Sinusoidal waves

$$\xi(x,t) = A\sin(kx - \omega t)$$

$$\psi(z,t) = B\cos(kz - \omega t))$$

$$E_x(y,t) = E_x\exp(i(ky - \omega t))$$

In general what are $\xi(x,t)$, $\psi(x,t)$?

$$p(x,t) = \bar{p} + \Delta p(x,t)$$
$$\rho(z,t) = \bar{\rho} + \Delta\rho(z,t)$$

Variation of pressure, density etc.

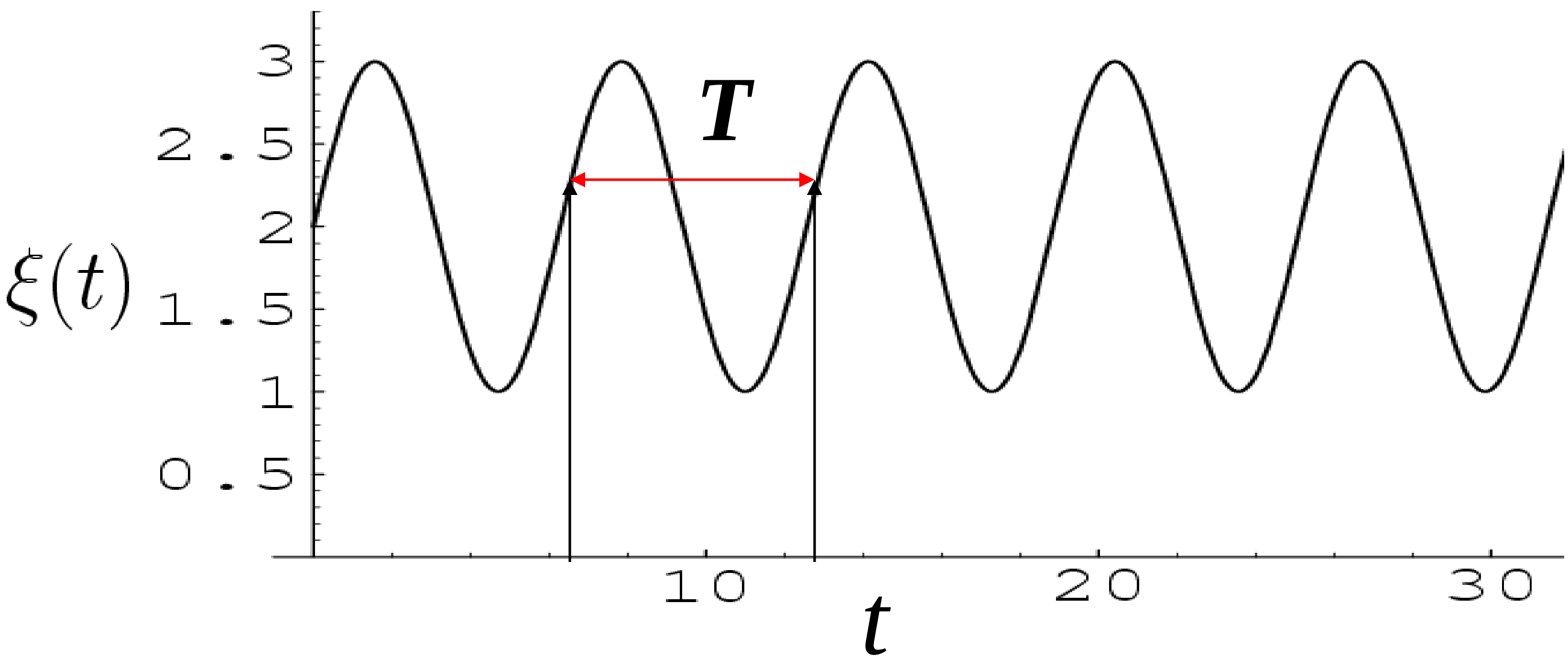
$$\xi(x,t) = \Delta p(x,t)$$
$$\psi(z,t) = \Delta\rho(z,t)$$

At, $x = x_0$

$$\xi(t) = C\exp(-i\omega t), \qquad C = A\exp(ikx_0)$$

$$\omega T = 2\pi, \qquad t = T$$

$$\xi(t) = \xi(t+T)$$

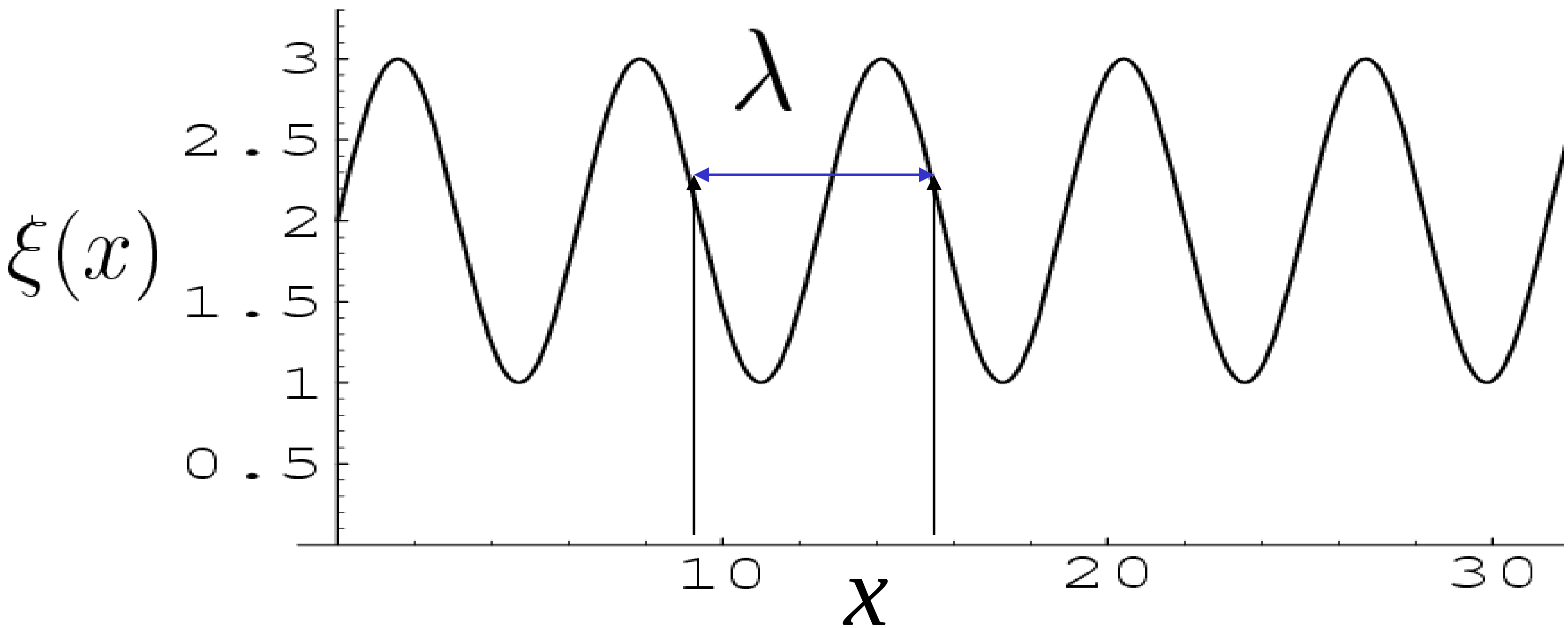


At, $t = t_0$

$$\xi(x) = D\exp(ikx), \qquad D = A\exp(-i\omega t_0)$$

$$k\lambda = 2\pi, \qquad x = \lambda$$

$$\xi(x) = \xi(x+\lambda)$$



$$\phi(x,t) = kx - \omega t$$

$$\phi = 0, \qquad x = 0,\ t = 0$$

New position of $\phi = 0,$ at Δt

$$k\Delta x - \omega \Delta t = 0$$

$$\Delta x = \frac{\omega}{k}\Delta t$$

Phase velocity =The speed with which the constant phase moves

$$\frac{\Delta x}{\Delta t} = v_p = \frac{\omega}{k}$$

$$\omega = \left|\frac{\partial \phi(x,t)}{\partial t}\right|$$

$$k = \left|\frac{\partial \phi(x,t)}{\partial x}\right|$$

Group velocity

$$\omega_1 = \omega + \Delta\omega \qquad k_1 = k + \Delta k$$

$$\omega_2 = \omega - \Delta\omega \qquad k_2 = k - \Delta k$$

$$\psi(x,t) = A\cos(k_1\,x - \omega_1\,t)$$

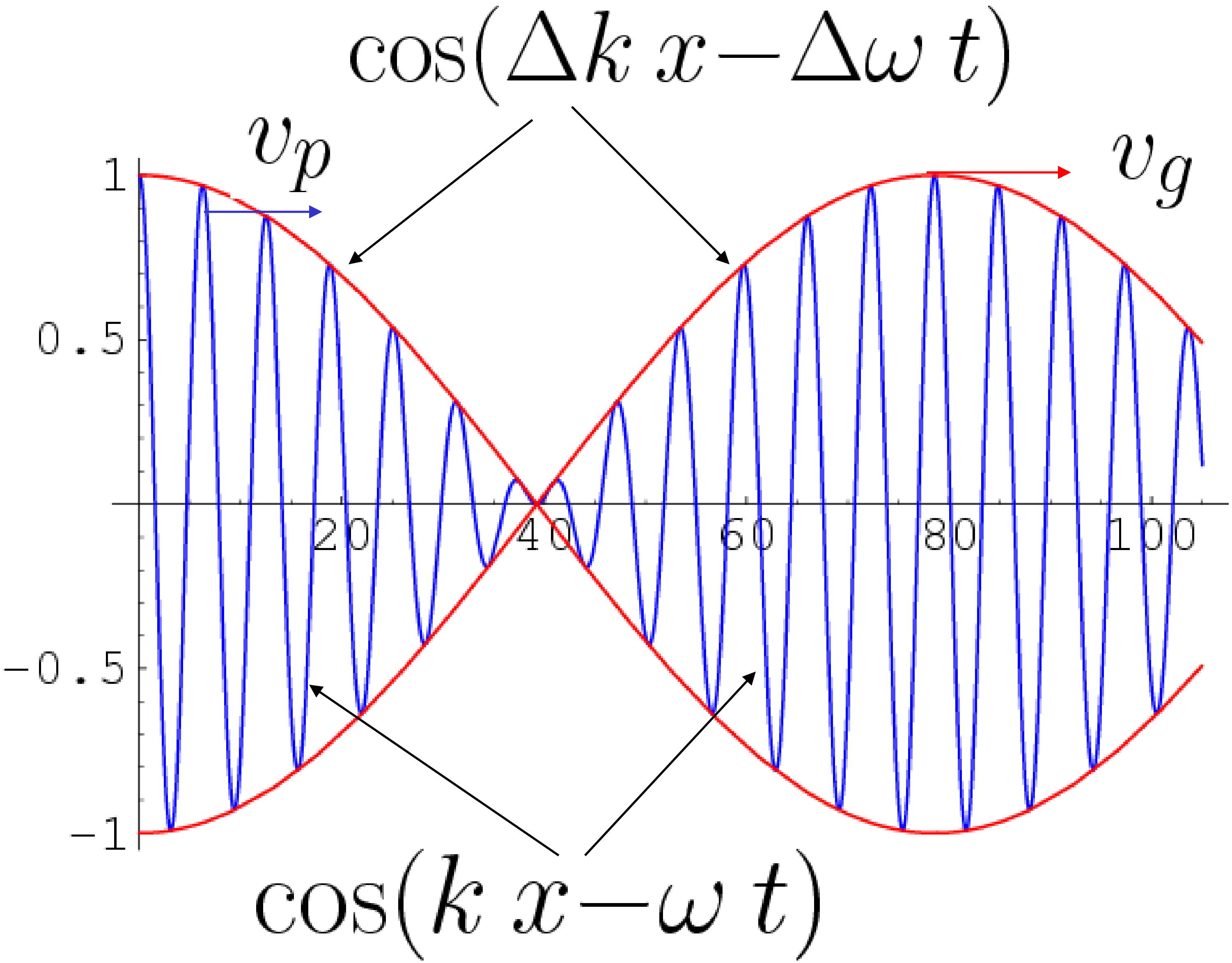
$$+A\cos(k_2\,x - \omega_2\,t)$$

$$\psi(x,t) = 2A\cos(k\,x - \omega\,t)\cdot\cos(\Delta k\,x - \Delta\omega\,t)$$

Phase velocity $$v_p = \frac{\omega}{k}$$

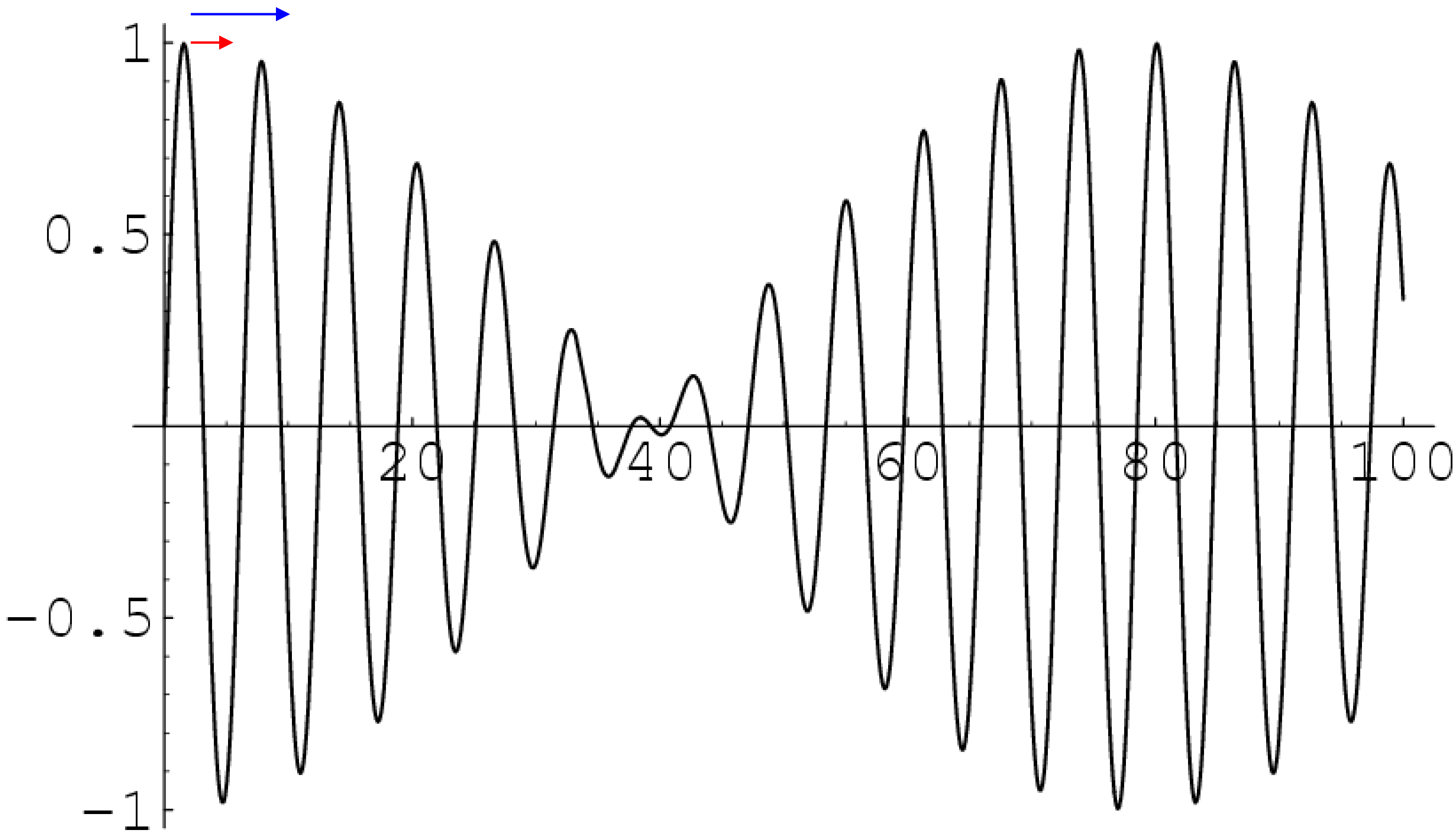
Group velocity $$v_g = \frac{\Delta\omega}{\Delta k} \longrightarrow \frac{\partial\omega}{\partial k}$$

$$\Delta k \longrightarrow 0$$



$\cos(\Delta k\, x - \Delta\omega\, t)$
v_p
v_g
1
0.5
-0.5
-1
20
40
60
80
100
$\cos(k\, x - \omega\, t)$

