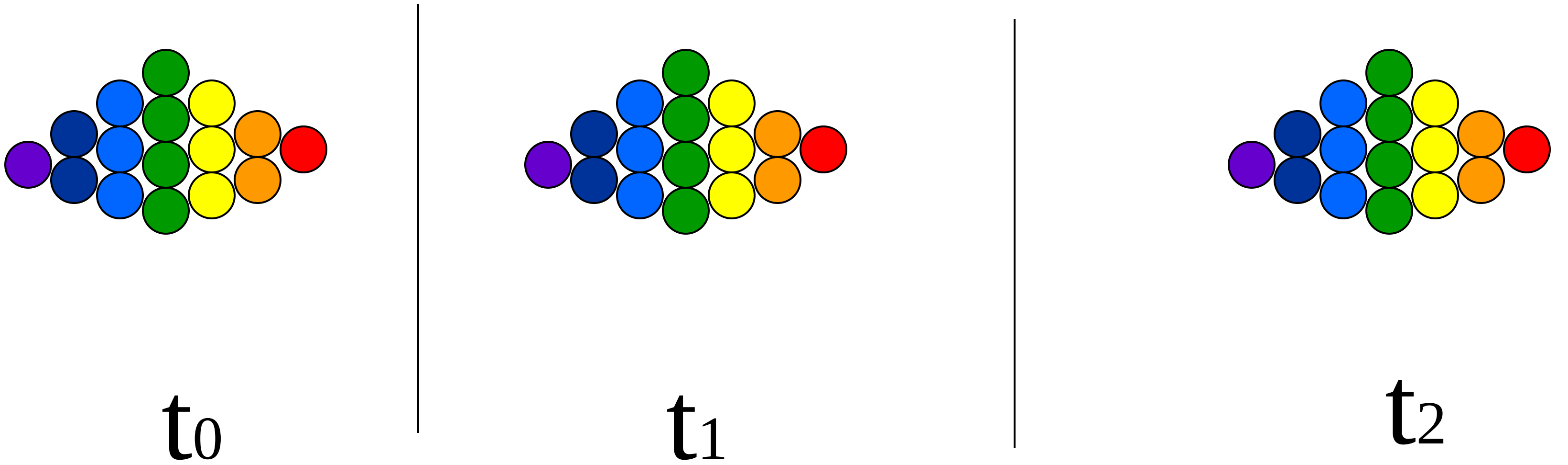
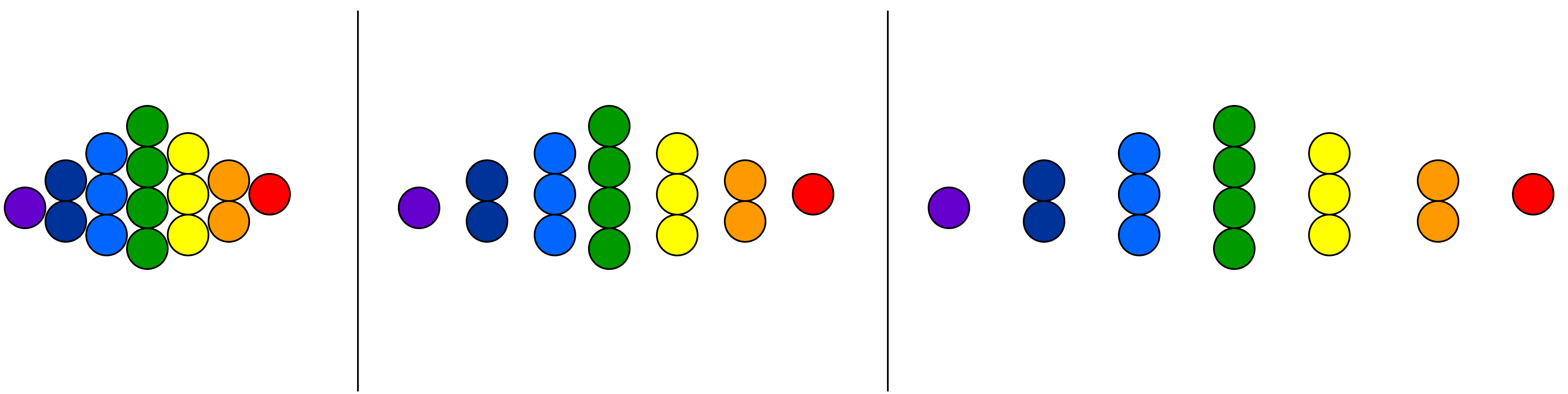
Non dispersive: All colours moving with same speed



