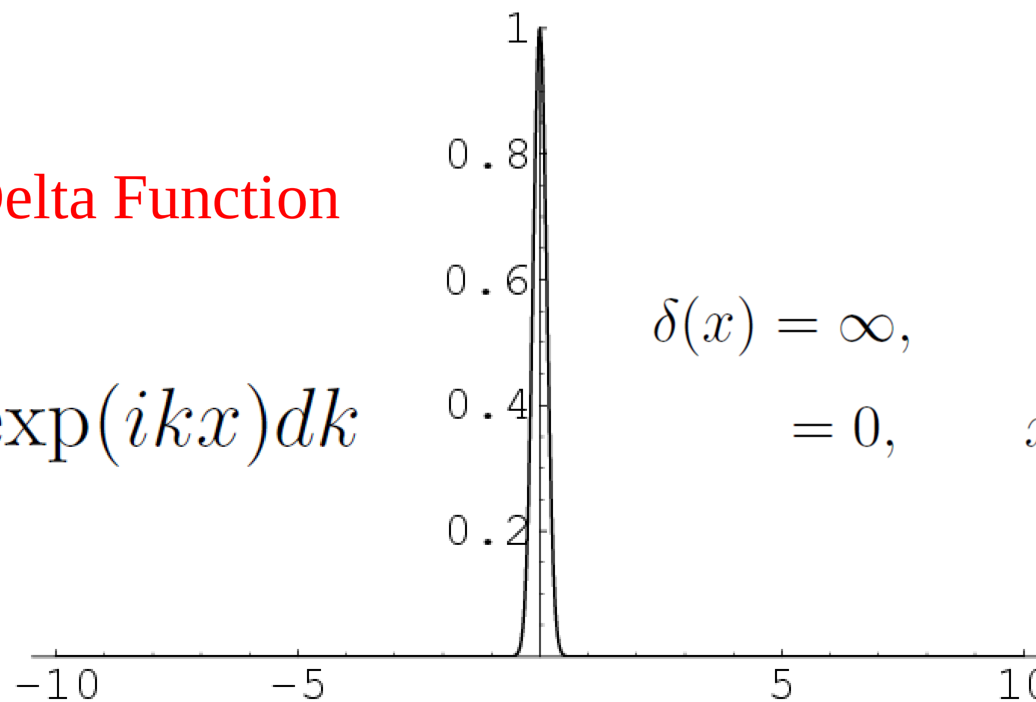
Cos x

F(k)