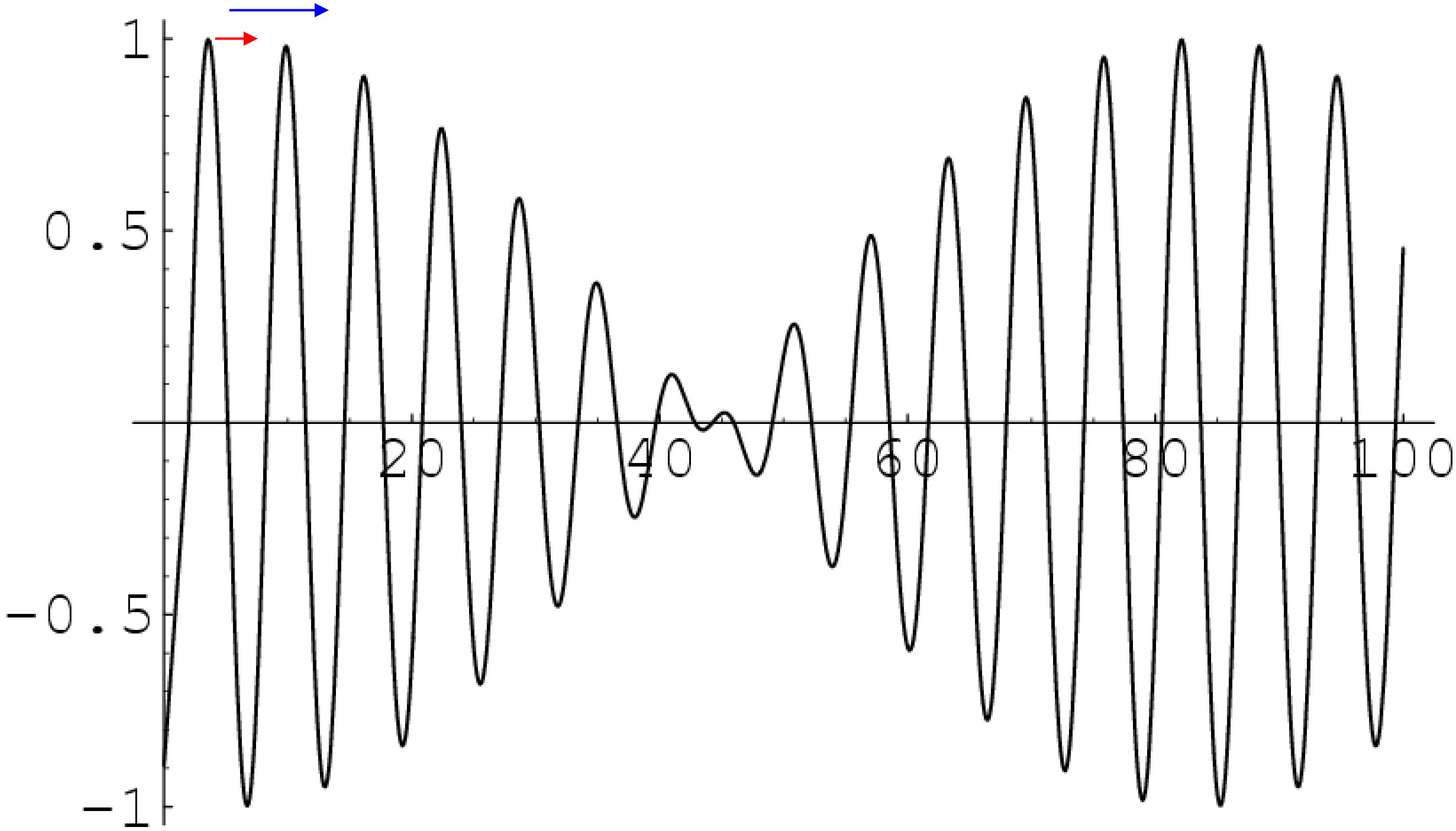
Vp < **V**g



$$\sin(1.00\,x - 2.0\,t)\cos(0.04\,x - 0.2\,t)$$

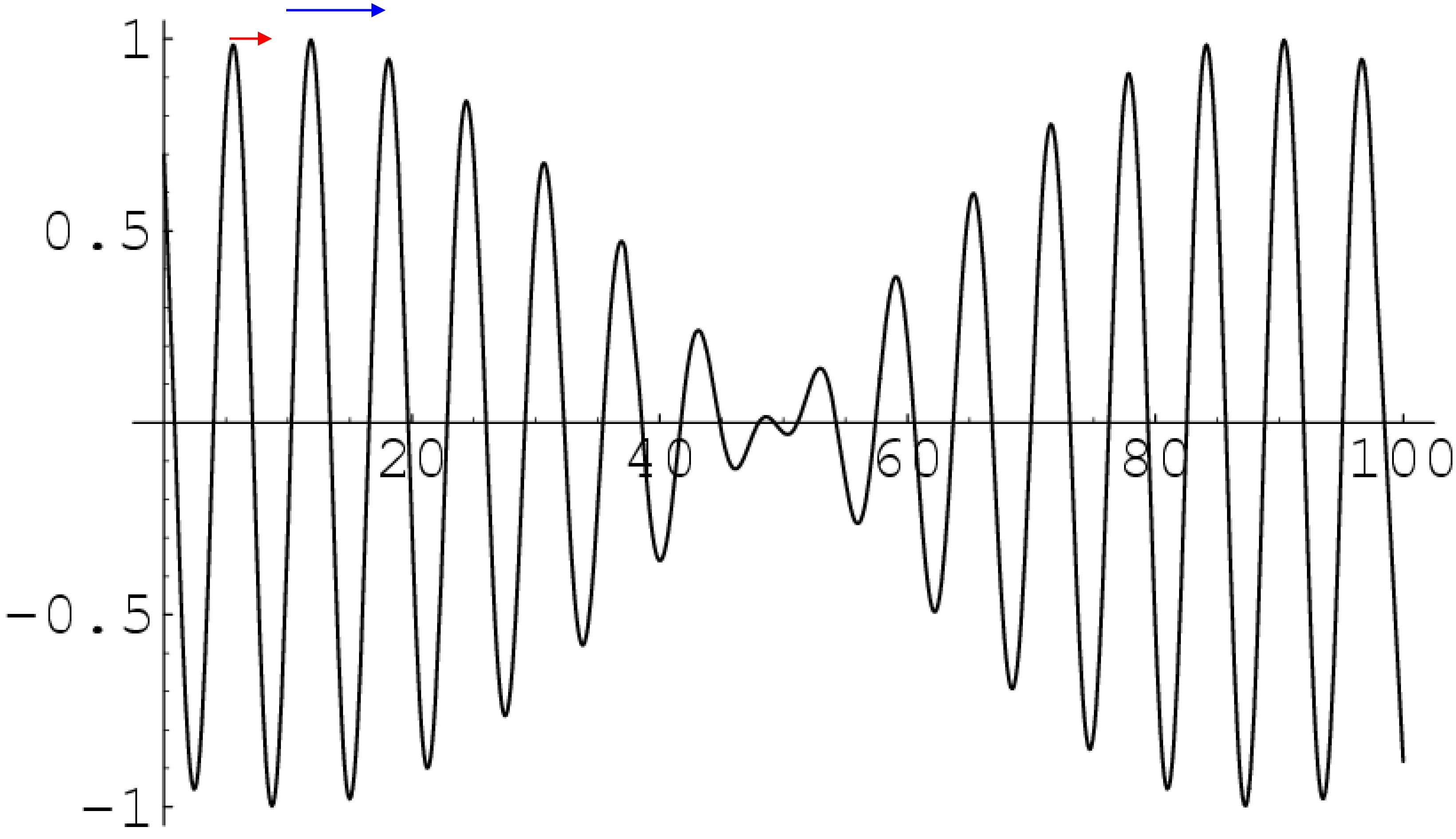
t = 0

$V_p < V_g$



t = 1 $\sin(1.00\ x - 2.0\ t)\cos(0.04\ x - 0.2\ t)$

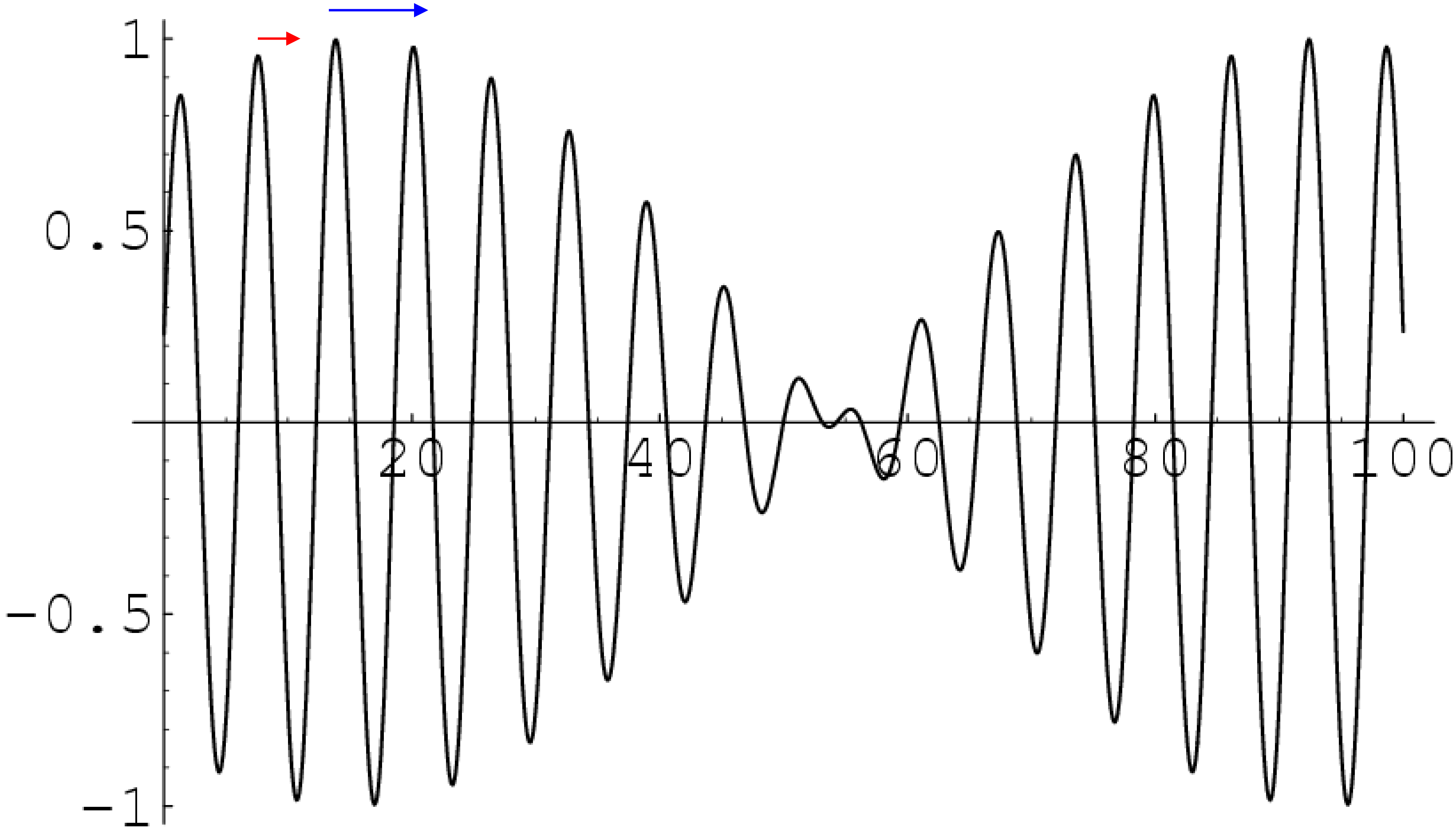
Vp < **V**g



t = 2

$$\sin(1.00\ x - 2.0\ t)\cos(0.04\ x - 0.2\ t)$$

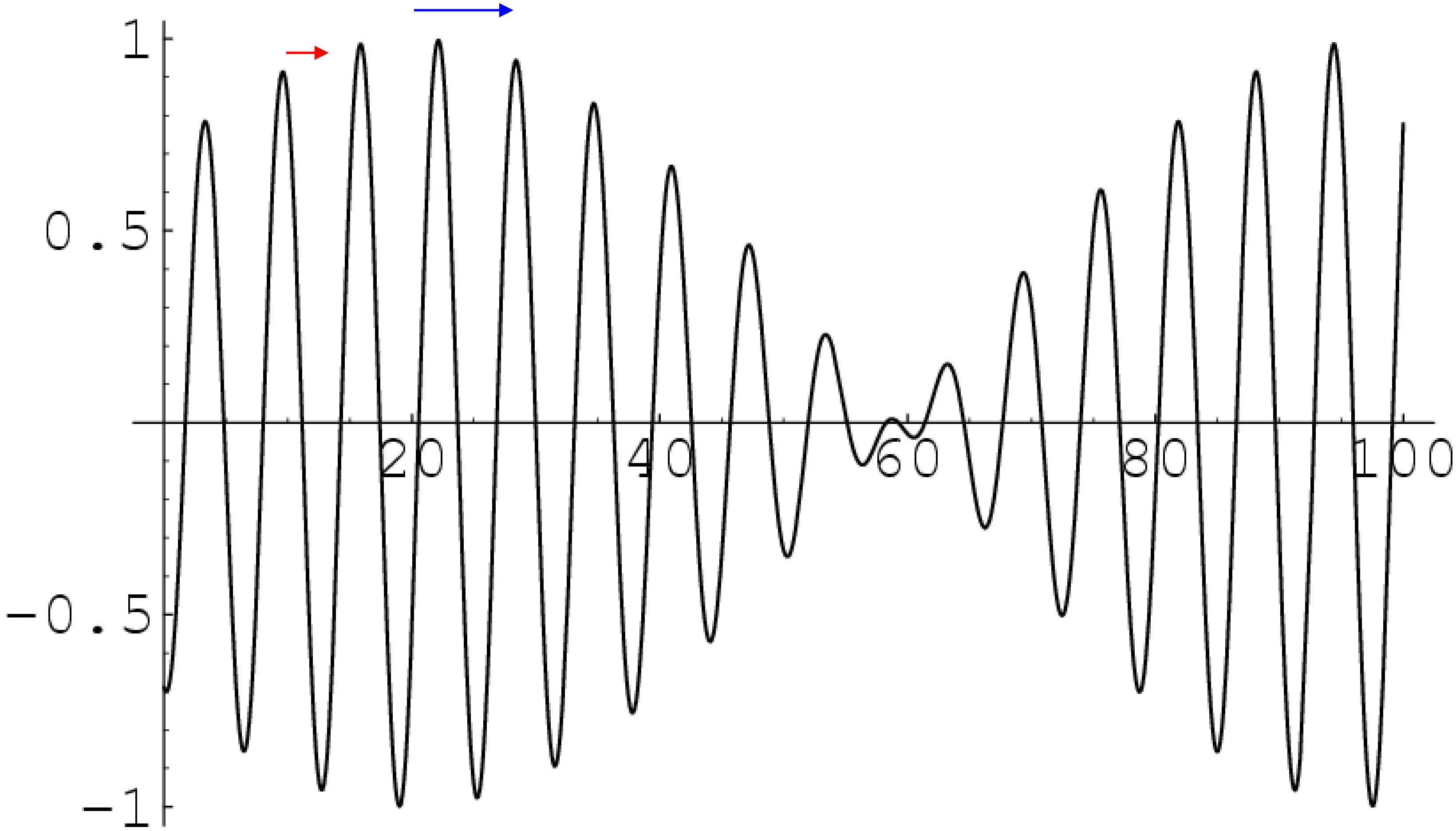
Vp < **V**g



t = 3

$$\sin(1.00\ x - 2.0\ t)\cos(0.04\ x - 0.2\ t)$$