Dispersive: Red moving faster than blue



Non dispersive waves v_p = Constant

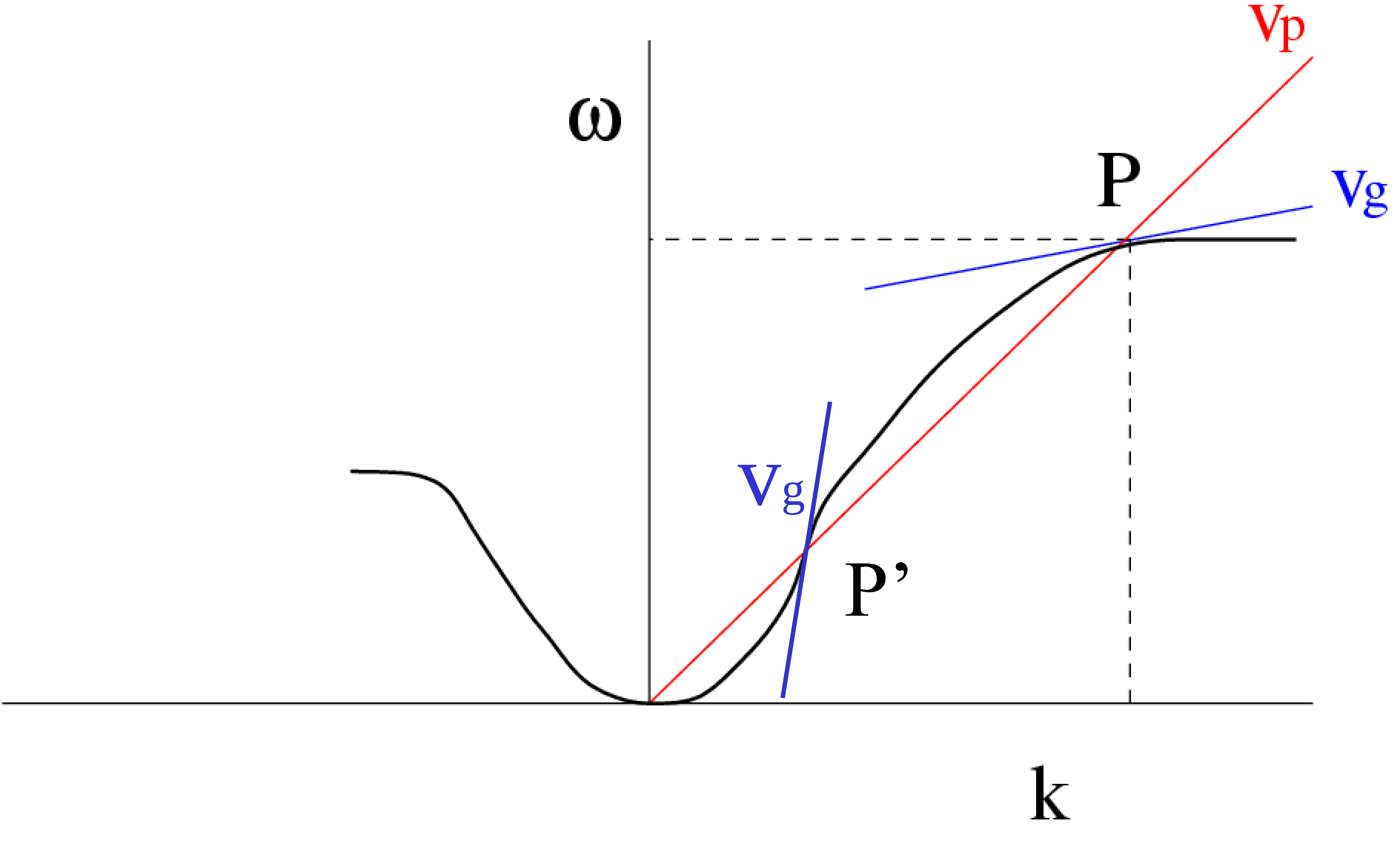
Signal is propagated without distortion

More generally v_p is a function of ω (or k)

$$v_g = v_p + k\frac{\partial v_p}{\partial k}$$

$$v_g = v_p - \lambda\frac{\partial v_p}{\partial \lambda}$$

Phase and Group velocity

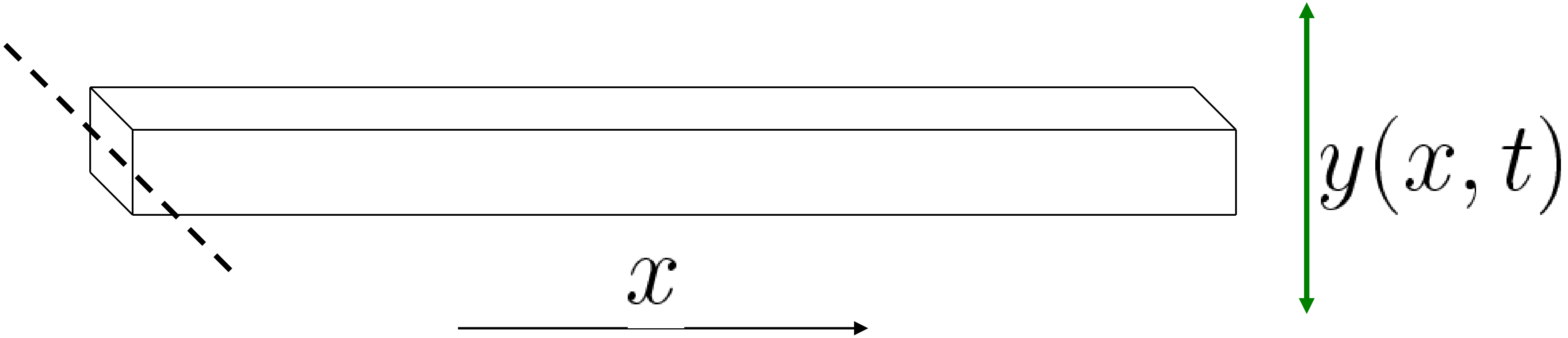


Lateral vibration in a bar

$$\frac{\partial^2 y(x,t)}{\partial t^2} + K^2 b^2 \frac{\partial^4 y(x,t)}{\partial x^4} = 0$$