$f(x)=$ Dirac Delta Function

$$\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(ikx)dk$$



$$\delta(x) = \infty, \qquad x = 0$$
$$= 0, \qquad x \neq 0$$

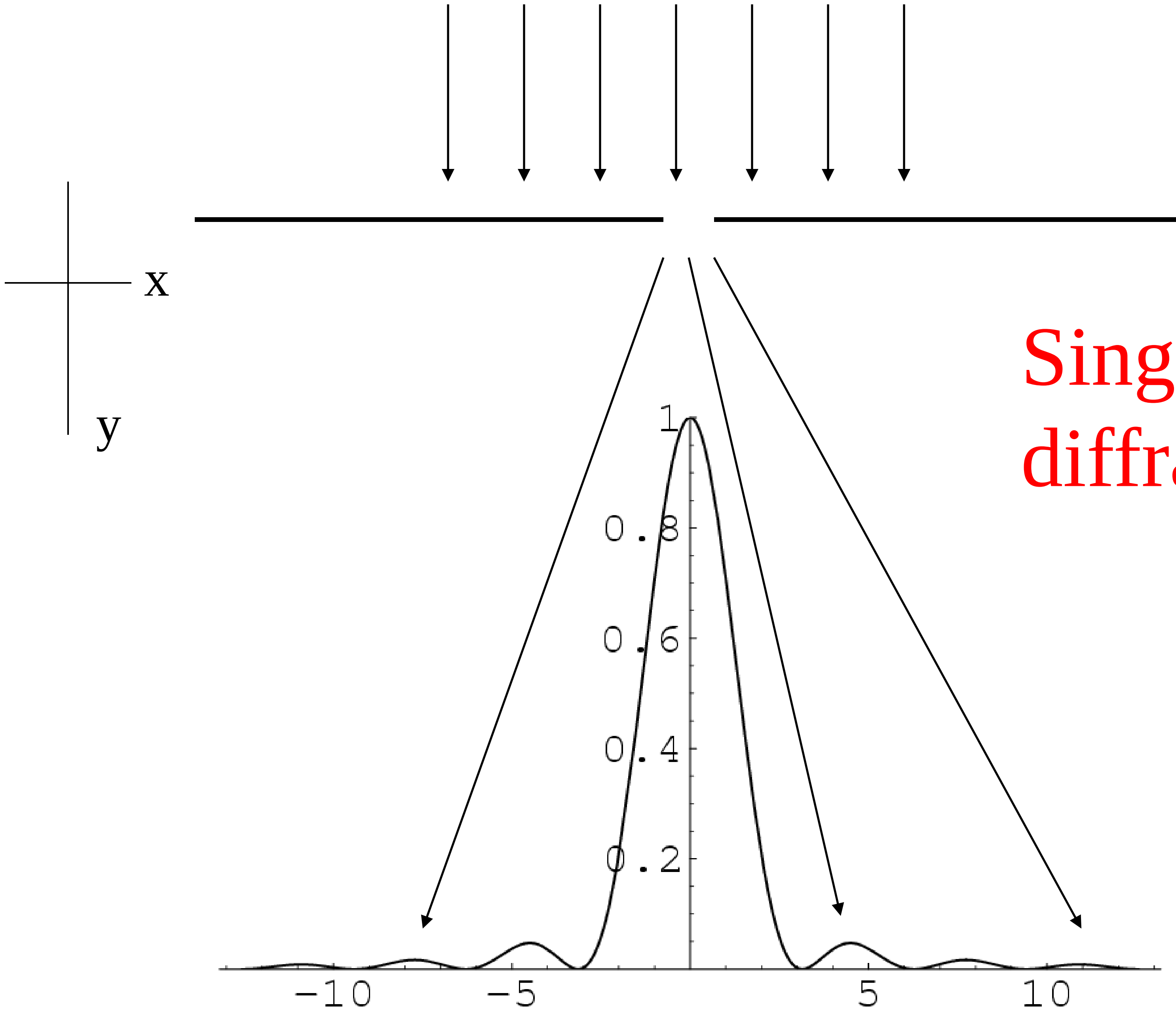