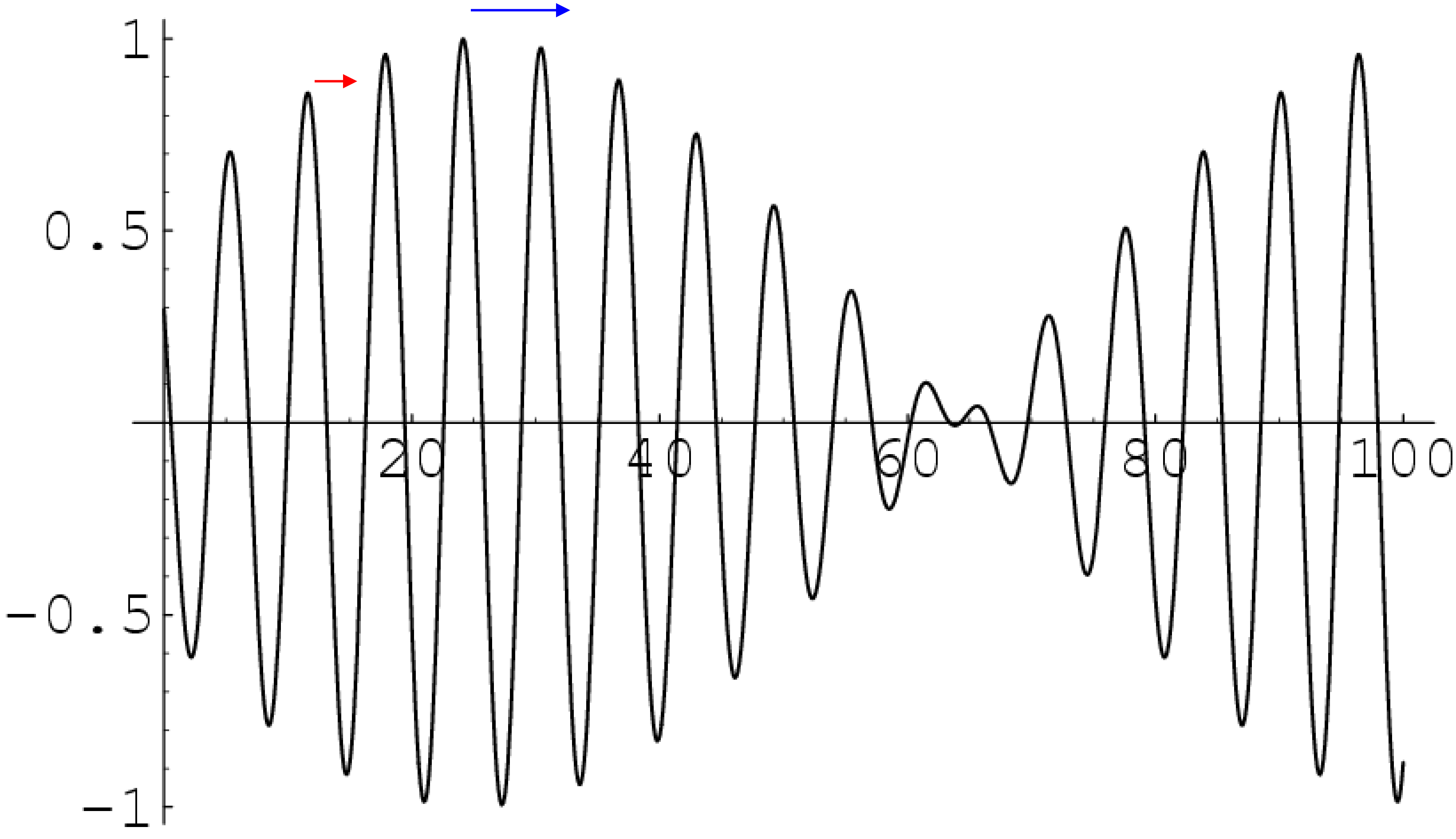
Vp < **V**g



t = 4

$$\sin(1.00\ x - 2.0\ t)\cos(0.04\ x - 0.2\ t)$$

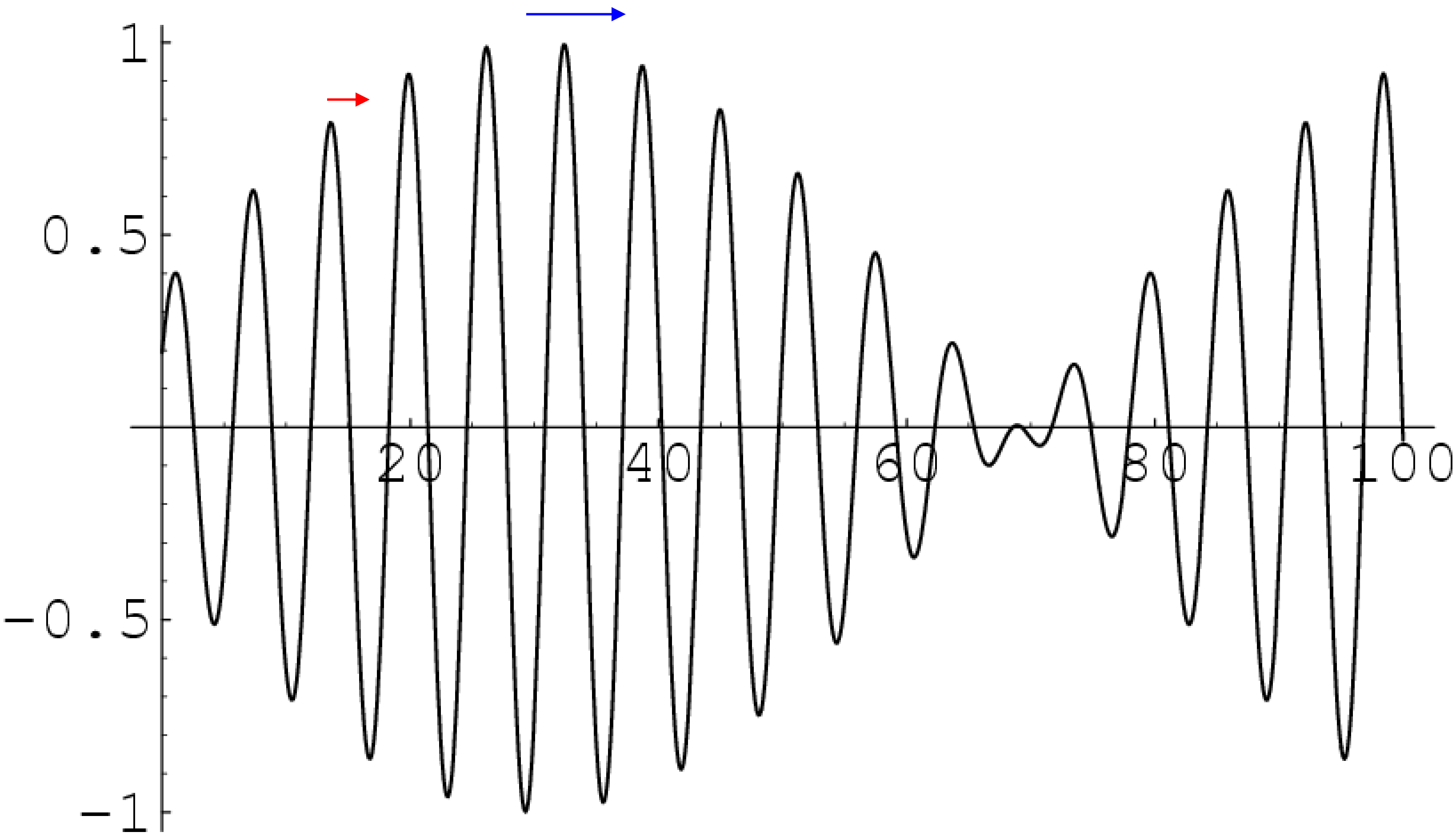
Vp < **V**g



t = 5

$$\sin(1.00\ x - 2.0\ t)\cos(0.04\ x - 0.2\ t)$$

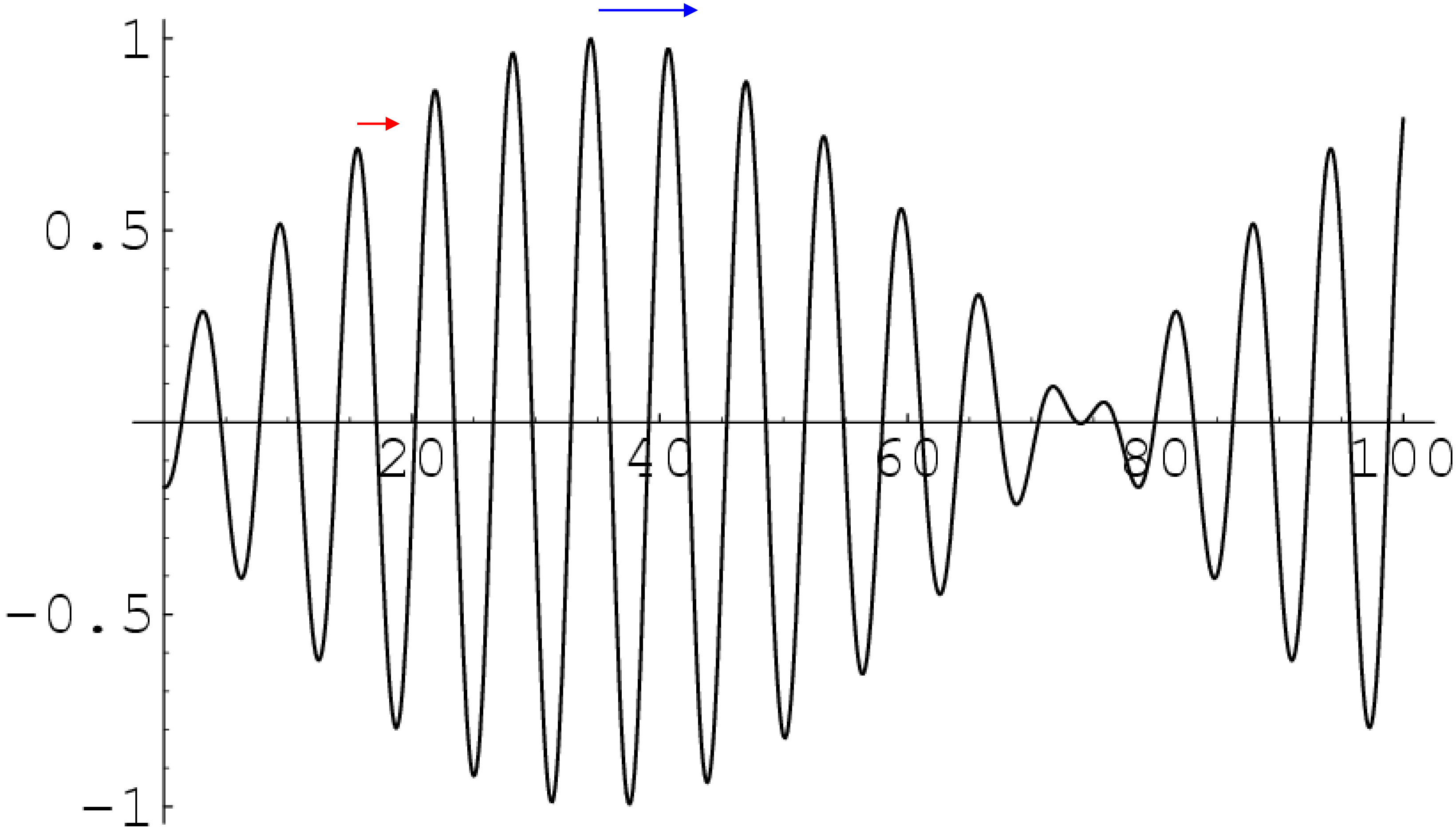
Vp < **V**g



t = 6

$$\sin(1.00\,x - 2.0\,t)\cos(0.04\,x - 0.2\,t)$$

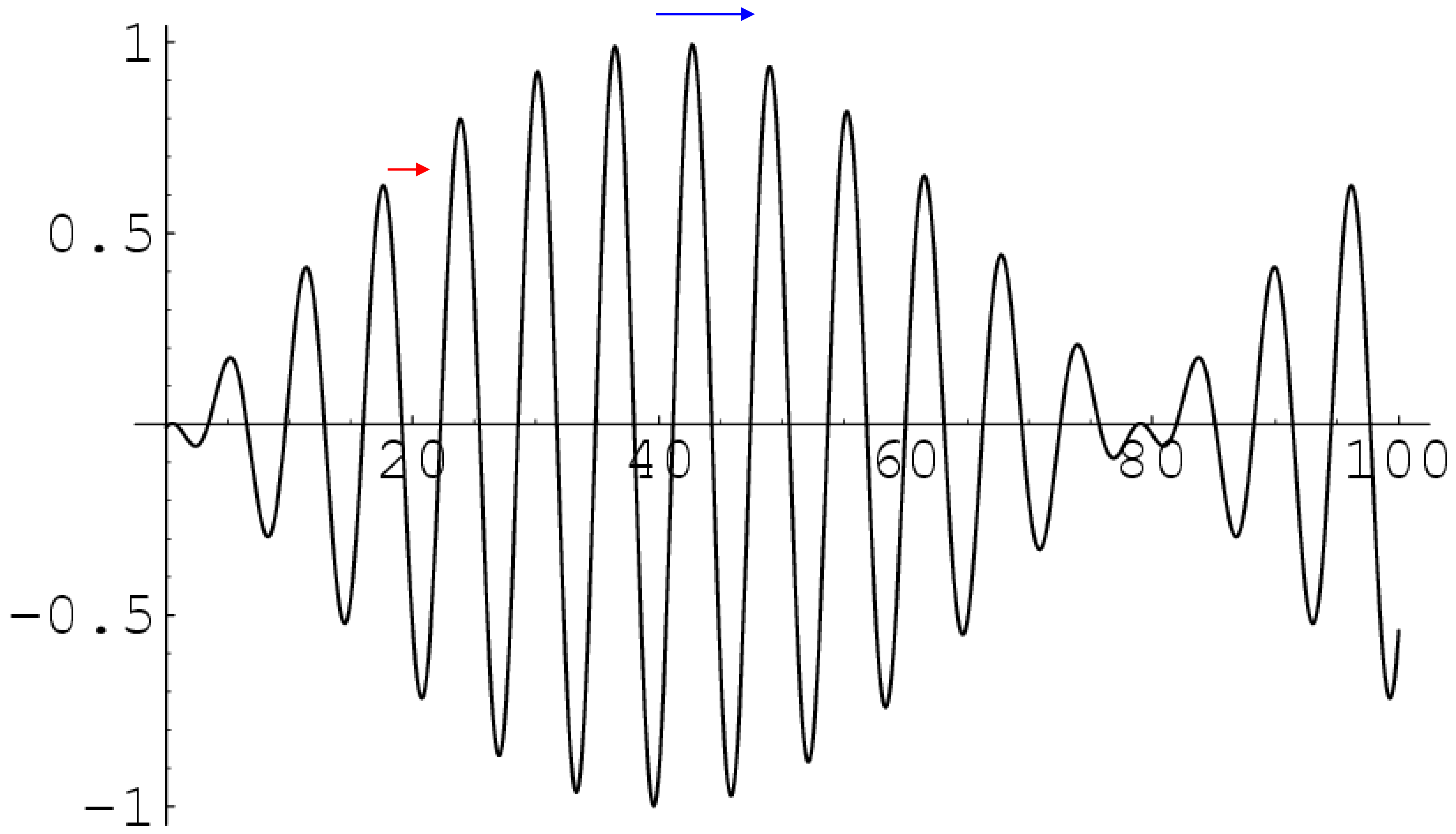
Vp < **V**g



t = 7

$$\sin(1.00\ x - 2.0\ t)\cos(0.04\ x - 0.2\ t)$$

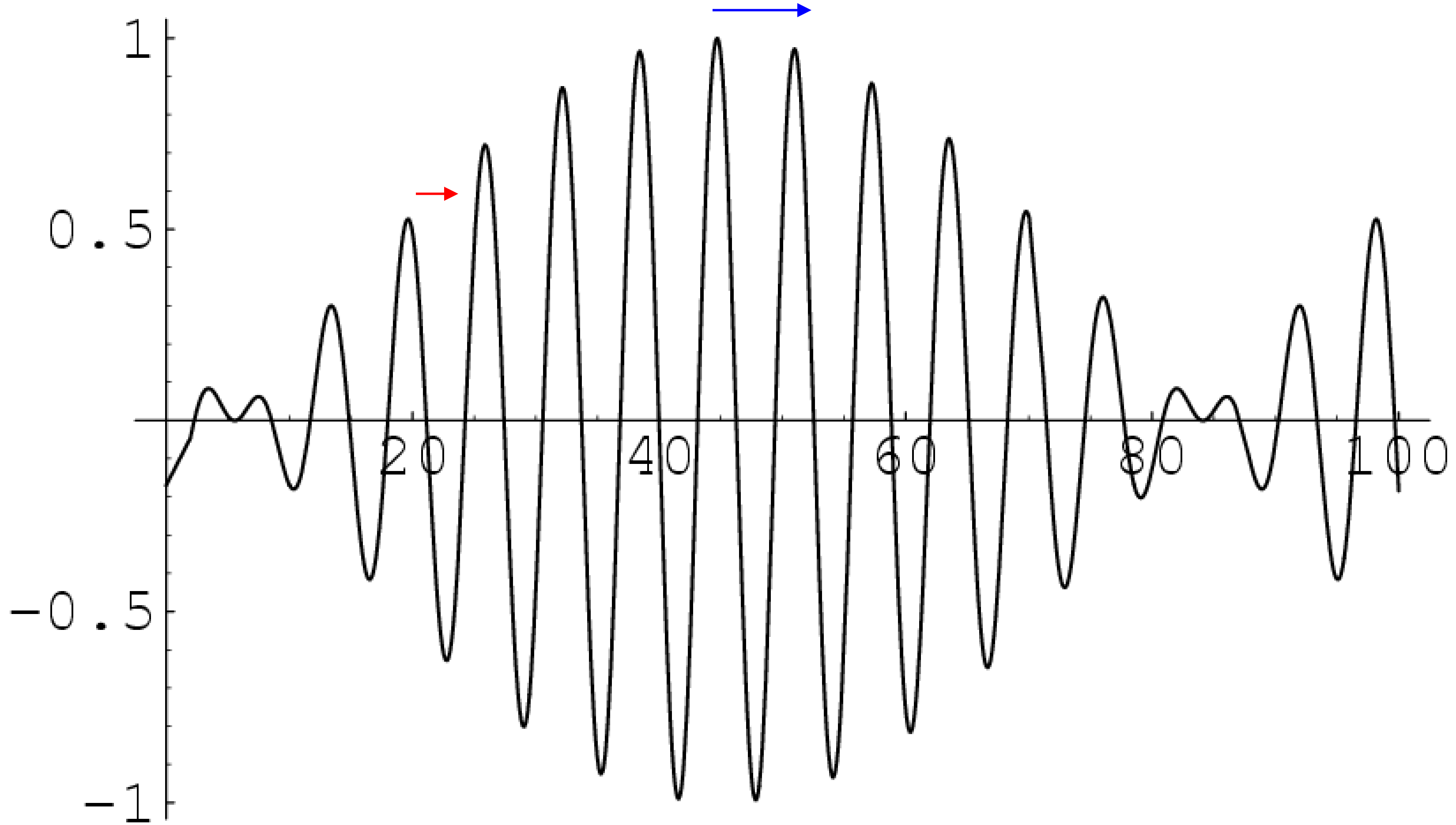