$K^2 = Y/\rho$,

b = radius of gyration of the cross section about the axis shown



Lateral vibrations in a bar

$$\omega^2 = K^2 b^2 k^4$$

$$v_p = Kbk \qquad v_g = 2v_p$$

$$v_p = Bk^{-n} \qquad v_g = v_p(1-n)$$

Electromagnetic waves in Ionosphere, wave guides

$$\omega^2 = \omega_0^2 + c^2 k^2$$

$$v_p > c \quad \Big| \quad v_g < c$$

$$v_p \cdot v_g = c^2$$

Water waves

$$\omega^2 = (gk + \frac{Tk^3}{\rho}) \tanh(kh)$$

$$v_p^2 = (\frac{g}{k} + \frac{Tk}{\rho}) \tanh(kh)$$

$$kh >> 1$$

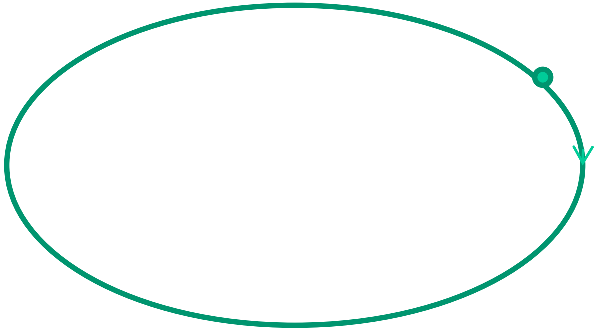
$$\tanh(kh) \approx 1$$

ρ = density

T = surface tension

h = depth of water

Motion of a water particle



$$k^2 = g\rho/T$$

g term and T term are comparable

$$\lambda_c = 2\pi\sqrt{T/(g\rho)}, \text{ At } 20^\circ \text{ C},$$

$$\lambda_c \approx 17mm, (T = 0.073\ N/m)$$

Long (gravity) waves in deep water

$$v_p^2 = \frac{g}{k} + \frac{Tk}{\rho}$$

$$\lambda \;>> \lambda_c$$

$$\tanh(kh) \approx 1$$

$$k^2 << \frac{g\rho}{T}$$

$$\lambda^2 >> \frac{4\pi^2 T}{g\rho}$$

$$v_p = \left(\frac{g}{k}\right)^{1/2}$$

$$v_g = \frac{1}{2} v_p$$

$$\tau = \lambda / v_p \approx \sqrt{2\pi\lambda / g}$$

$$t = t_0 + \frac{L}{v_p} = t_0 + \frac{L\tau}{\lambda}$$

$$\tau \approx \frac{2\pi L}{g(t - t_0)}$$

$$-\frac{d\tau}{dt} \approx \frac{2\pi L}{g(t - t_0)^2} = \frac{g\tau^2}{2\pi L}$$

Storm struck at time t_0 and at L distance away

Short ripples under surface tension

$\lambda_c >> \lambda$ and $\tanh(kh) \approx 1$

$$v_p = \left(\frac{Tk}{\rho}\right)^{1/2}$$

$$v_g = \frac{3}{2} v_p$$

Gravity waves in shallow water (Tsunami)

$$kh << 1$$

$$\tanh(kh) \approx kh - \frac{1}{3}(kh)^3 + \frac{2}{15}(kh)^5 \cdots$$

$$\omega^2 \approx ghk^2(1 - \frac{1}{3}k^2h^2)$$

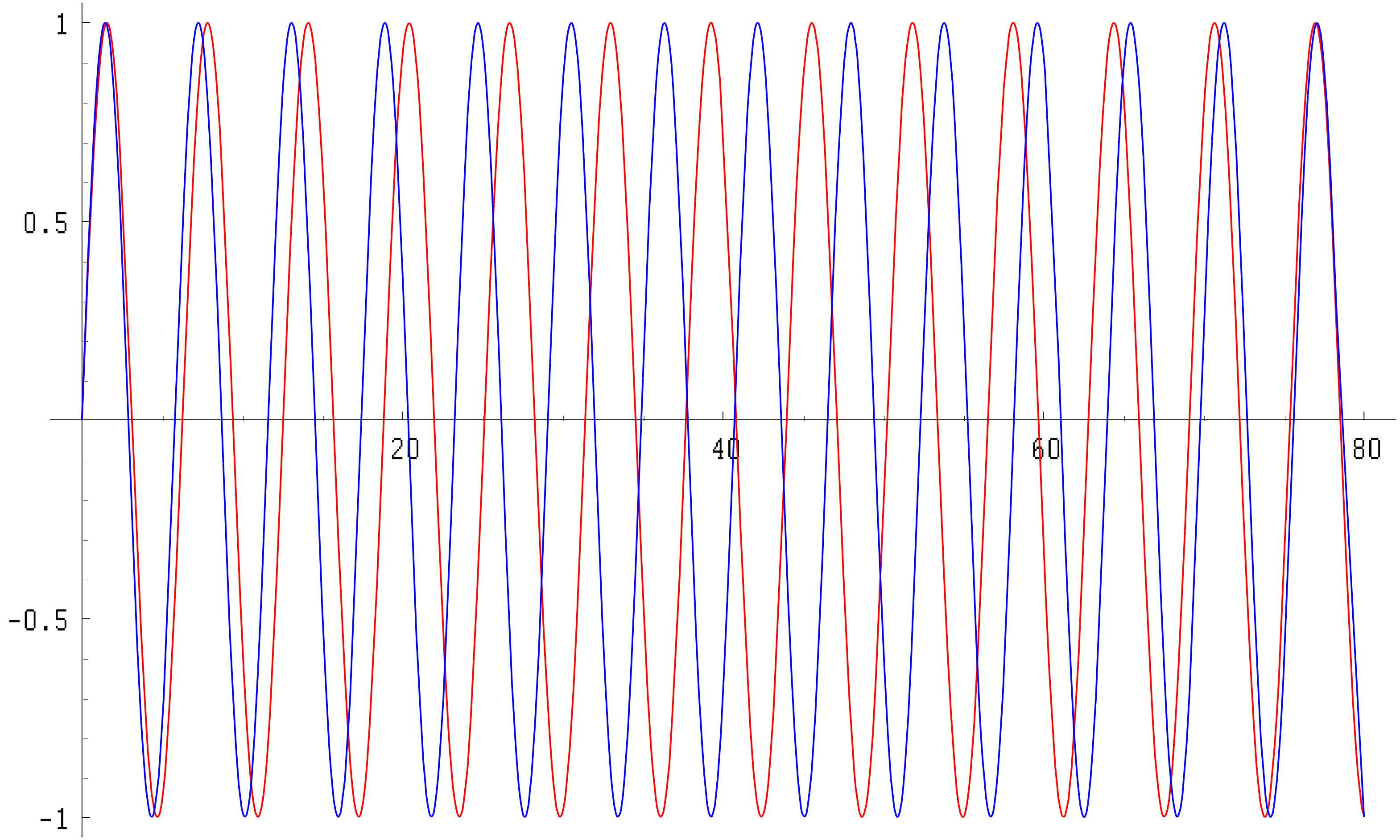
$$\omega \approx \sqrt{gh}k - \frac{1}{6}\sqrt{gh}h^2k^3$$