Time-energy and Co-ordinate –momentum uncertainty

$$\Delta\omega\Delta t \geq 1 \qquad \Delta k\Delta x \geq 1$$

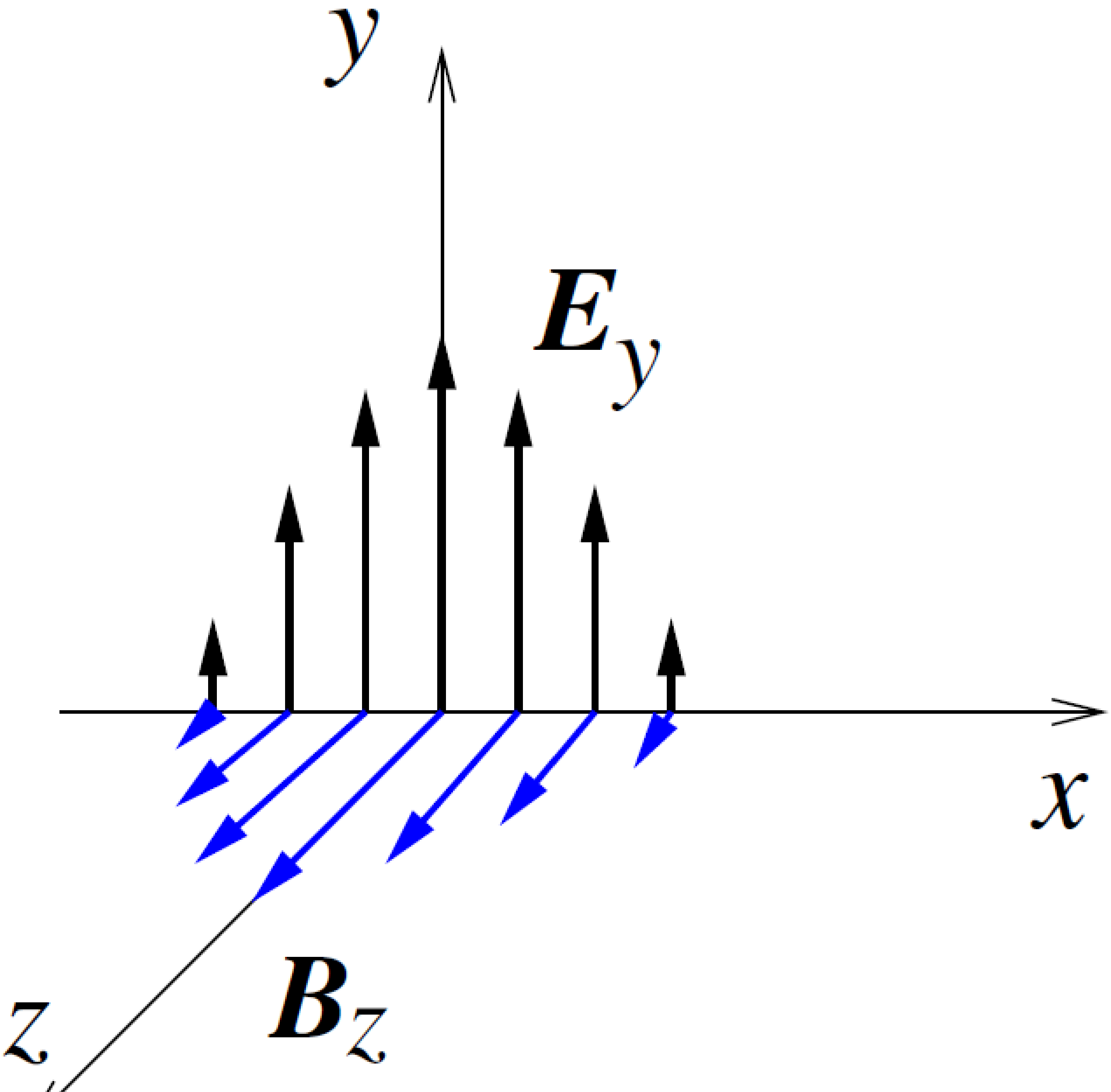
$$\hbar\Delta\omega\Delta t \geq \hbar \qquad \hbar\Delta k\Delta x \geq \hbar$$