Vp < **V**g



t = 8

$$\sin(1.00\,x - 2.0\,t)\cos(0.04\,x - 0.2\,t)$$

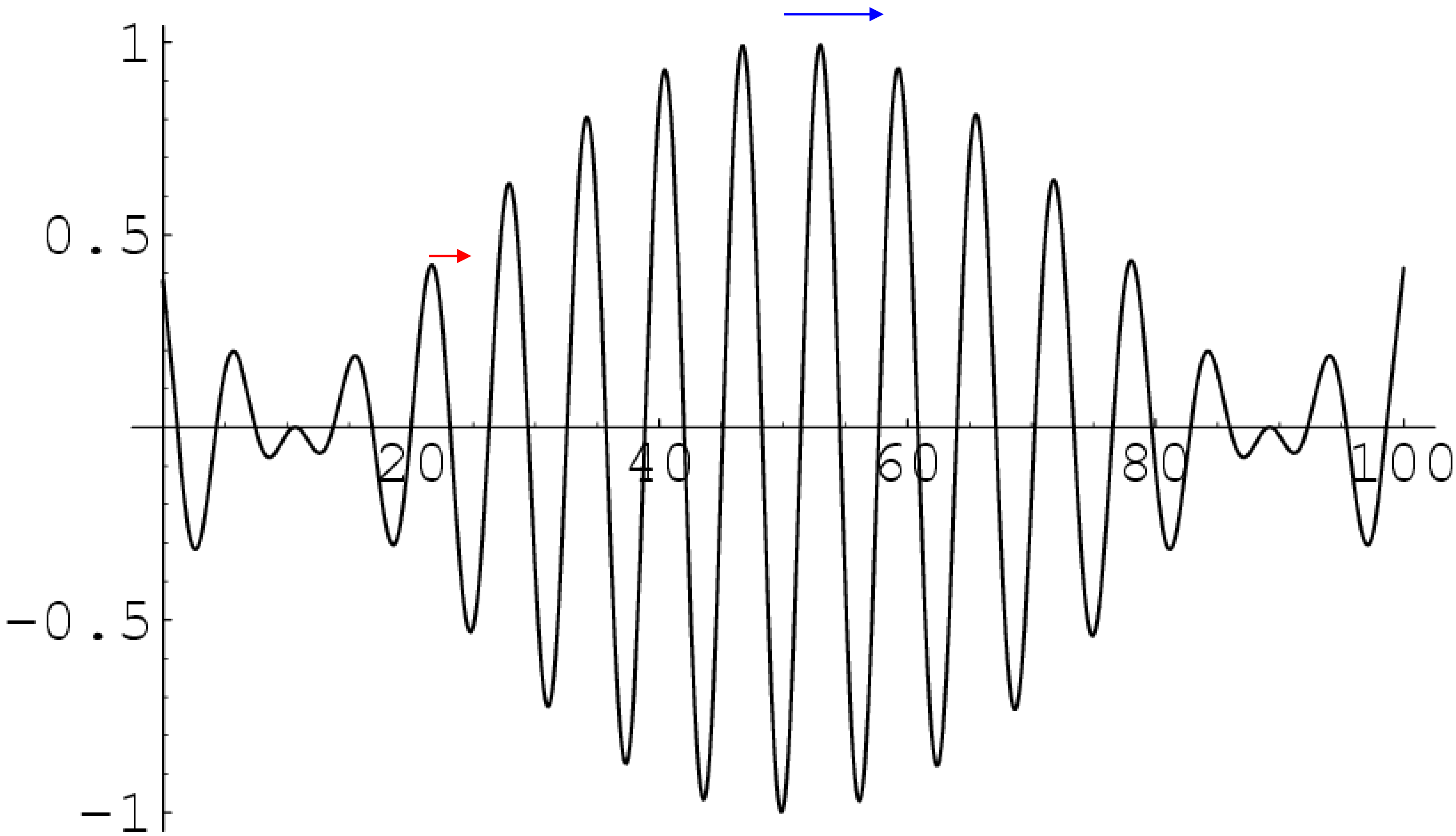
Vp < **V**g



t = 9

$$\sin(1.00\ x - 2.0\ t)\cos(0.04\ x - 0.2\ t)$$

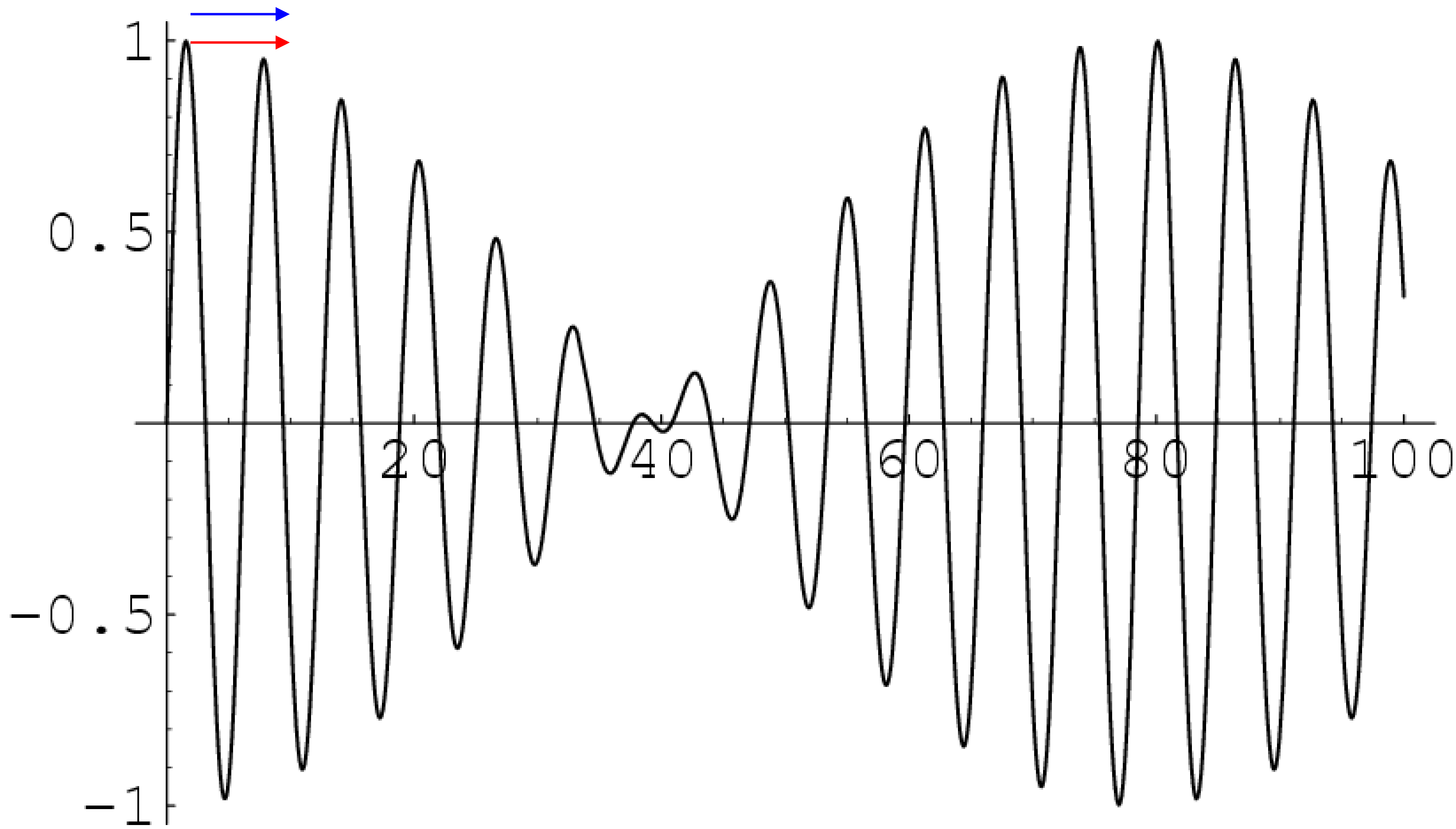
Vp < **V**g



t = 10

$$\sin(1.00\ x - 2.0\ t)\cos(0.04\ x - 0.2\ t)$$

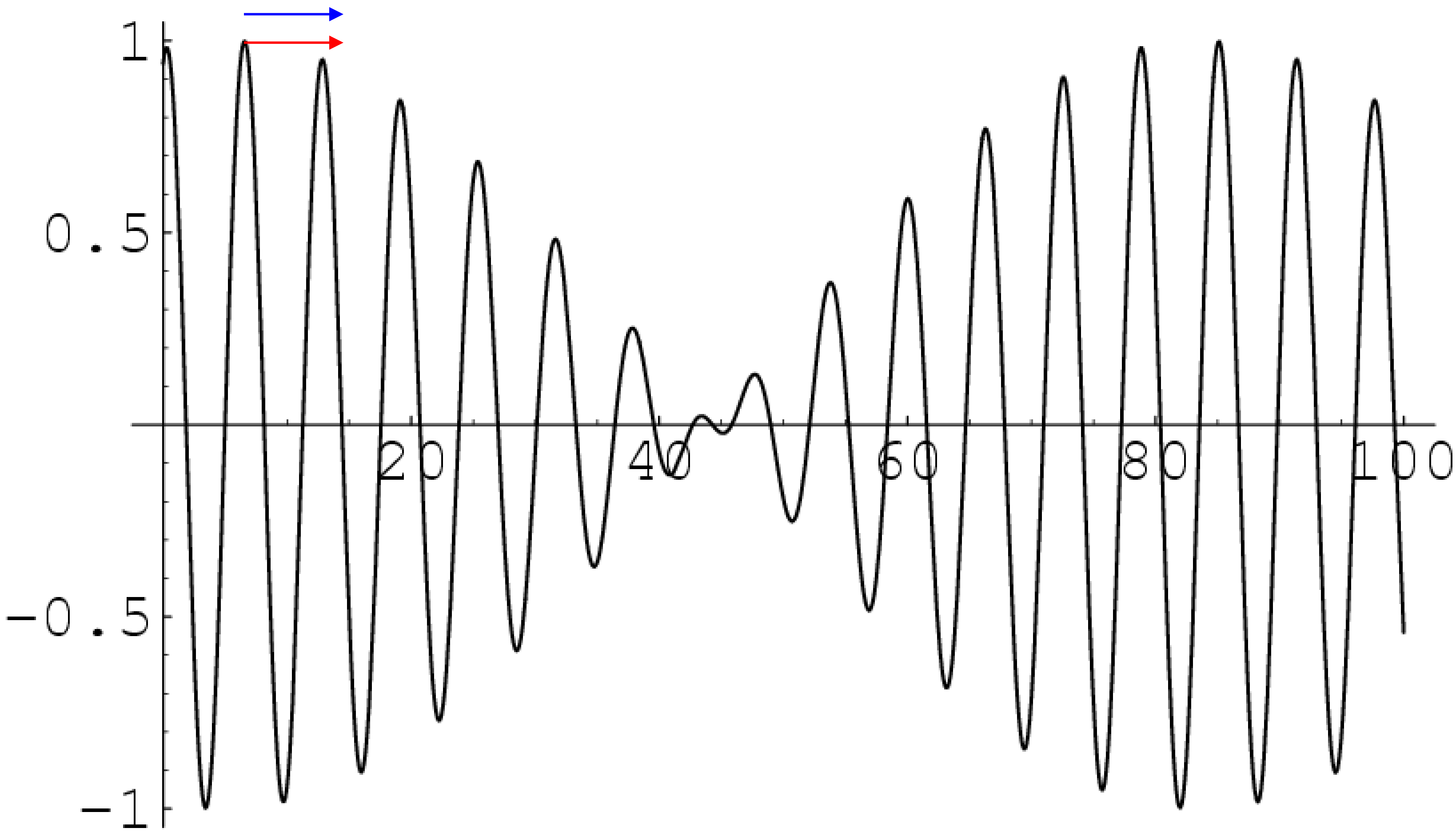
Vp = **V**g



t = 0

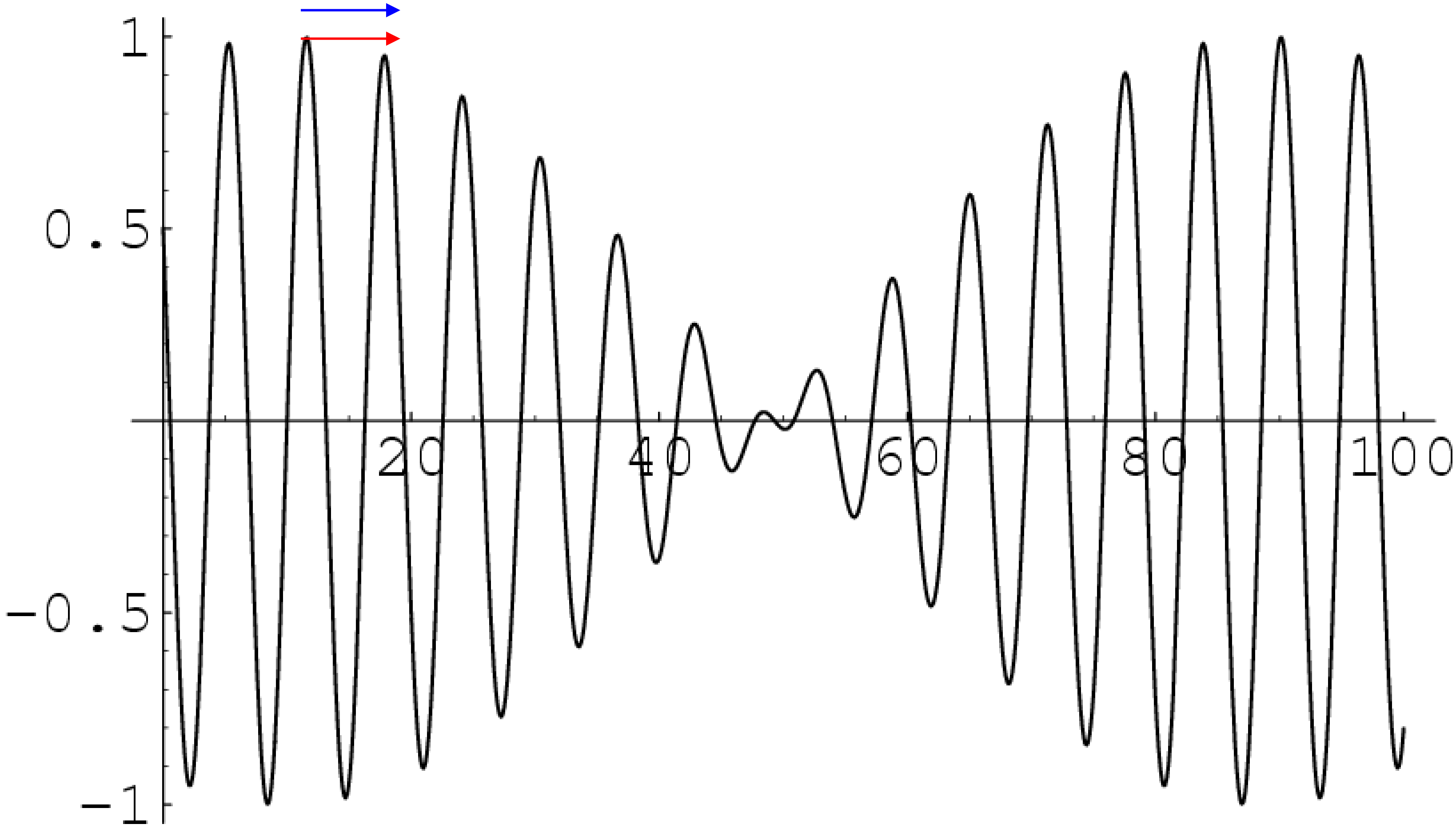
$$\sin(1.00\ x - 5.0\ t)\cos(0.04\ x - 0.2\ t)$$