$$v_p = v_g \approx \sqrt{gh} = c$$

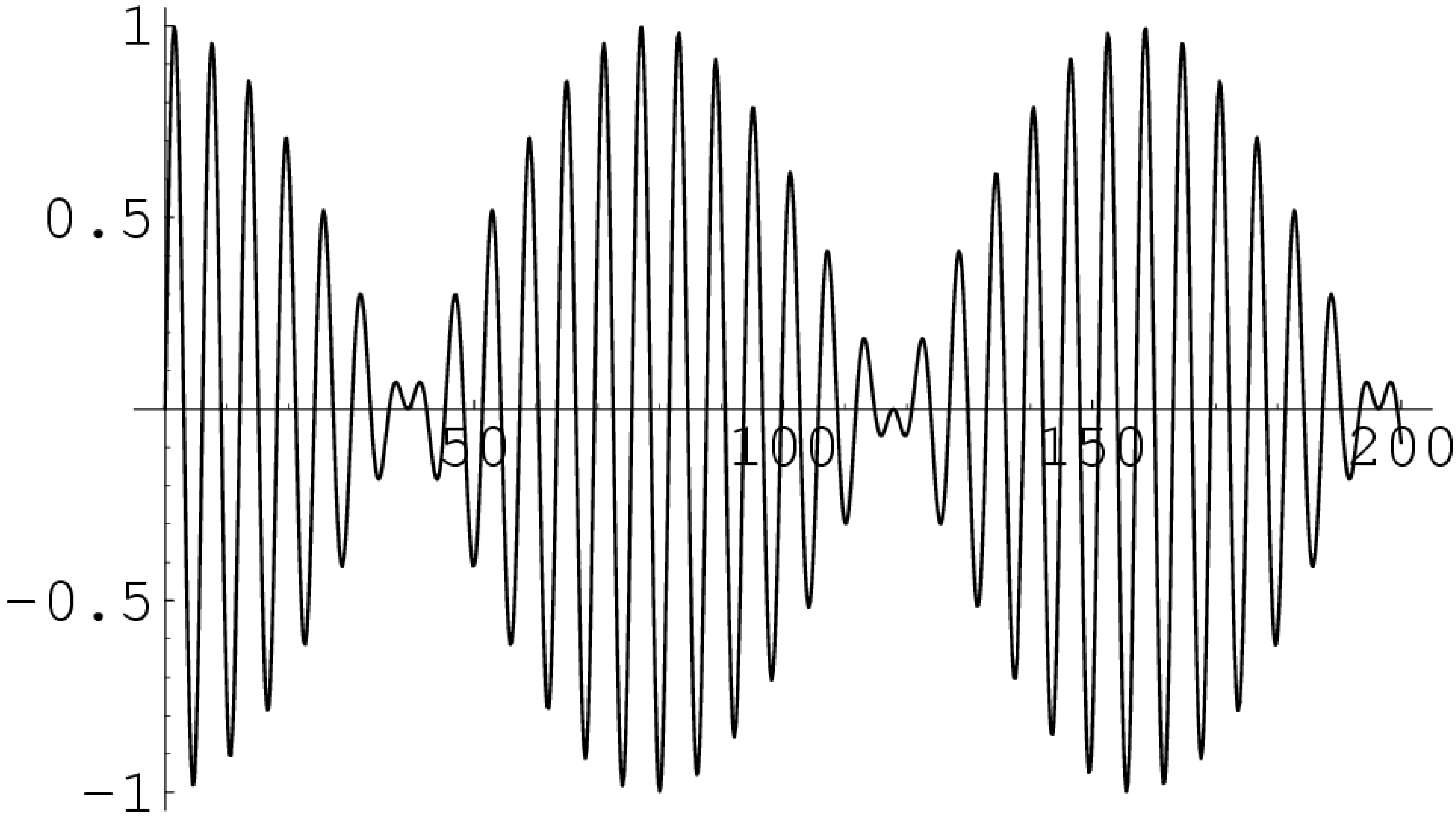
Superposition of waves and wave packet formation