$$\Delta E\Delta t \geq \hbar \qquad \Delta p\Delta x \geq \hbar$$

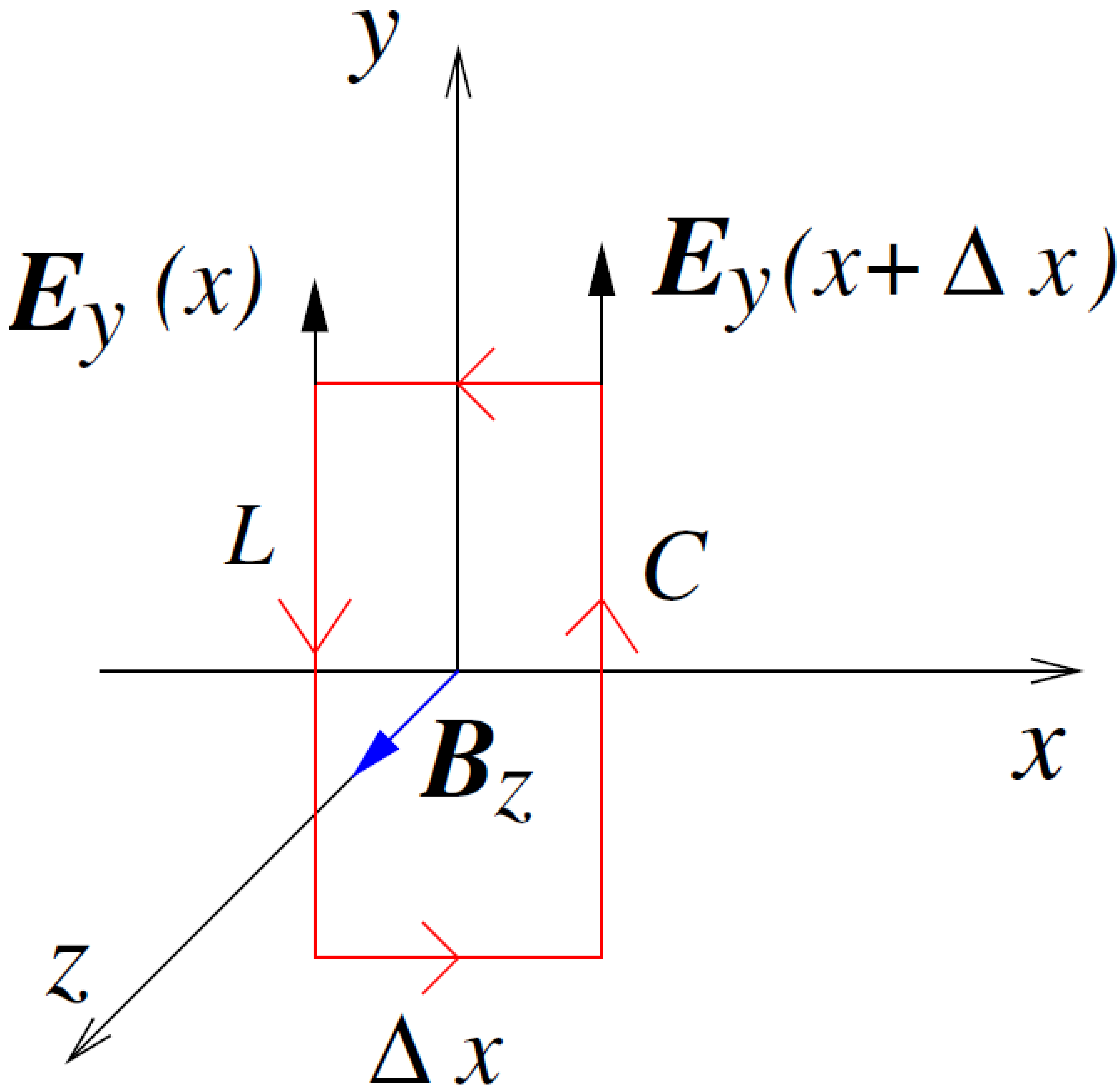
$$\hbar = h/2\pi$$



Single slit diffraction



y
E_y
x
B_z
z



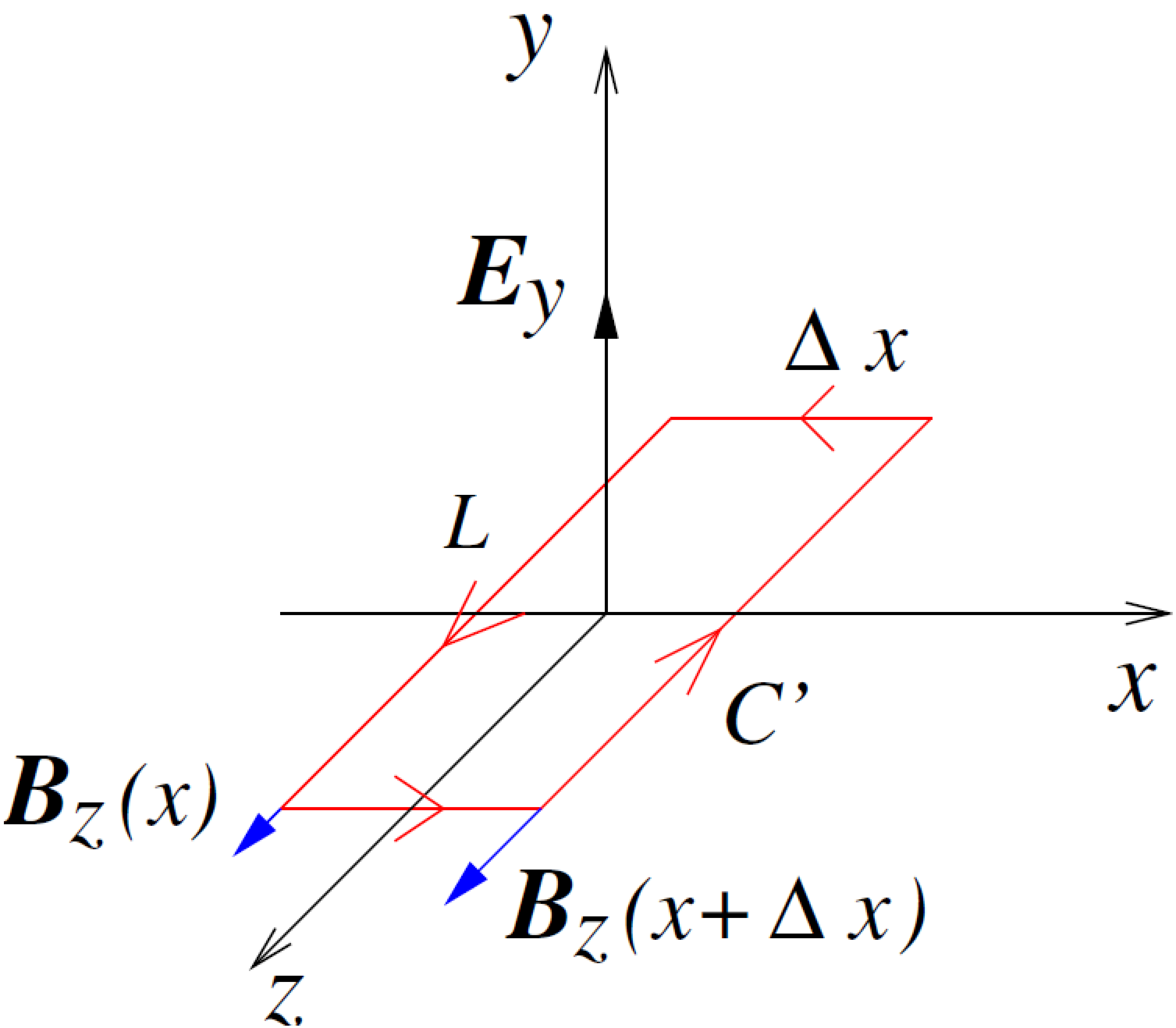
y
$E_y(x)$
$E_y(x+\Delta x)$
L
C
x
B_z
z
Δx