Vp = **V**g



t = 1 $\sin(1.00\ x - 5.0\ t)\cos(0.04\ x - 0.2\ t)$

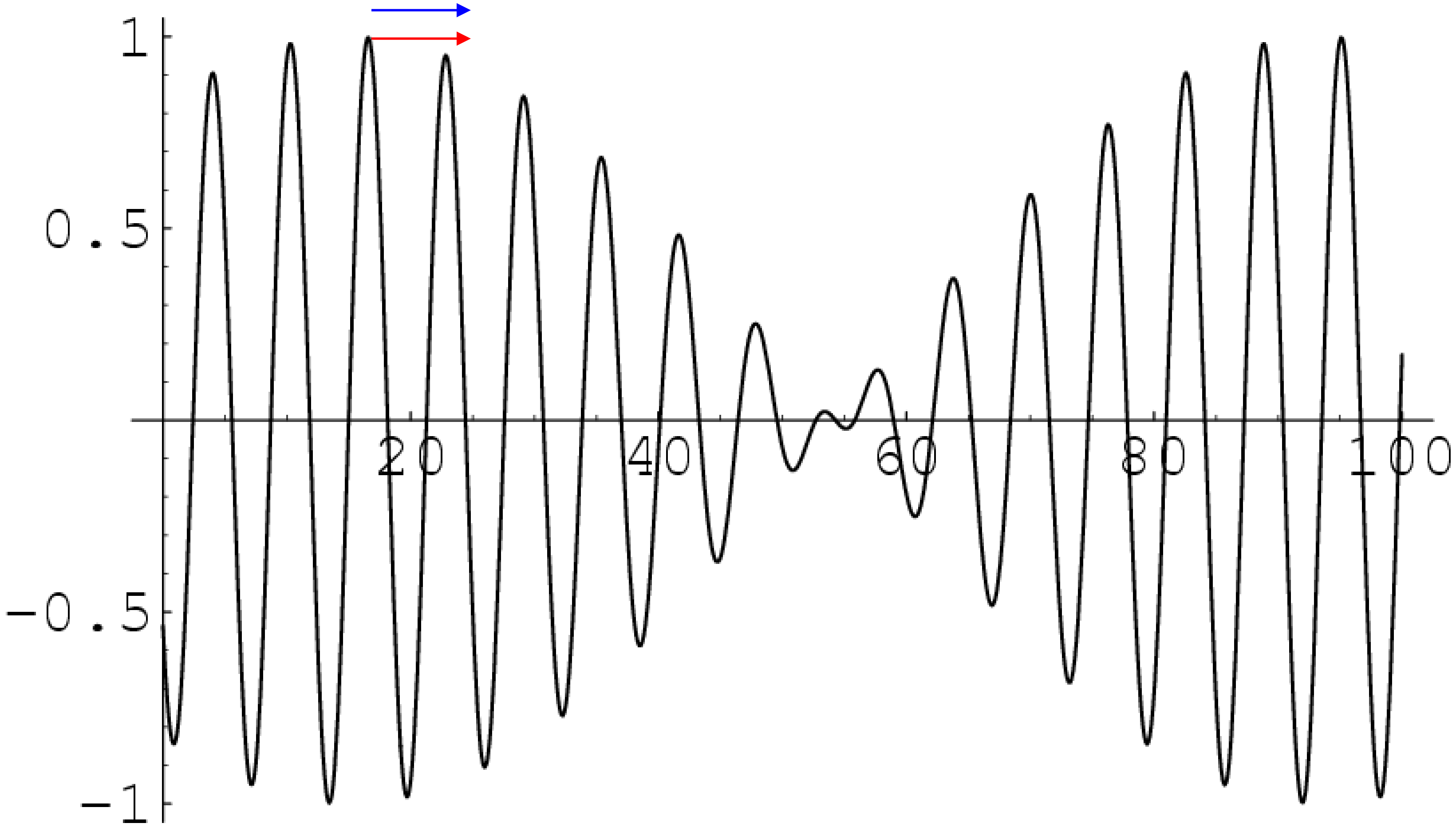
Vp = **V**g



t = 2

$$\sin(1.00\,x - 5.0\,t)\cos(0.04\,x - 0.2\,t)$$

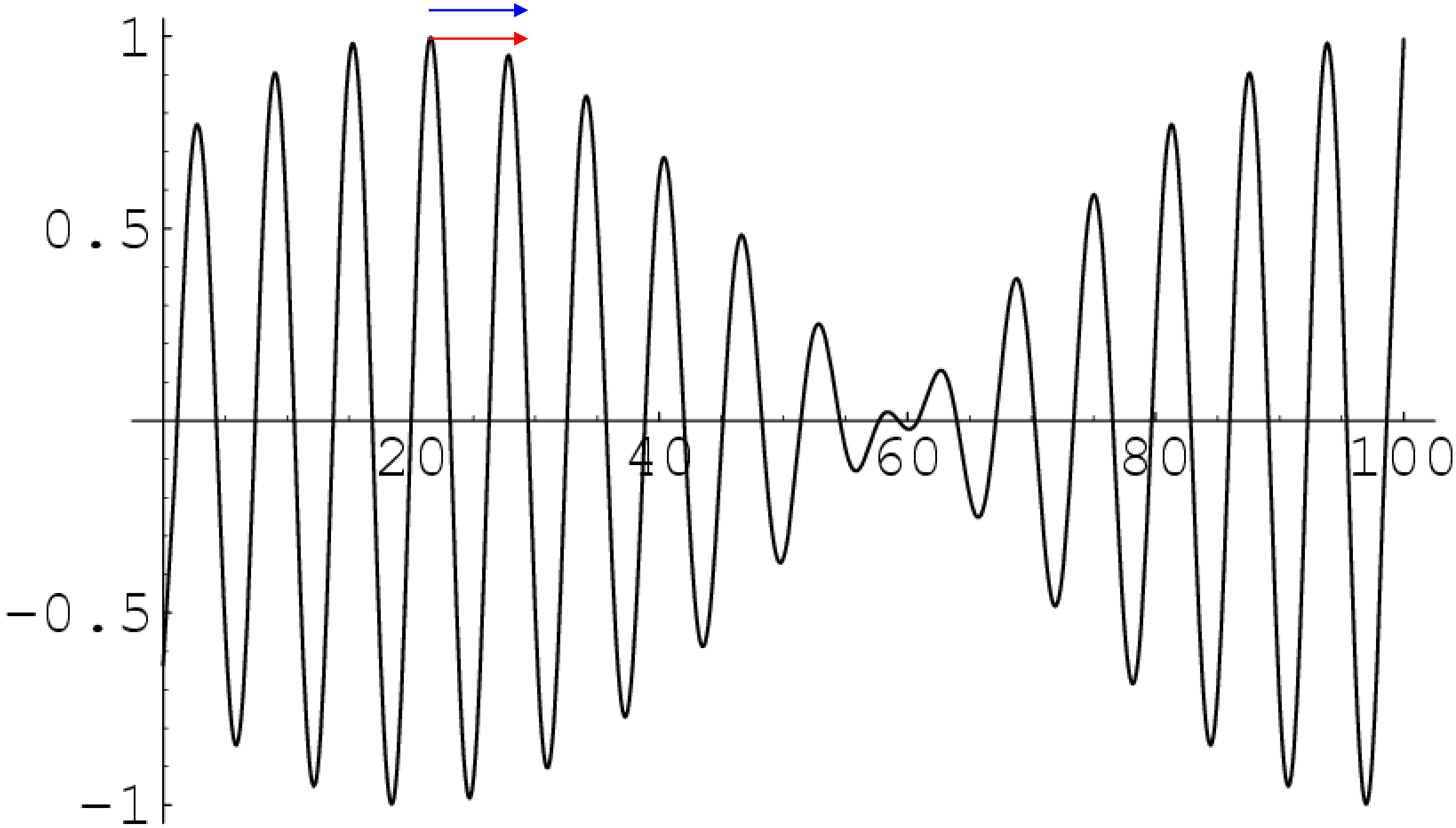
$\mathbf{V}_p = \mathbf{V}_g$



t = 3

$$\sin(1.00\ x - 5.0\ t)\cos(0.04\ x - 0.2\ t)$$

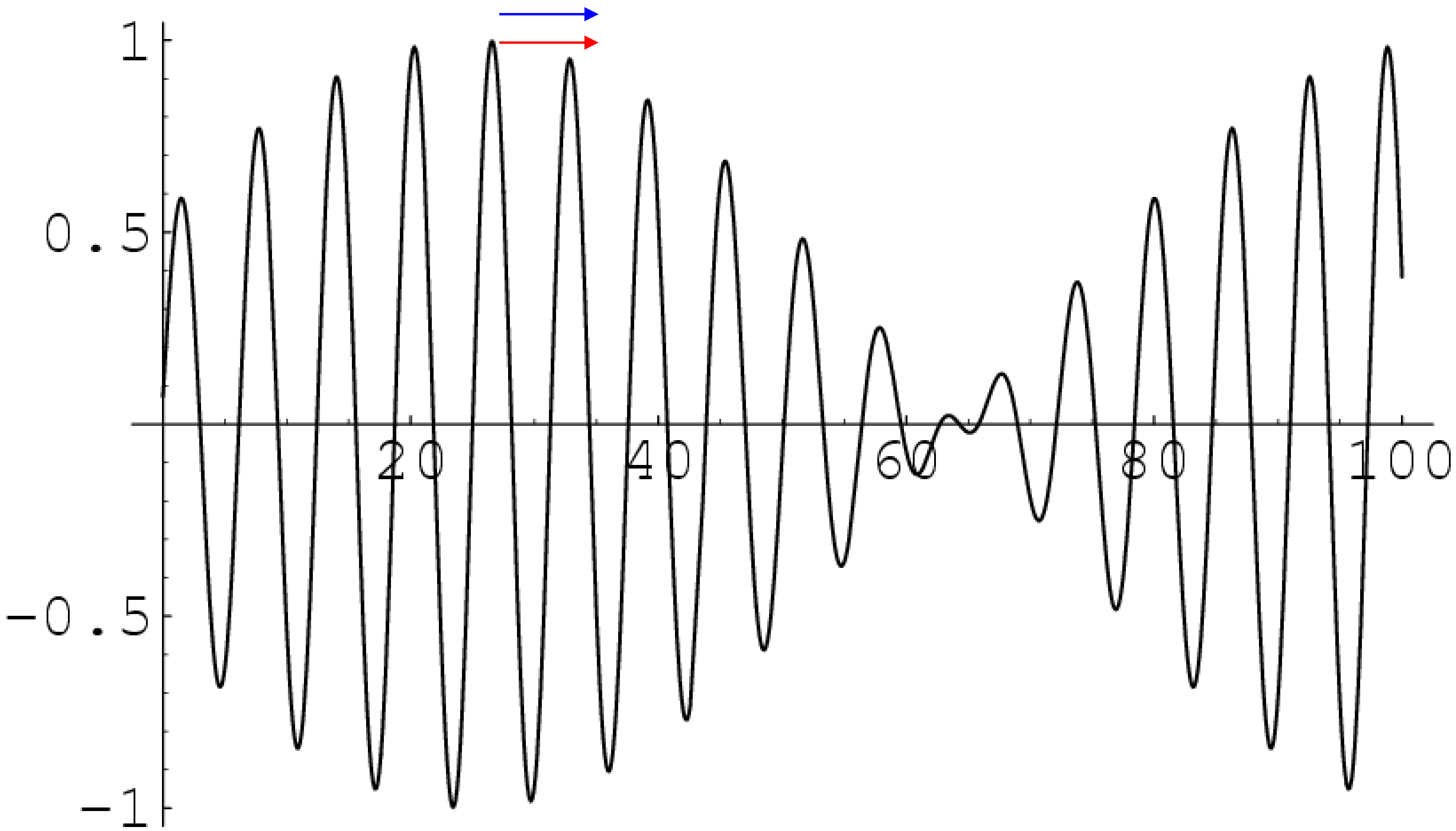
Vp = **V**g



t = 4

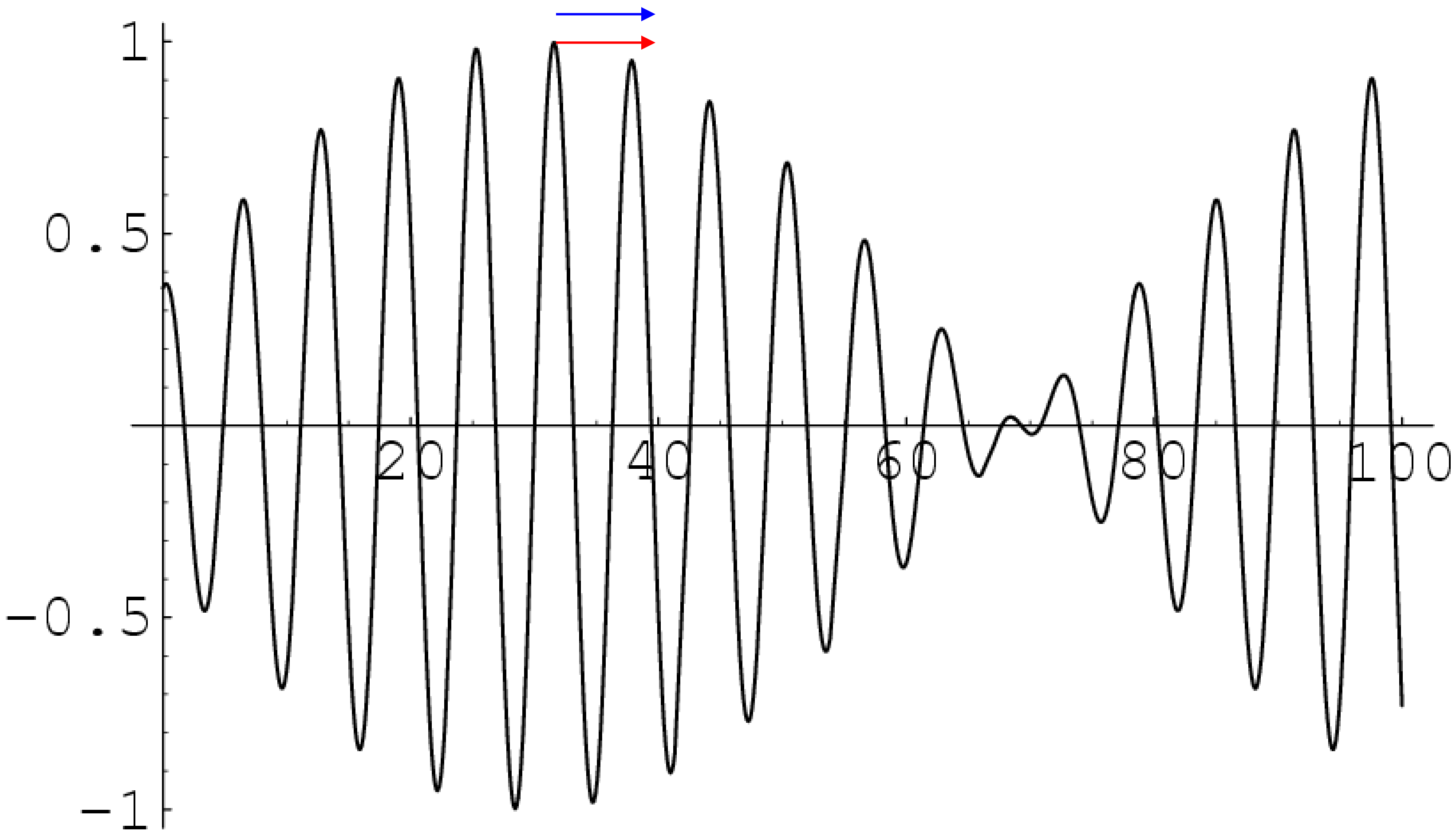
$$\sin(1.00\ x - 5.0\ t)\cos(0.04\ x - 0.2\ t)$$

Vp = Vg



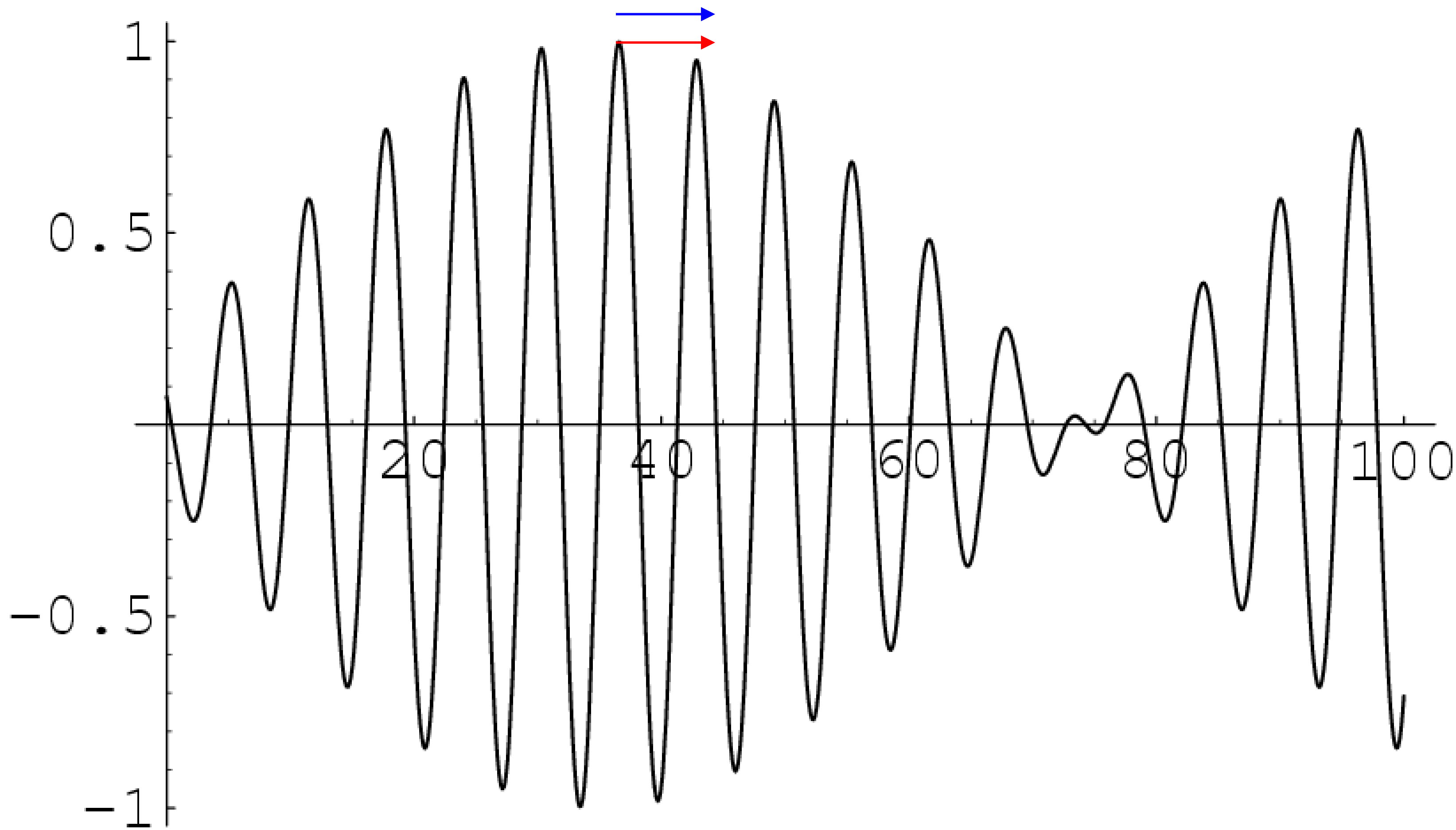
t = 5 $\sin(1.00\ x - 5.0\ t)\cos(0.04\ x - 0.2\ t)$

