The visibility based Tapered Gridded Estimator (TGE)

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How to quantify the sky fluctuations?

1.Angular Power Spectrum C_ℓ

2. Power Spectrum $P(\mathbf{k}_\perp, k_\parallel)$

Angular Power Spectrum

Any *brightness temperature fluctuation* on the sky are usually described by an expansion in spherical harmonics.

$$\delta T(\nu, \hat{\mathbf{n}}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}(\nu) Y_{\ell m}(\hat{\mathbf{n}})$$

The *Angular Power Spectrum* defined as :

$$C_l = < a_{lm}(\nu) a_{lm}{}^{*}(\nu) >$$

Power Spectrum

$$\langle \Delta\tilde{T}(\mathbf{k})\,\Delta\tilde{T}^*(\mathbf{k}^{'})\rangle = (2\pi)^3\delta_D^3(\mathbf{k}-\mathbf{k}^{'})P(\mathbf{k}_\perp, k_{||})$$

How to estimate these using observables?

$$\mathcal{V}(\mathbf{U},\nu) = \mathcal{S}(\mathbf{U},\nu) + \mathcal{N}(\mathbf{U},\nu)$$

↑

Entire Sky Signal

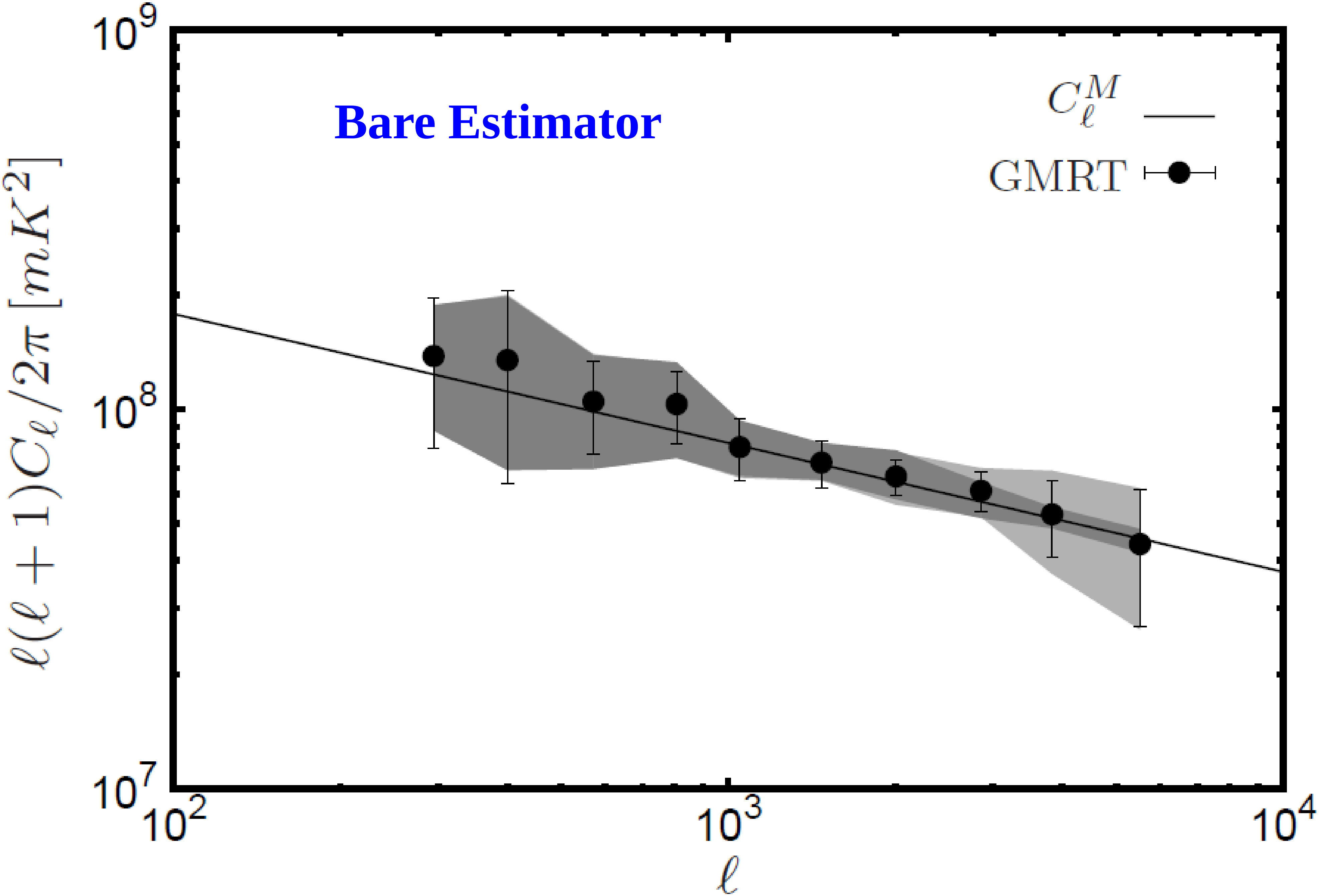
Two Visibility Correlation:

$$V_{2ij} \equiv \langle \mathcal{V}_i \mathcal{V}_j^* \rangle = V_0 \, e^{-|\Delta \mathbf{U}_{ij}|^2/\sigma_0^2} \, C_{\ell_i} + \delta_{ij} 2\sigma_n^2$$

$$S_2(\mathbf{U}, \mathbf{U} + \Delta\mathbf{U}) = \frac{\pi\theta_0^2}{2} \left(\frac{\partial B}{\partial T}\right)^2 \exp\left[-\left(\frac{\Delta U}{\sigma_0}\right)^2\right] C_\ell$$

$$V_0 = \frac{\pi\theta_0^2}{2} \left(\frac{\partial B}{\partial T}\right)^2$$

Noise bias can be avoided by excluding self-correlation term.

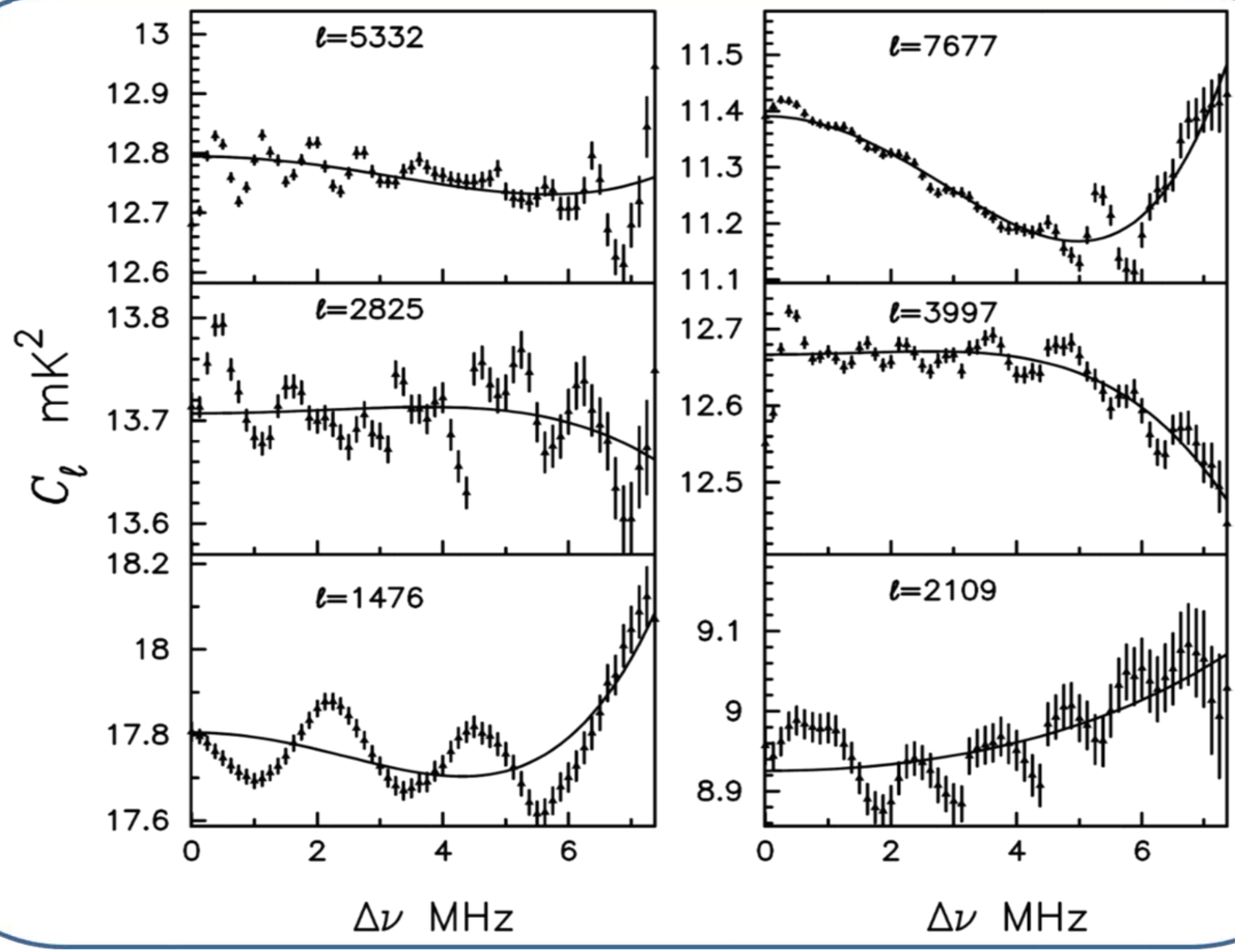


Bare Estimator
C_ℓ^M
GMRT
10^9
10^8
10^7
10^2
10^3
10^4
$\ell(\ell+1)C_\ell/2\pi\,[mK^2]$
ℓ

Disadvantage:

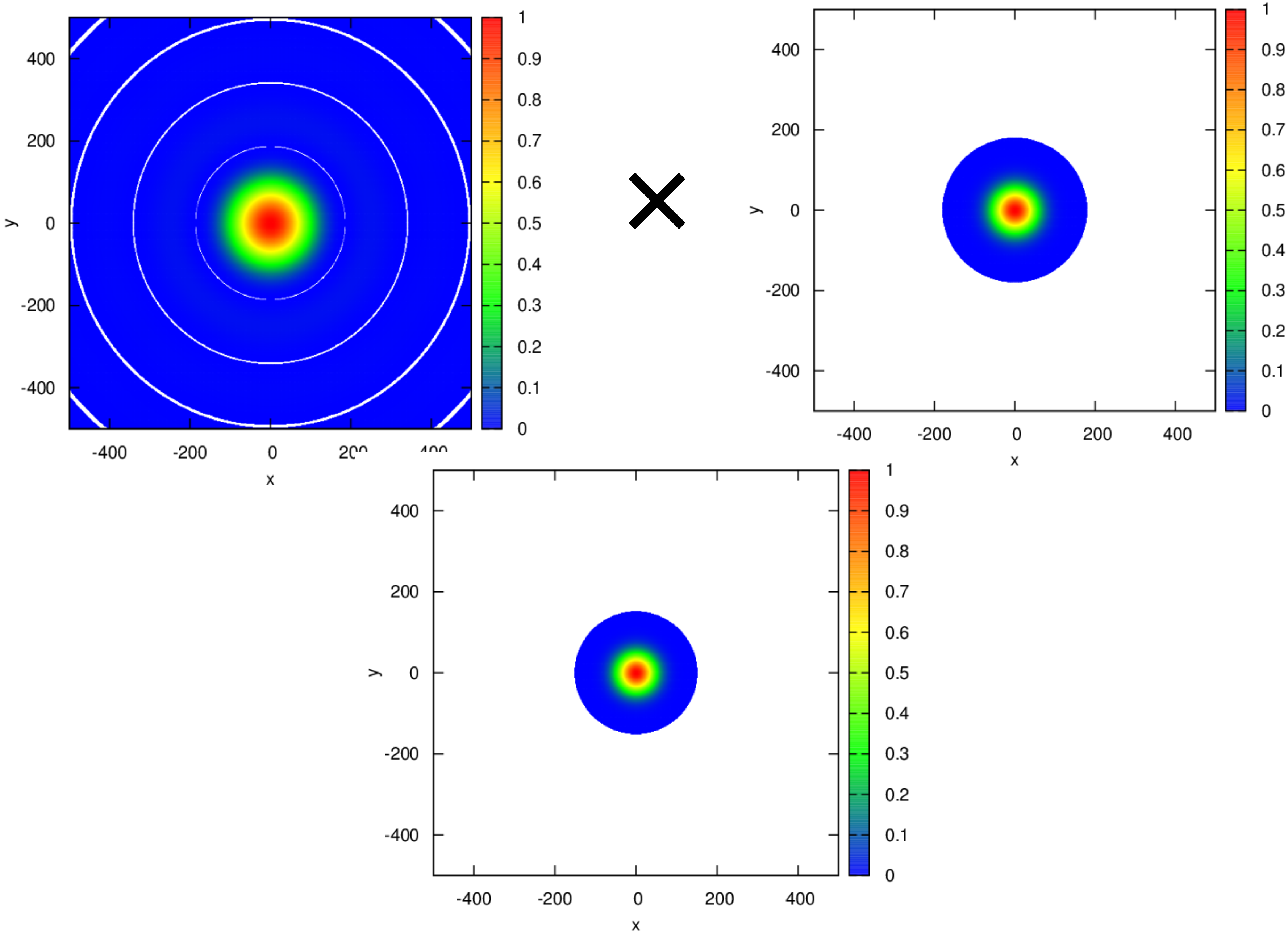
The Bare Estimator deals directly with the visibilities and the computational time for the pairwise correlation scales proportional to N^2, where N is the total number of visibilities in the data.