$y(t) = \sin t \qquad 0 < t < 80$

$y(t) = \sin 1.08\, t \qquad 0 < t < 80$

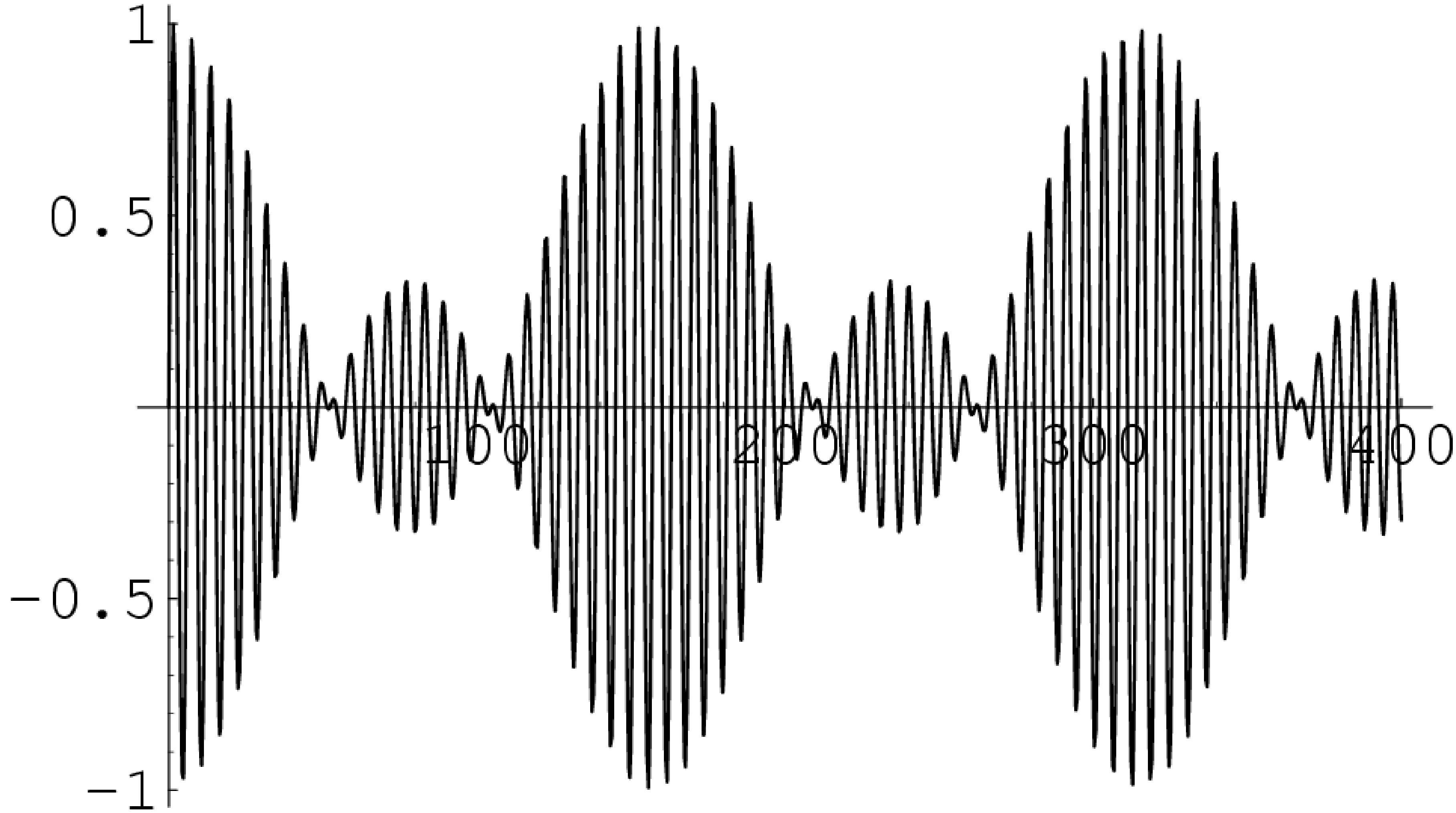


$$y(t) = [\sin t + \sin (1.08\, t)]/2 \quad 0 < t < 200$$



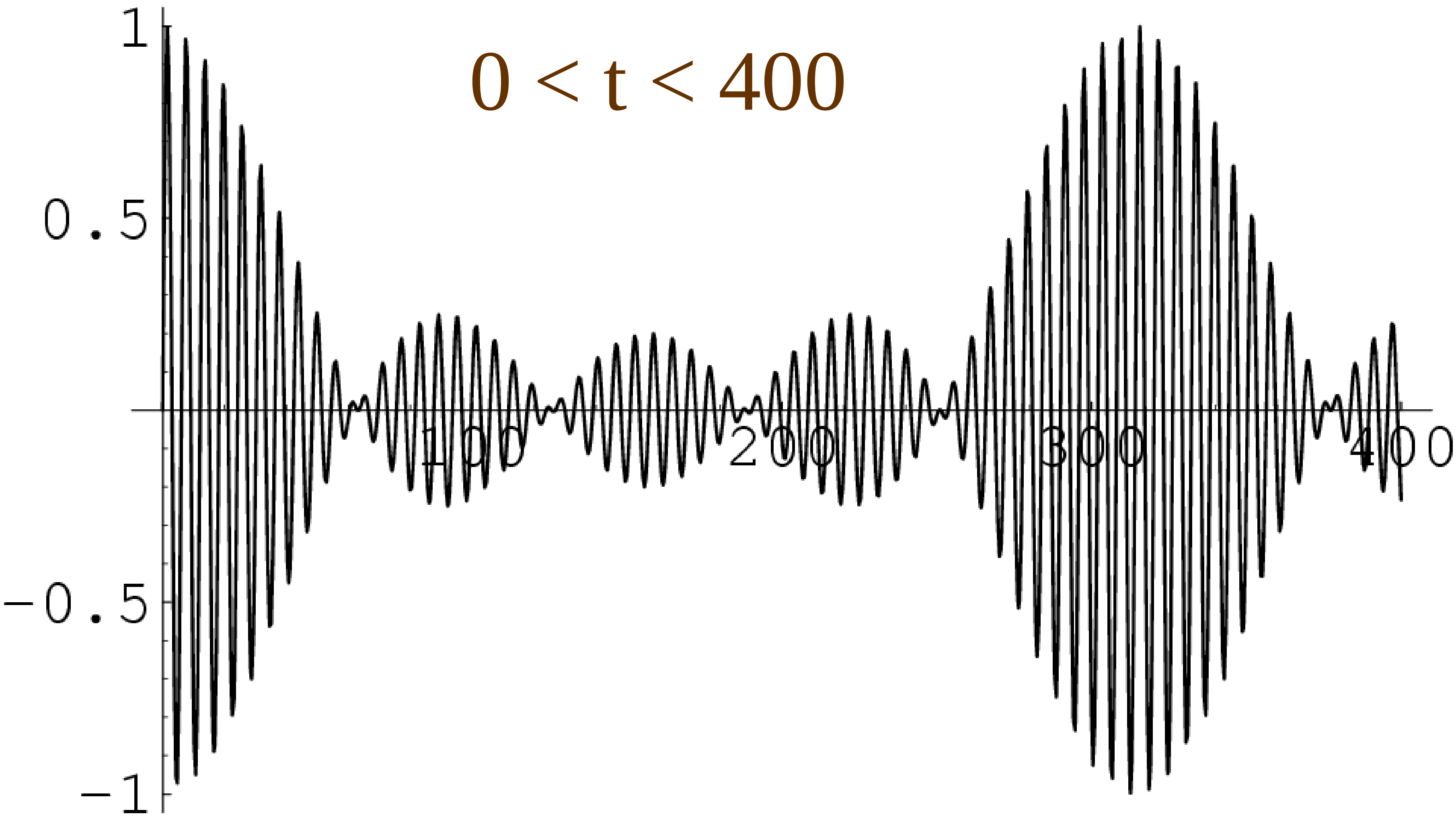
$$y(t) = [\text{Sin } t + \text{Sin}(1.04\ t) + \text{Sin}\ (1.08\ t)]/3$$