$V_p = V_g$



t = 6 $\sin(1.00\ x - 5.0\ t)\cos(0.04\ x - 0.2\ t)$

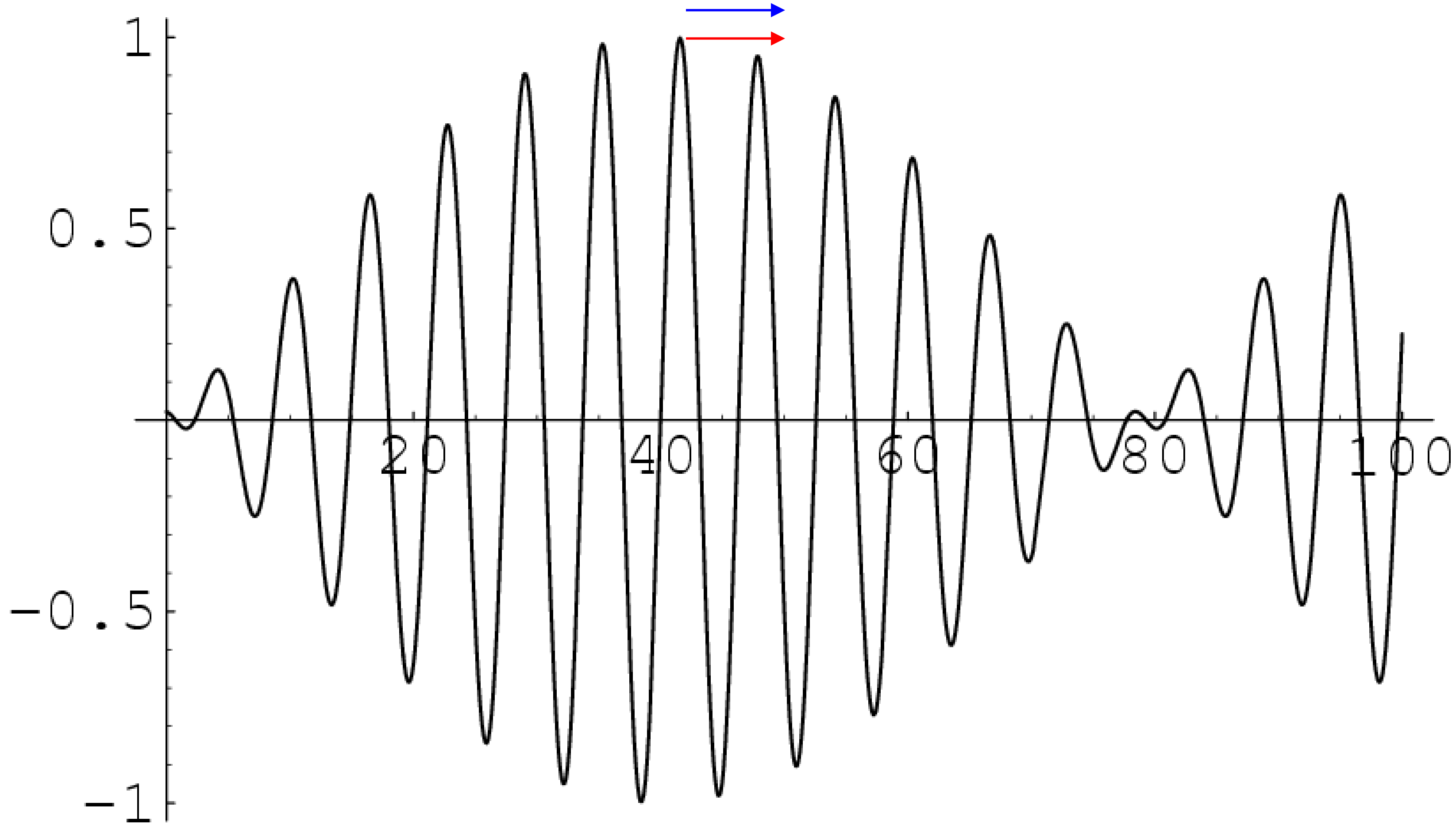
$V_p = V_g$



t = 7

$$\sin(1.00\ x - 5.0\ t)\cos(0.04\ x - 0.2\ t)$$

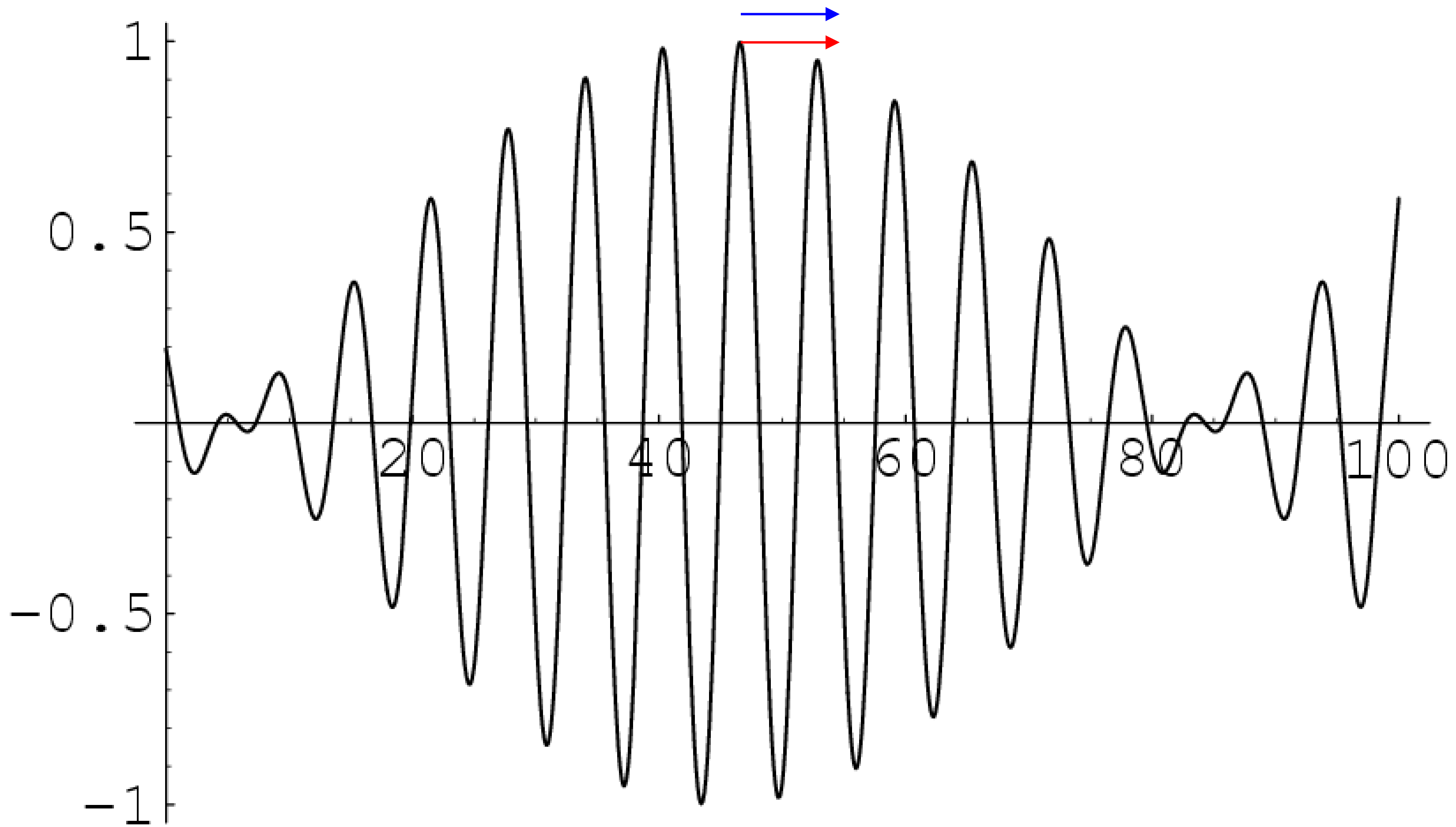
Vp = **V**g



t = 8

$$\sin(1.00\ x - 5.0\ t)\cos(0.04\ x - 0.2\ t)$$

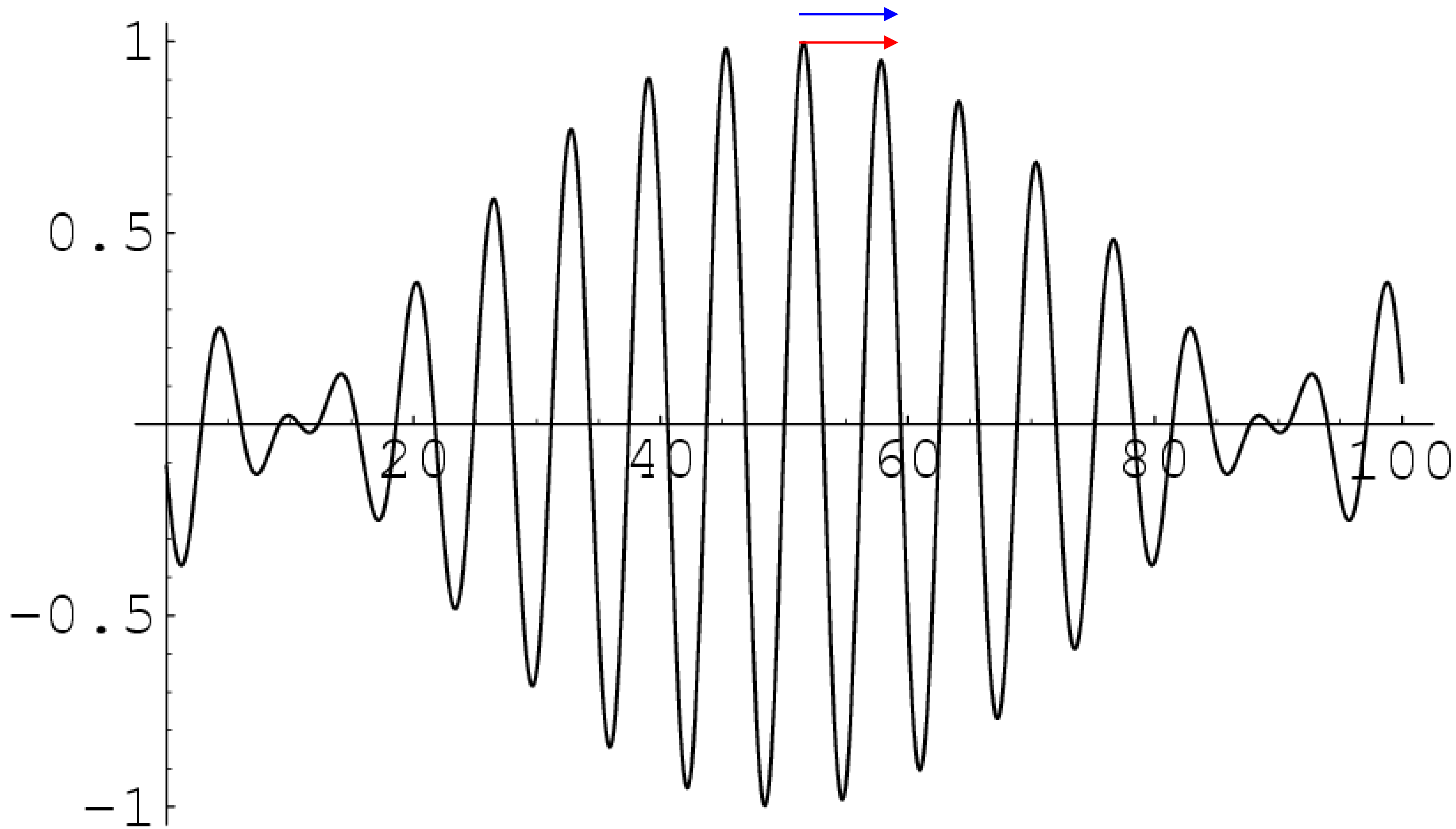
Vp = **V**g



t = 9

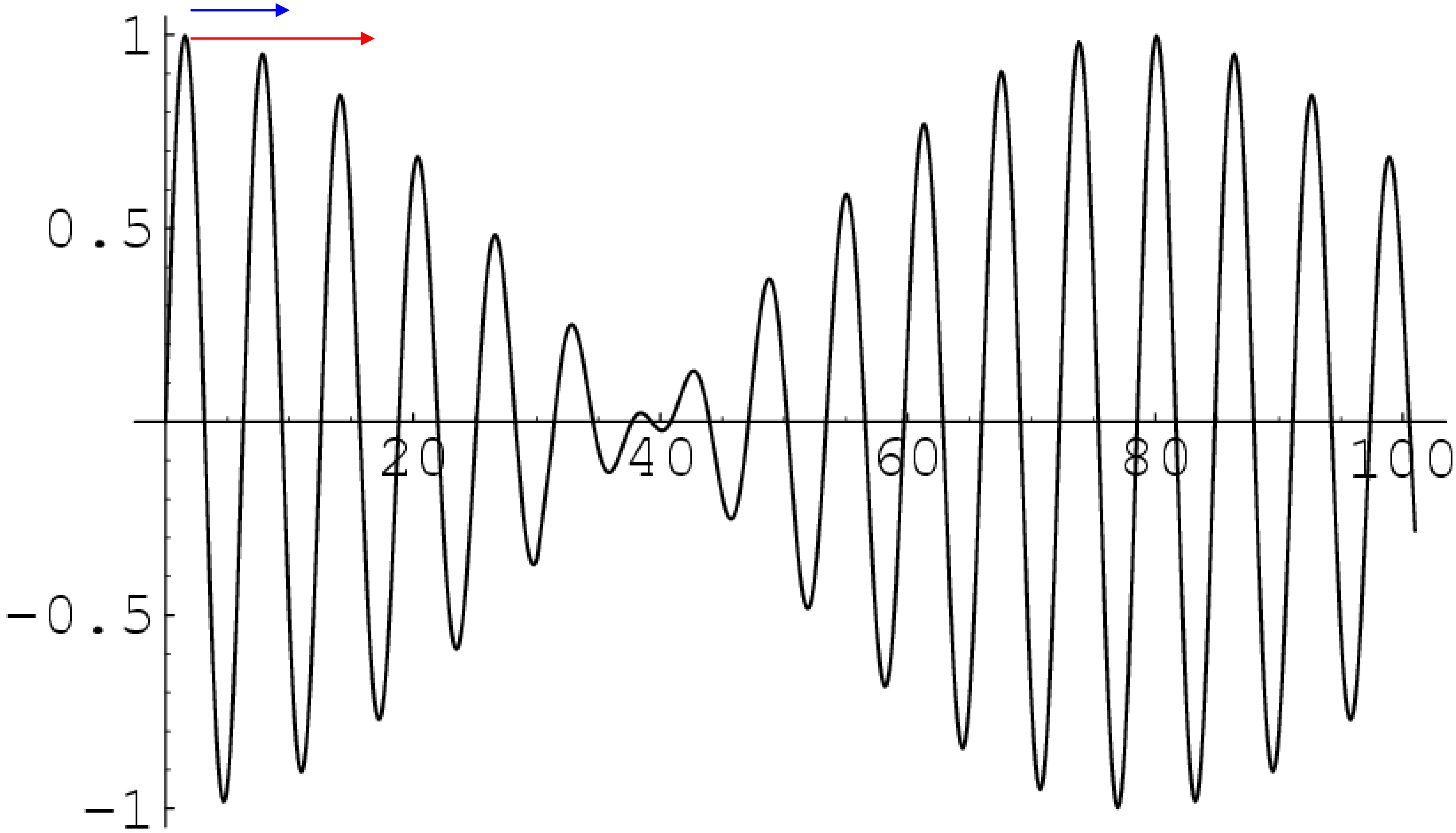
$$\sin(1.00\ x - 5.0\ t)\cos(0.04\ x - 0.2\ t)$$

Vp = **V**g



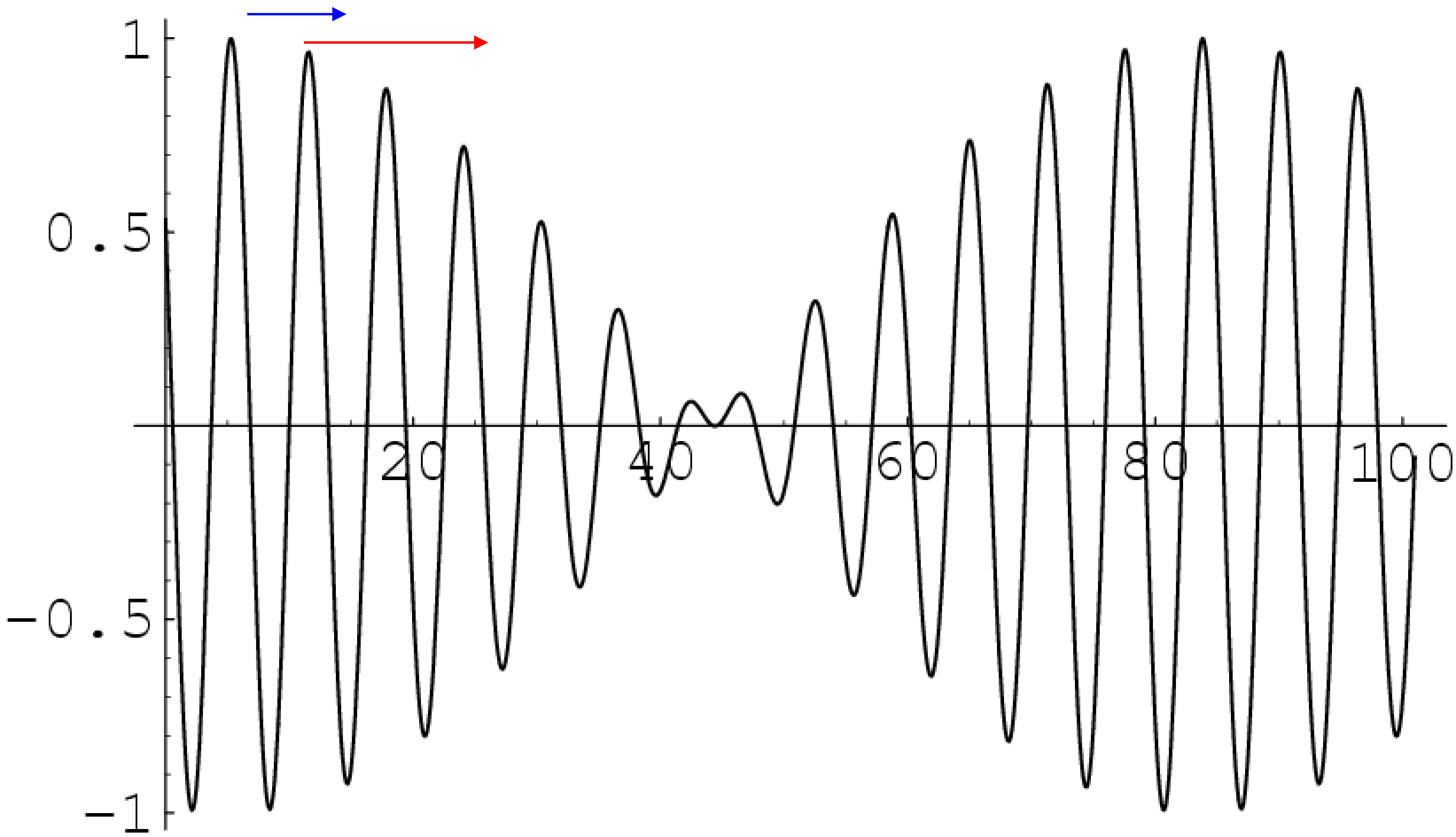
t = 10 $\sin(1.00\ x - 5.0\ t)\cos(0.04\ x - 0.2\ t)$