Any bright sources located near the null or the sidelobes of the PB produce oscillatory pattern along $\Delta\nu$ in the measured $C_l(\Delta\nu)$.



Ghosh *et al.* 2011

Tapered Gridded Estimator

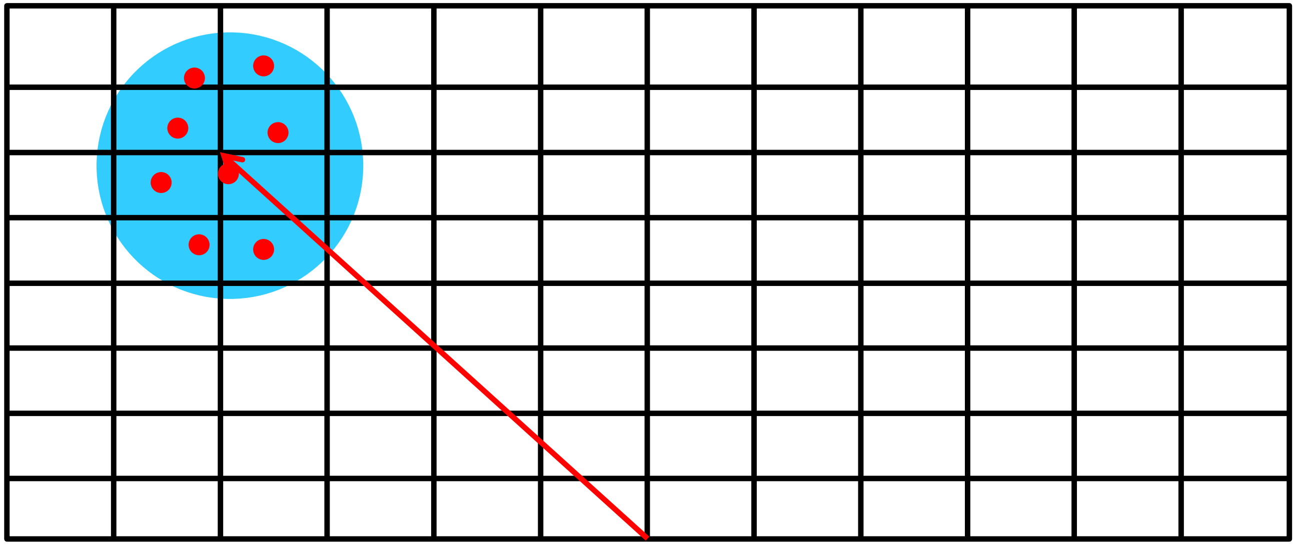


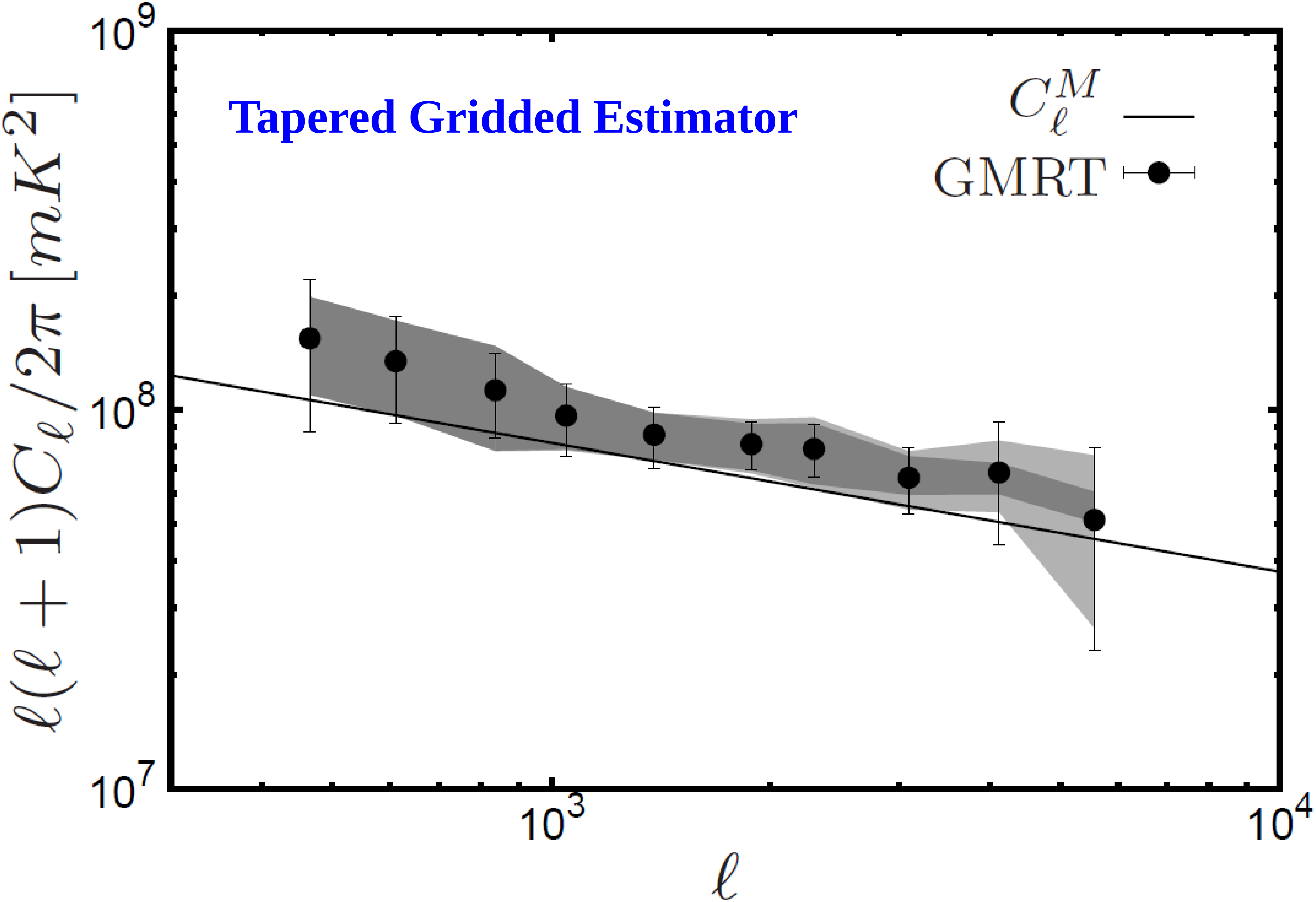
Tapered Gridded Estimator

We define tapered Gridded Estimator as,

$$\hat{E}_g = \frac{(\mathcal{V}_{cg}\mathcal{V}^*_{cg} - \sum_i \mid \tilde{w}(\mathbf{U}_g - \mathbf{U}_i) \mid^2 \mid \mathcal{V}_i \mid^2)}{(\mid K_{1g} \mid^2 V_1 - K_{2gg} V_0)}$$

$$\mathcal{V}_c(\mathbf{U}) = \tilde{w}(\mathbf{U}) \otimes \mathcal{V}(\mathbf{U})$$