$$0 < t < 400$$



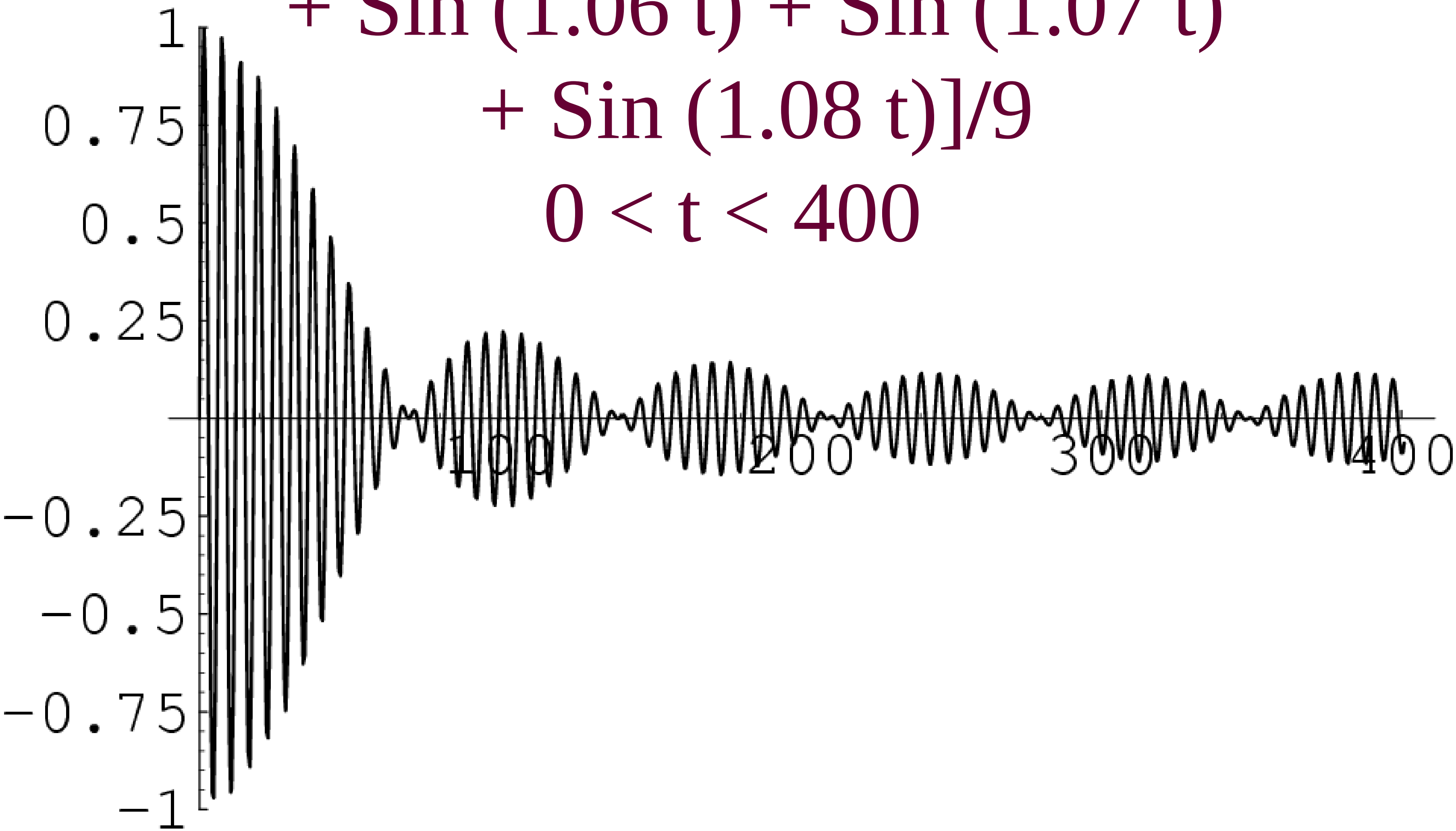
$$y(t) = [\sin t + \sin(1.02\, t) + \sin(1.04\, t) + \sin(1.06\, t) + \sin(1.08\, t)]/5$$