Vp > **V**g



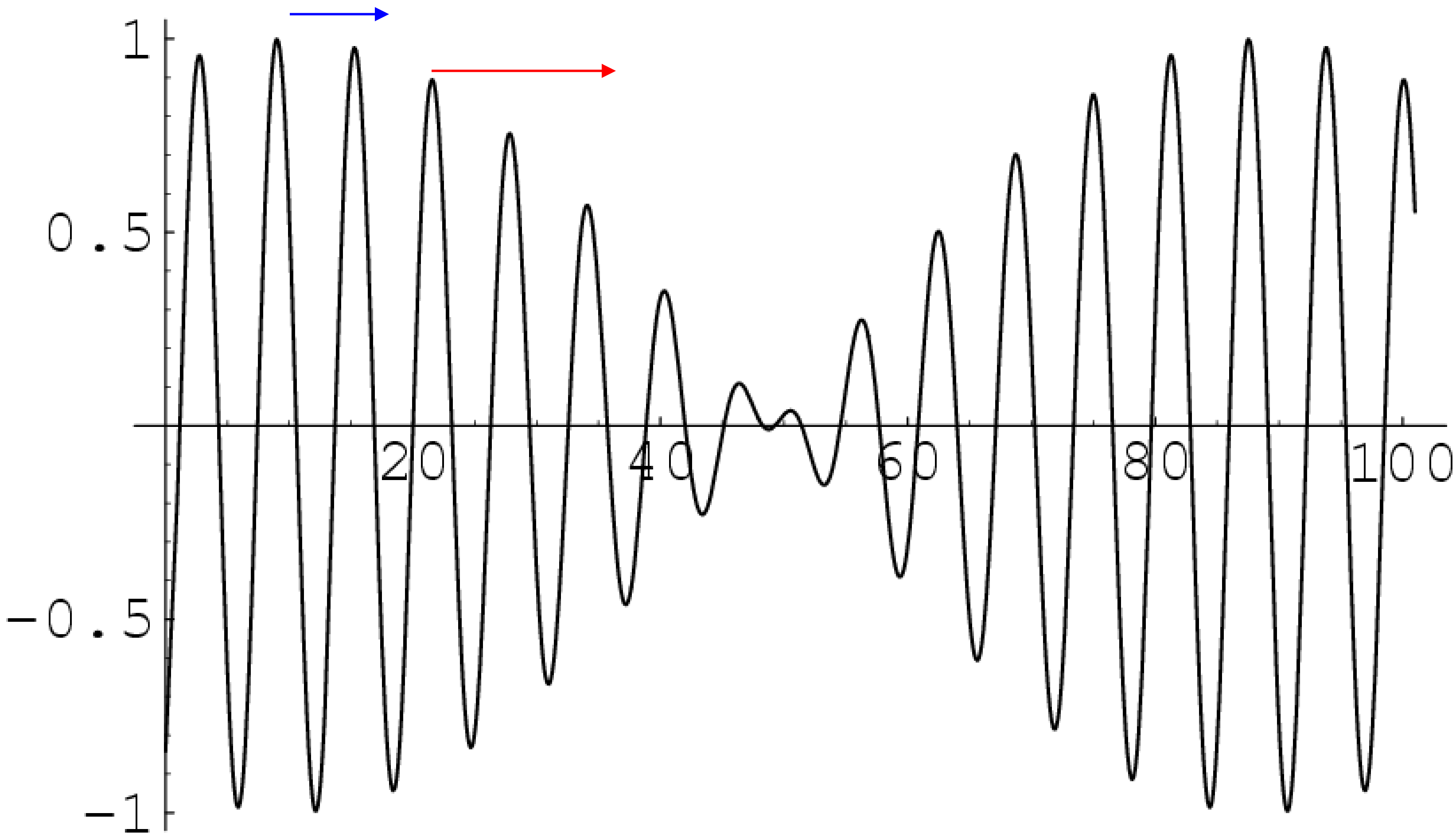
t = 0 $\sin(1.00\ x - 10.0\ t)\cos(0.04\ x - 0.2\ t)$

Vp > **V**g



t = 1 $\sin(1.00\,x - 10.0\,t)\cos(0.04\,x - 0.2\,t)$

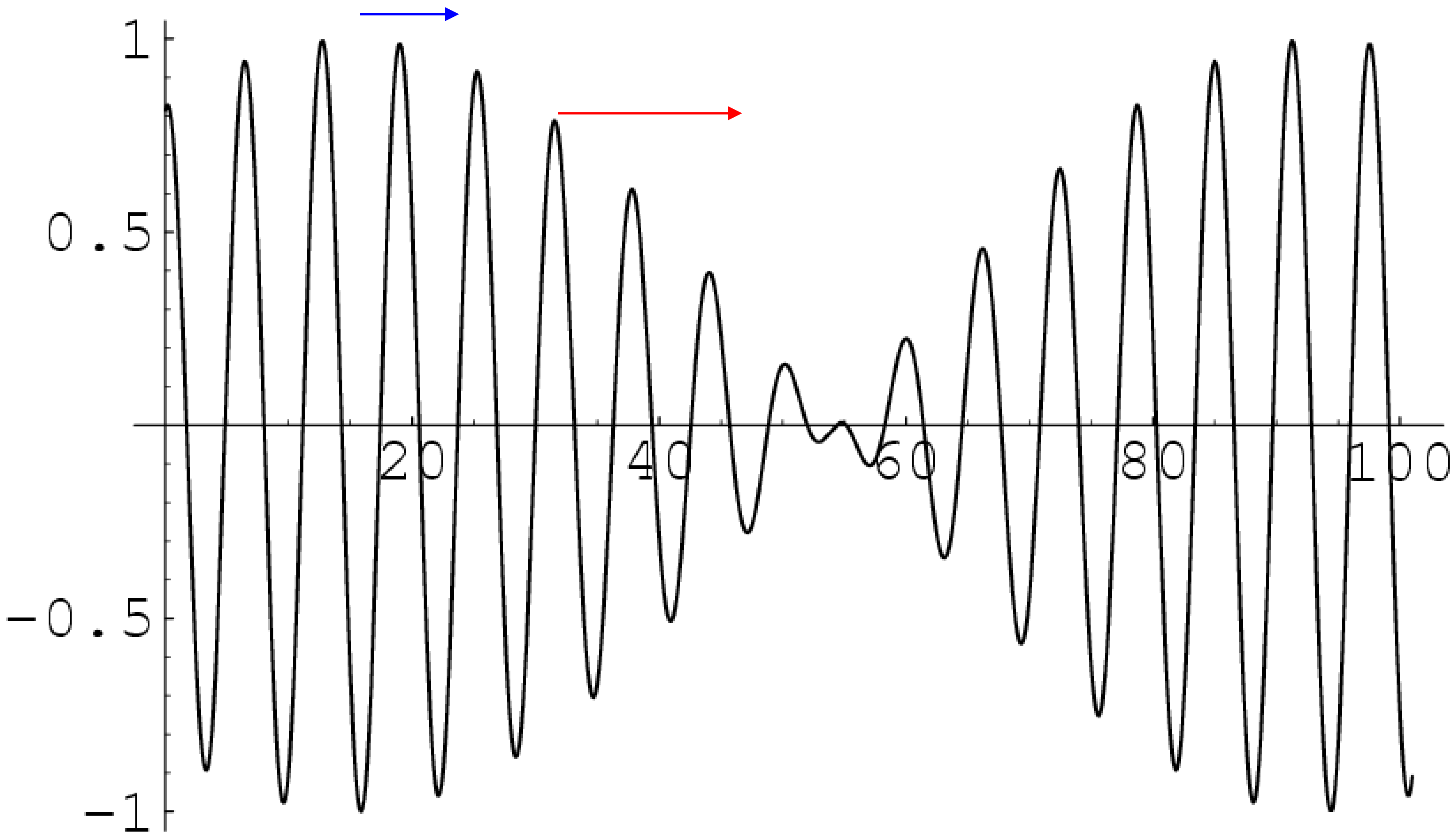
Vp > **V**g



t = 2

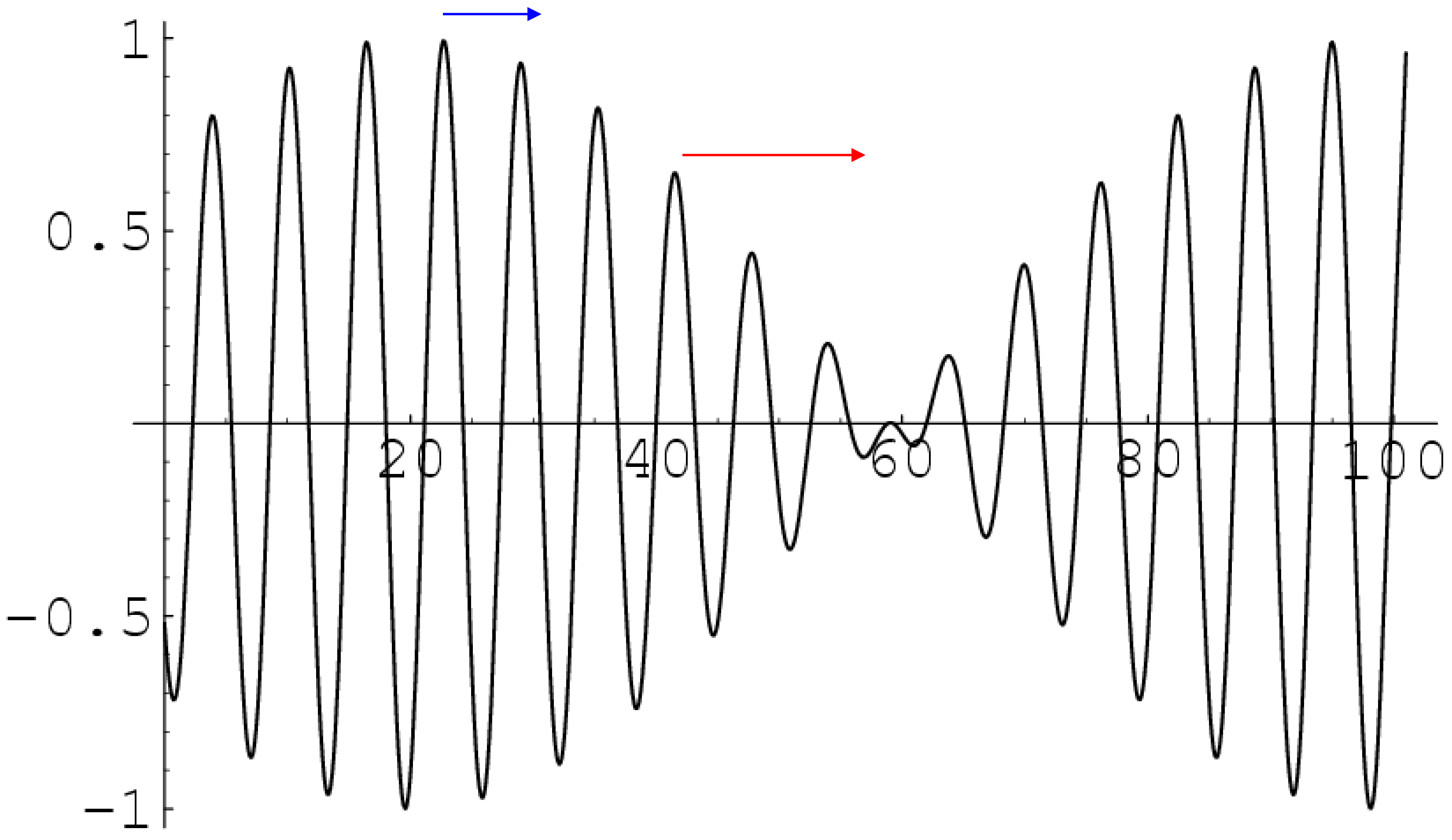
$$\sin(1.00\ x - 10.0\ t)\cos(0.04\ x - 0.2\ t)$$

Vp > **V**g



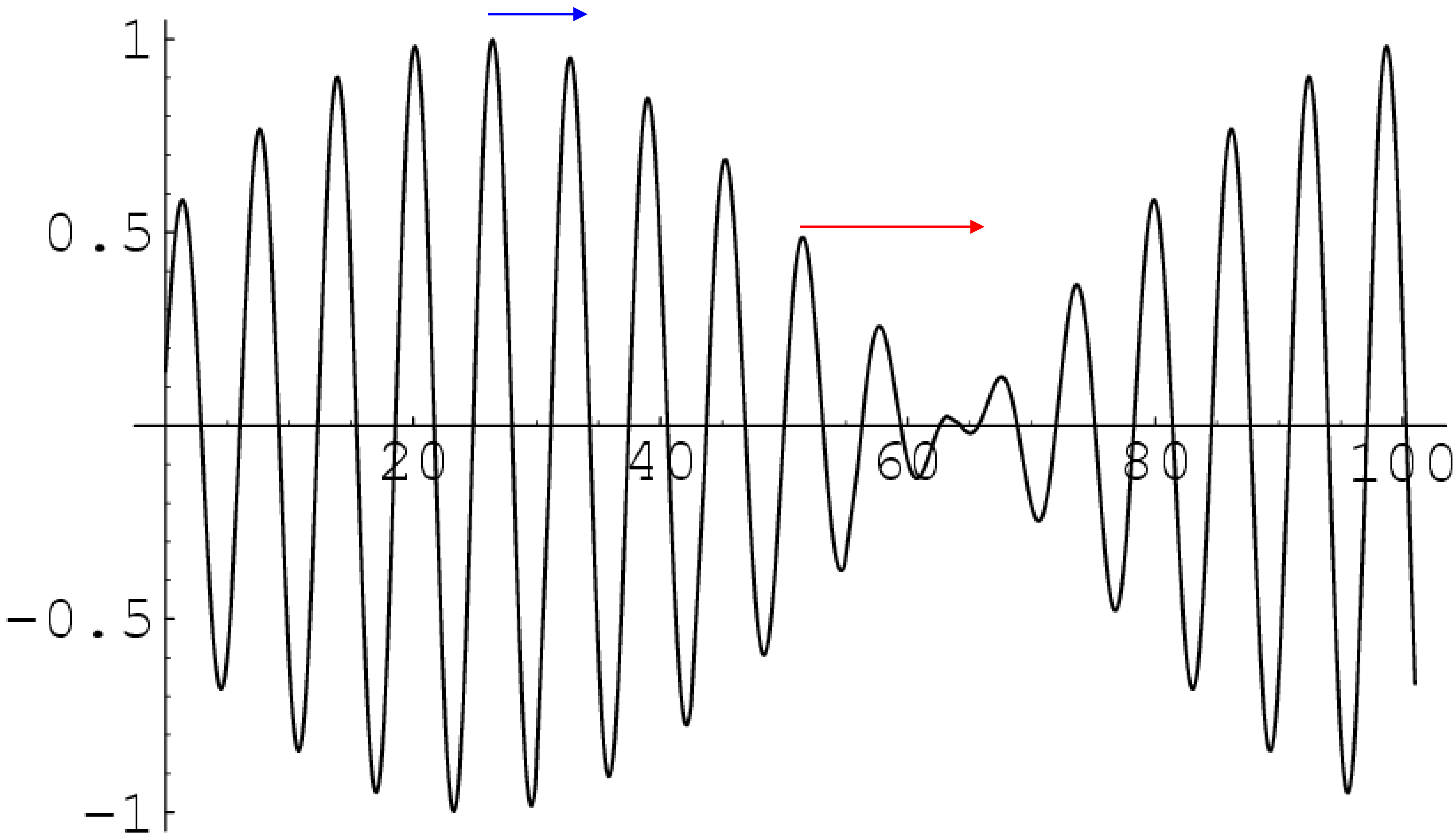
t = 3 $\sin(1.00\ x - 10.0\ t)\cos(0.04\ x - 0.2\ t)$

Vp > **V**g



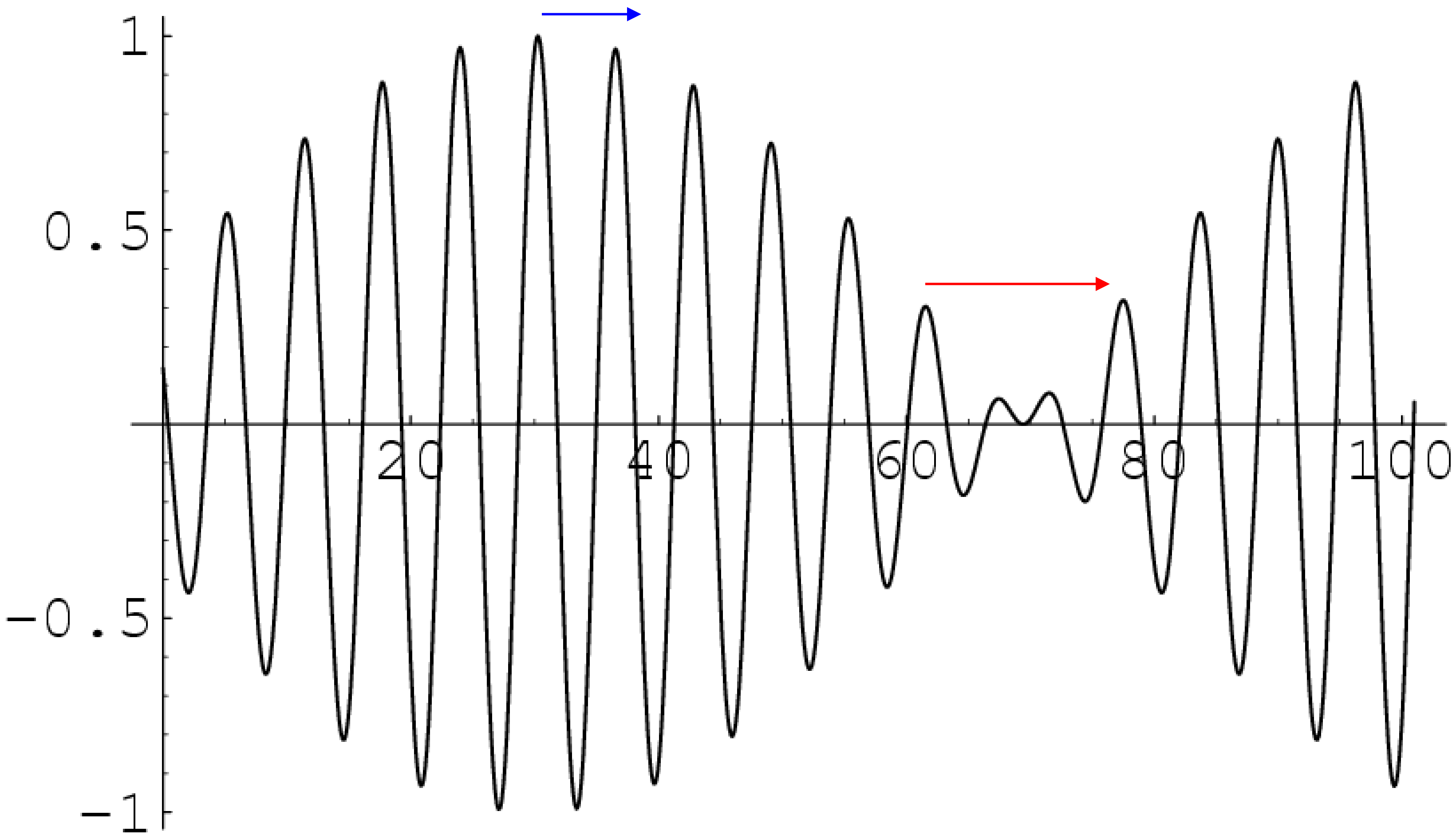
t = 4 $\sin(1.00\ x - 10.0\ t)\cos(0.04\ x - 0.2\ t)$