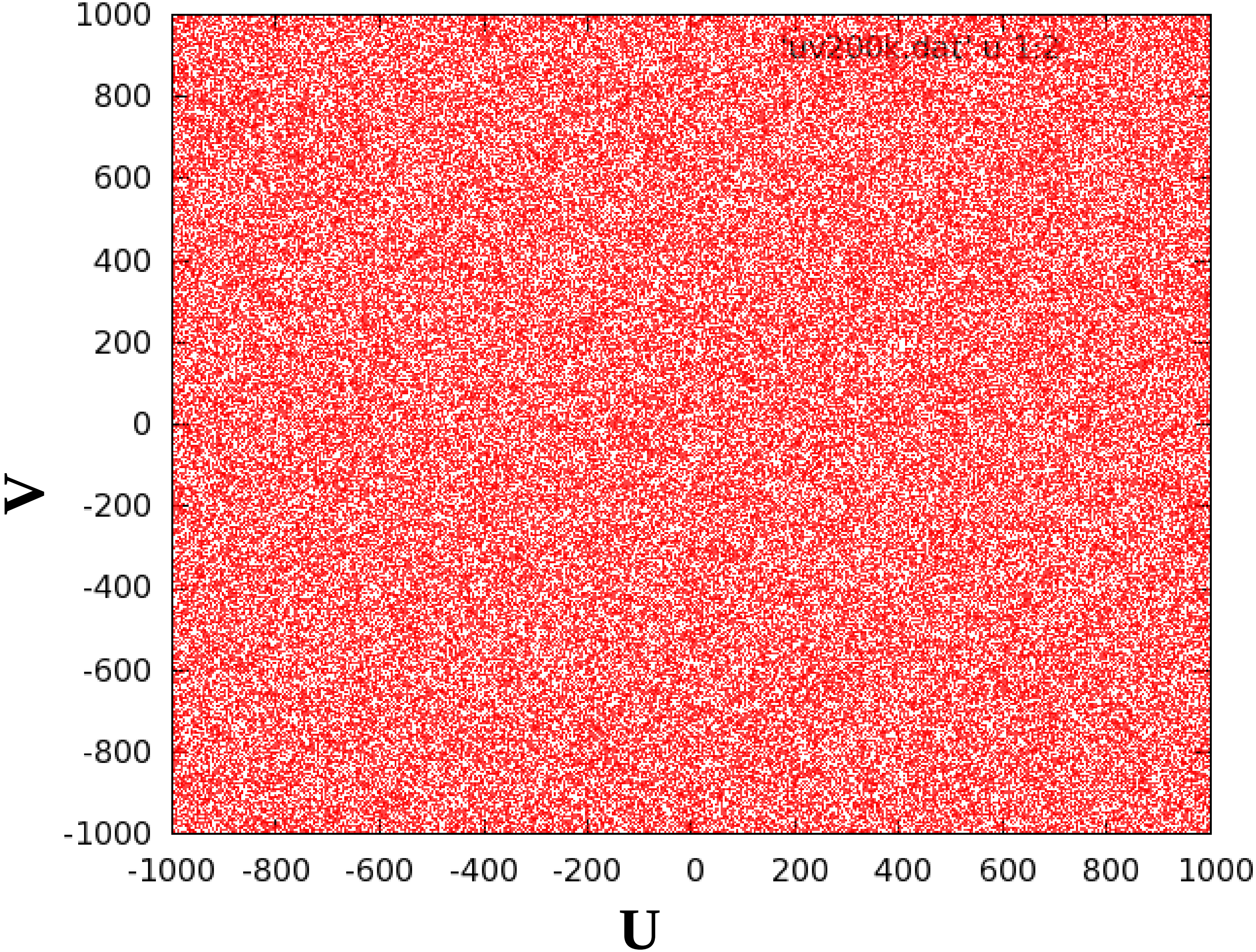


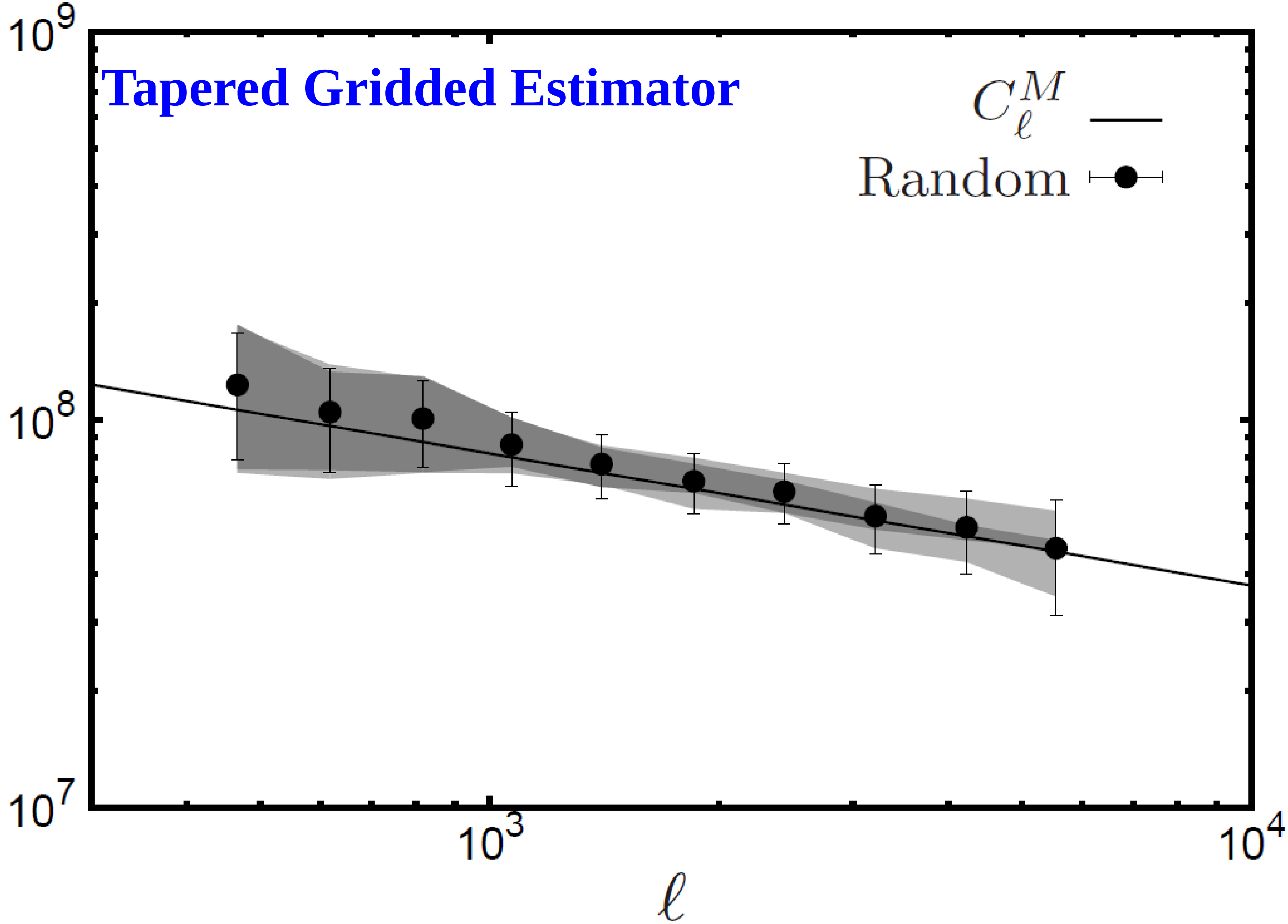


Tapered Gridded Estimator
C_ℓ^M
GMRT
$\ell(\ell+1)C_\ell/2\pi\ [mK^2]$
10^9
10^8
10^7
10^3
10^4
ℓ

Why Overestimate?