$0 < t < 400$



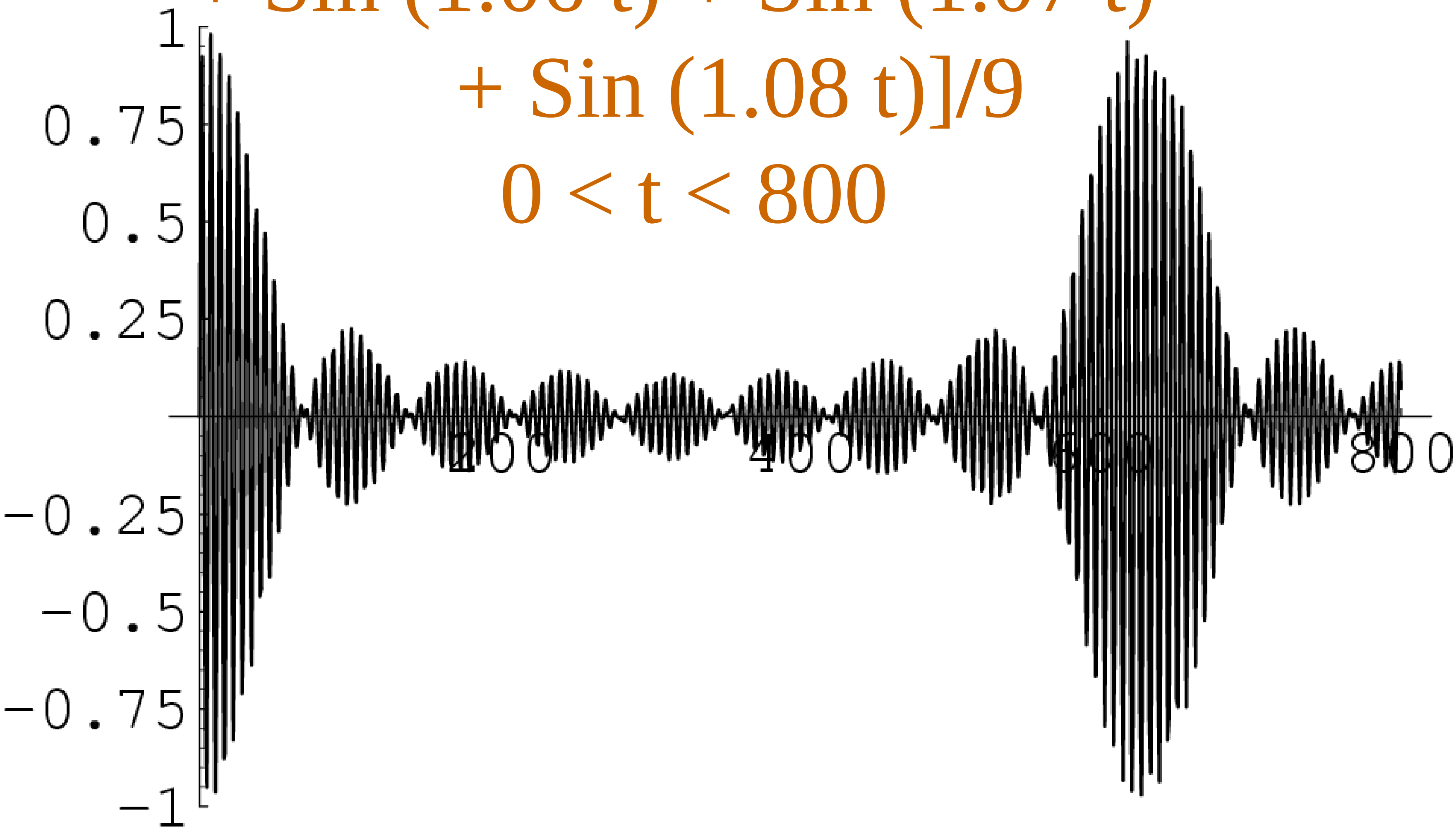
$$y(t) = [\sin t + \sin(1.01\, t) + \sin(1.02\, t) + \sin(1.03\, t) + \sin(1.04\, t) + \sin(1.05\, t) + \sin(1.06\, t) + \sin(1.07\, t) + \sin(1.08\, t)]/9$$