Vp > **V**g



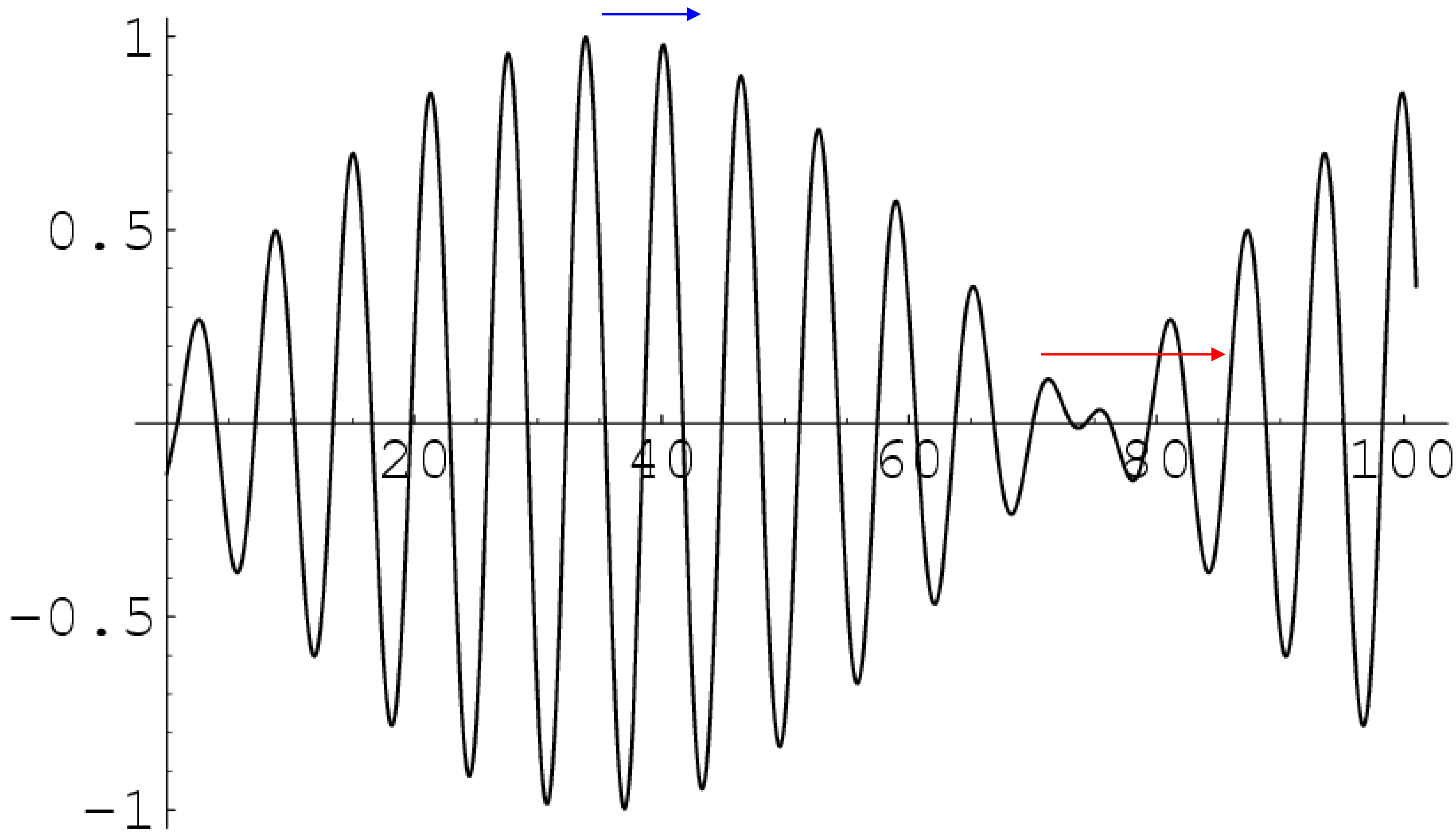
t = 5 $\sin(1.00\ x - 10.0\ t)\cos(0.04\ x - 0.2\ t)$

Vp > **V**g



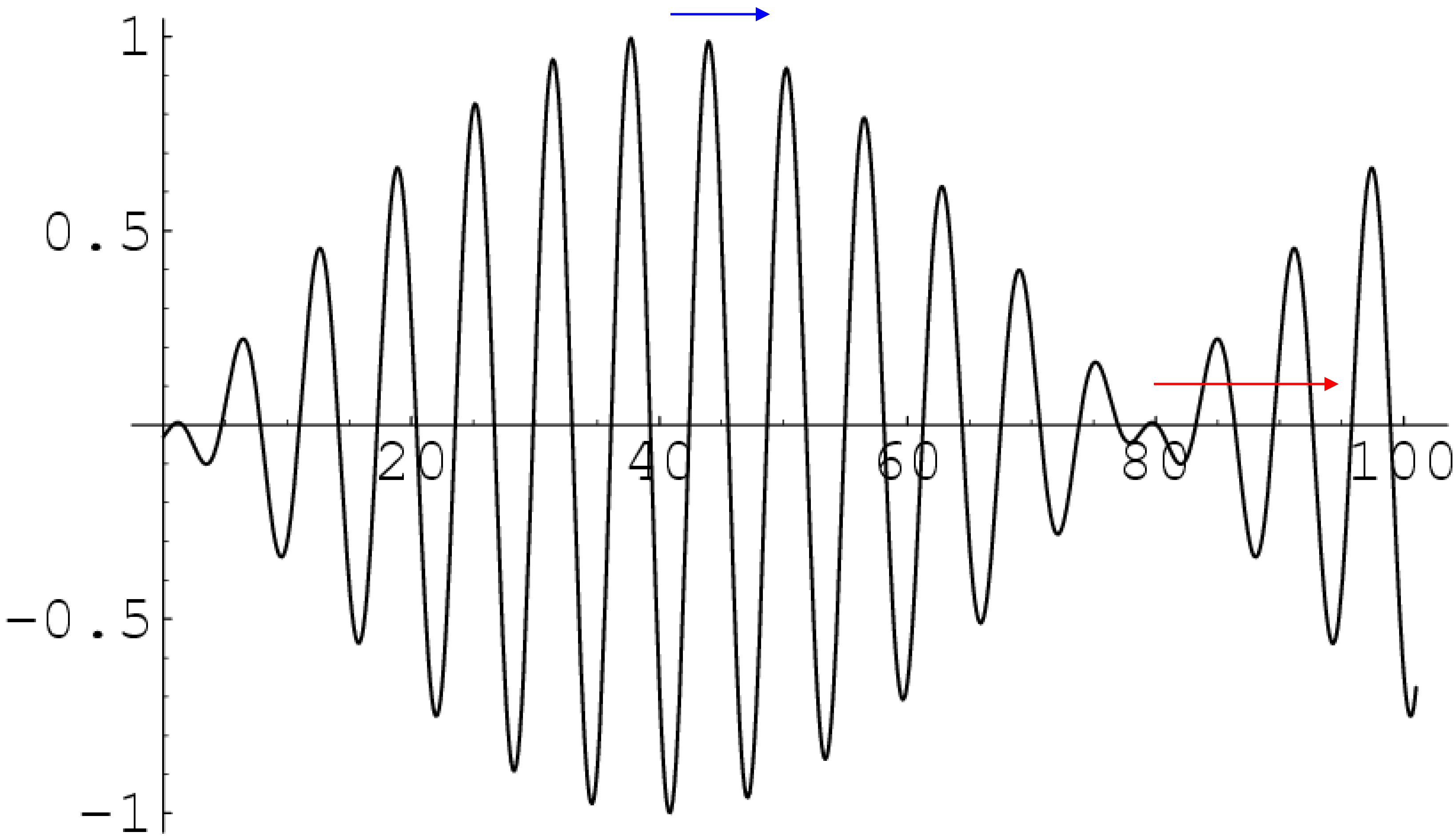
t = 6 $\sin(1.00\ x - 10.0\ t)\cos(0.04\ x - 0.2\ t)$

Vp > **V**g



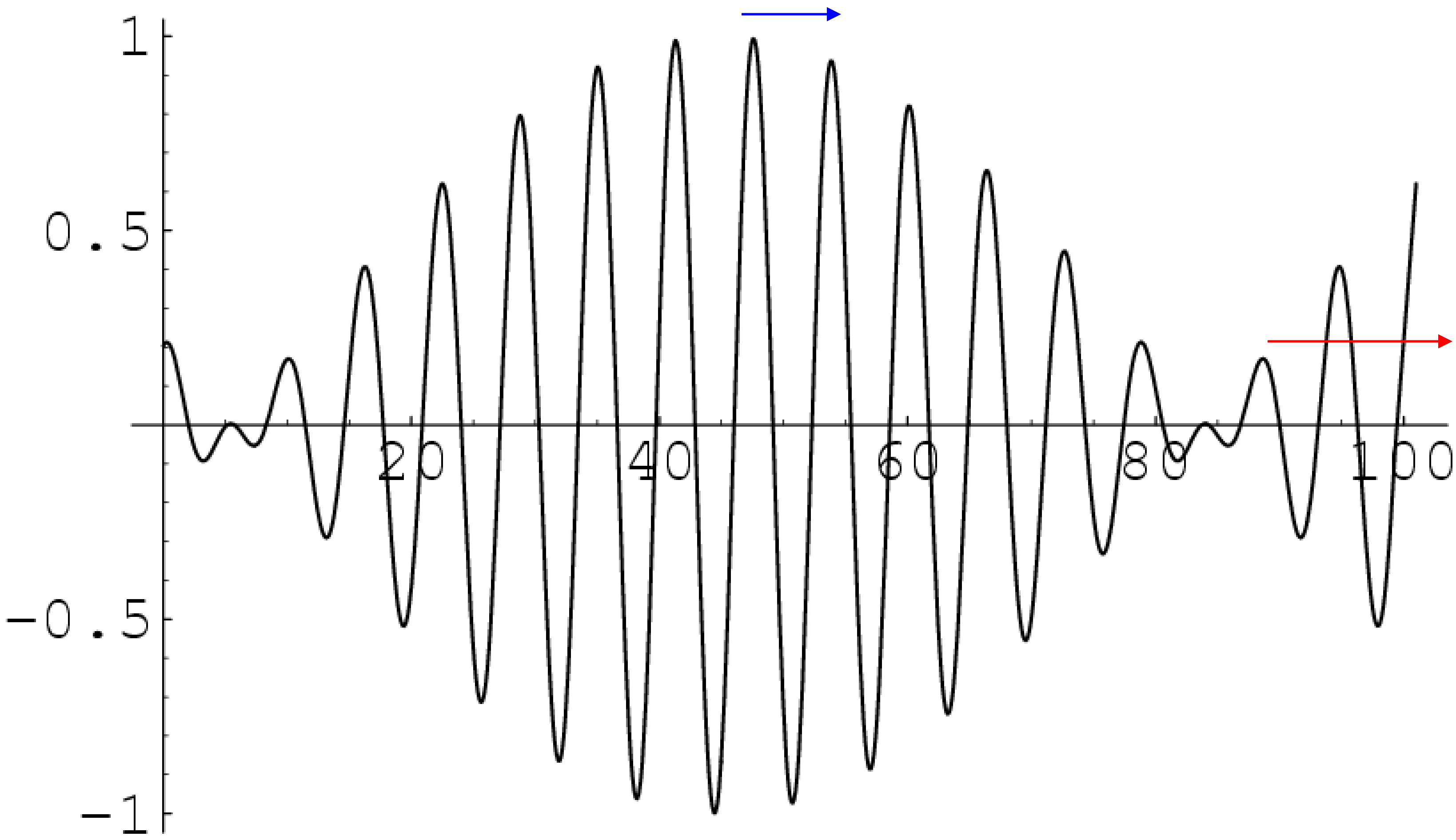
t = 7 $\quad \sin(1.00\ x - 10.0\ t)\cos(0.04\ x - 0.2\ t)$

Vp > **V**g



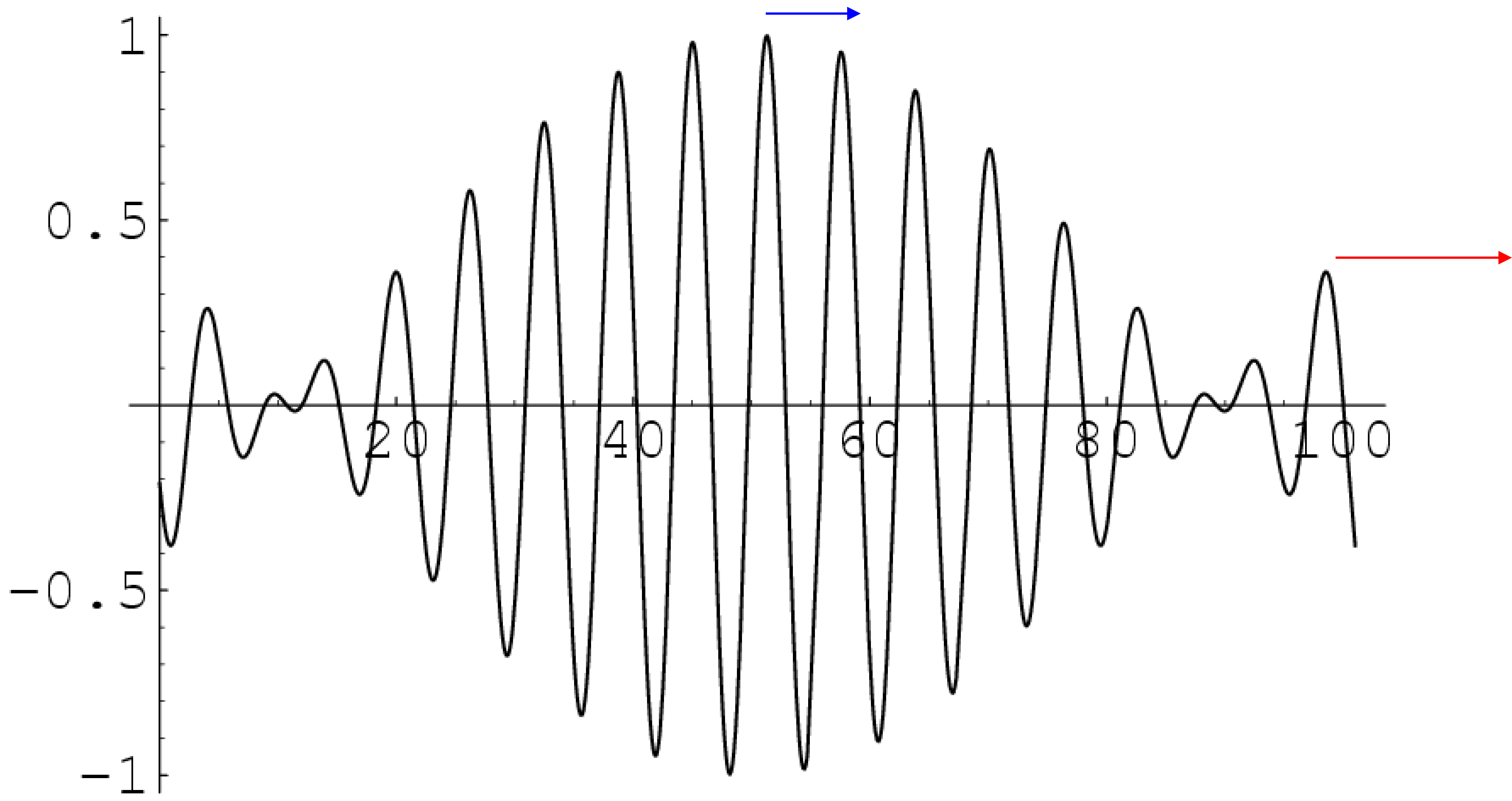
t = 8 $\sin(1.00\ x - 10.0\ t)\cos(0.04\ x - 0.2\ t)$

Vp > **V**g



t = 9 $\sin(1.00\ x - 10.0\ t)\cos(0.04\ x - 0.2\ t)$

Vp > **V**g



t = 10 $\sin(1.00\ x - 10.0\ t)\cos(0.04\ x - 0.2\ t)$

Wave packets

$$0.7\sin[(\omega + 2\Delta\omega)t - (k + 2\Delta k)x]$$

$$+0.49\sin[(\omega + \Delta\omega)t - (k + \Delta k)x]$$

$$+\sin[\omega t - kx]$$

$$+0.49\sin[(\omega - \Delta\omega)t - (k - \Delta k)x]$$

$$+0.7\sin[(\omega - 2\Delta\omega)t - (k - 2\Delta k)x]$$

$t=0$