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int\int_S \mathbf{B} \cdot d\mathbf{s}$$

$$[E_y(x + \Delta x) - E_y(x)]\, L = -\frac{\partial B_z}{\partial t} L \Delta x$$

$$\Delta x \to 0$$

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$


y
E_y
Δ x
L
x
C'
B_Z(x)
B_Z(x+Δ x)
z

$$\oint_{C'} \mathbf{B} \cdot d\mathbf{l} = \epsilon_0 \mu_0 \frac{\partial}{\partial t} \int\int_S \mathbf{E} \cdot d\mathbf{s}$$

$$[-B_z(x + \Delta x) + B_z(x)]\, L = \epsilon_0 \mu_0 \frac{\partial E_y}{\partial t} L \Delta x$$

$$\Delta x \rightarrow 0$$

$$-\frac{\partial B_z}{\partial x} = \epsilon_0 \mu_0 \frac{\partial E_y}{\partial t}$$

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2}$$

$$c^2 = \frac{1}{\epsilon_0 \mu_0}$$

$$\frac{\partial^2 B_z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 B_z}{\partial t^2}$$

Electromagnetic waves in vacuum

Del operator

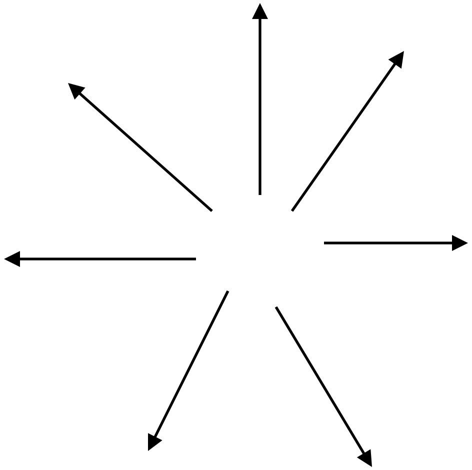
$$\nabla \equiv \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$$

Gradient of a scalar field

$$\nabla \phi(x, y, z, t) = \mathbf{i}\frac{\partial \phi}{\partial x} + \mathbf{j}\frac{\partial \phi}{\partial y} + \mathbf{k}\frac{\partial \phi}{\partial z}$$

Divergence of a vector field

$$\nabla \cdot \mathbf{v}(x, y, z, t) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

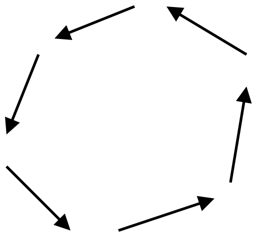


Curl of a vector field

$$\nabla \times \mathbf{v}(x,y,z,t) = \mathbf{i}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)$$

$$+\mathbf{j}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \mathbf{k}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

$$\nabla \times \mathbf{v} = \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{pmatrix}$$



The Laplacian

$$\nabla^2 \phi = \nabla \cdot \nabla \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$$

Identities

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$

Show

$$\nabla \times (\nabla \phi) = 0$$

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

Maxwell's equation in vacuum

Gauss's laws

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

Faraday's law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Ampere's law (modified)

$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$