$$0 < t < 400$$

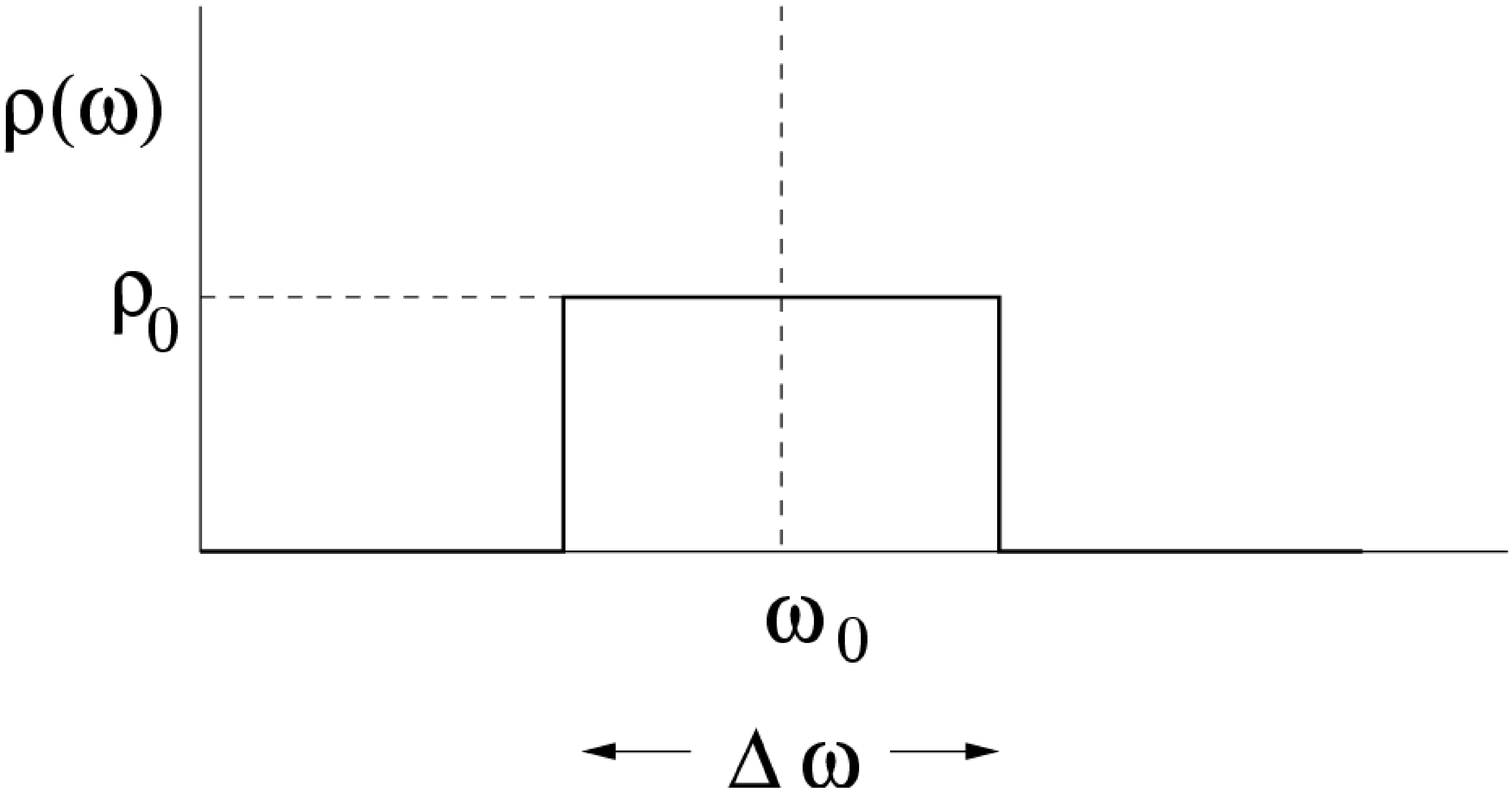


$$y(t) = [\sin t + \sin(1.01\,t) + \sin(1.02\,t) + \sin(1.03\,t) + \sin(1.04\,t) + \sin(1.05\,t) + \sin(1.06\,t) + \sin(1.07\,t) + \sin(1.08\,t)]/9$$

$$0 < t < 800$$



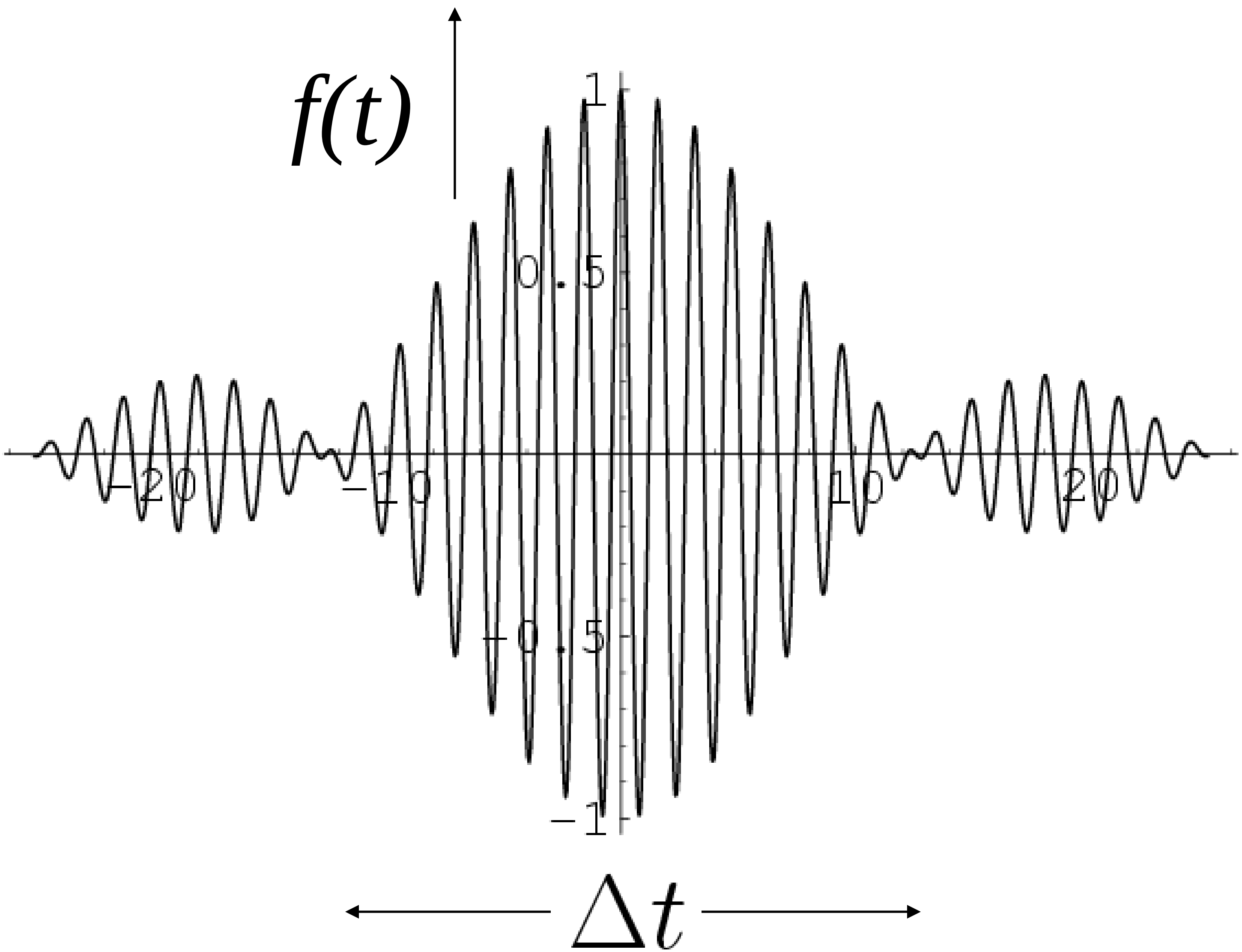
Wave packet



$$f(t) = \int_{\omega_0-\Delta\omega/2}^{\omega_0+\Delta\omega/2} \rho_0 \exp(i\omega t)\ d\omega$$

$$f(t) = \rho_0 \Delta\omega \left(\frac{\sin\beta}{\beta}\right) \exp(i\omega_0 t)$$

$$\beta \equiv \Delta\omega t/2$$



f(t)
1
0.5
−0.5
−1
−20
−10
10
20
Δt

Uncertainty Product

$$\Delta\omega \cdot \Delta t \approx 4\pi$$

$$\Delta\omega \cdot \Delta t = c\Delta k \cdot \Delta t = \Delta k \cdot \Delta x \approx \pi$$