Random UV Distribution

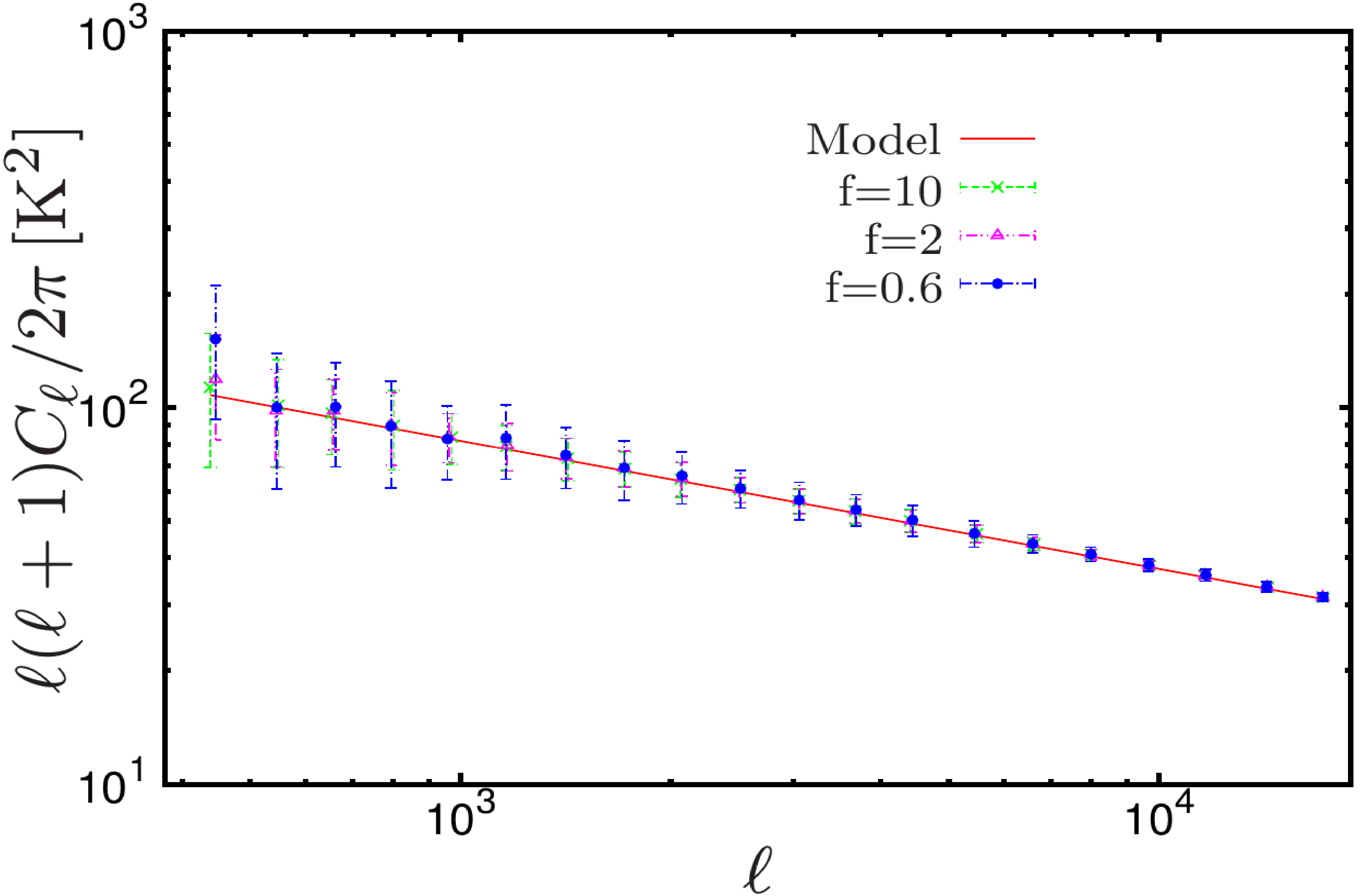


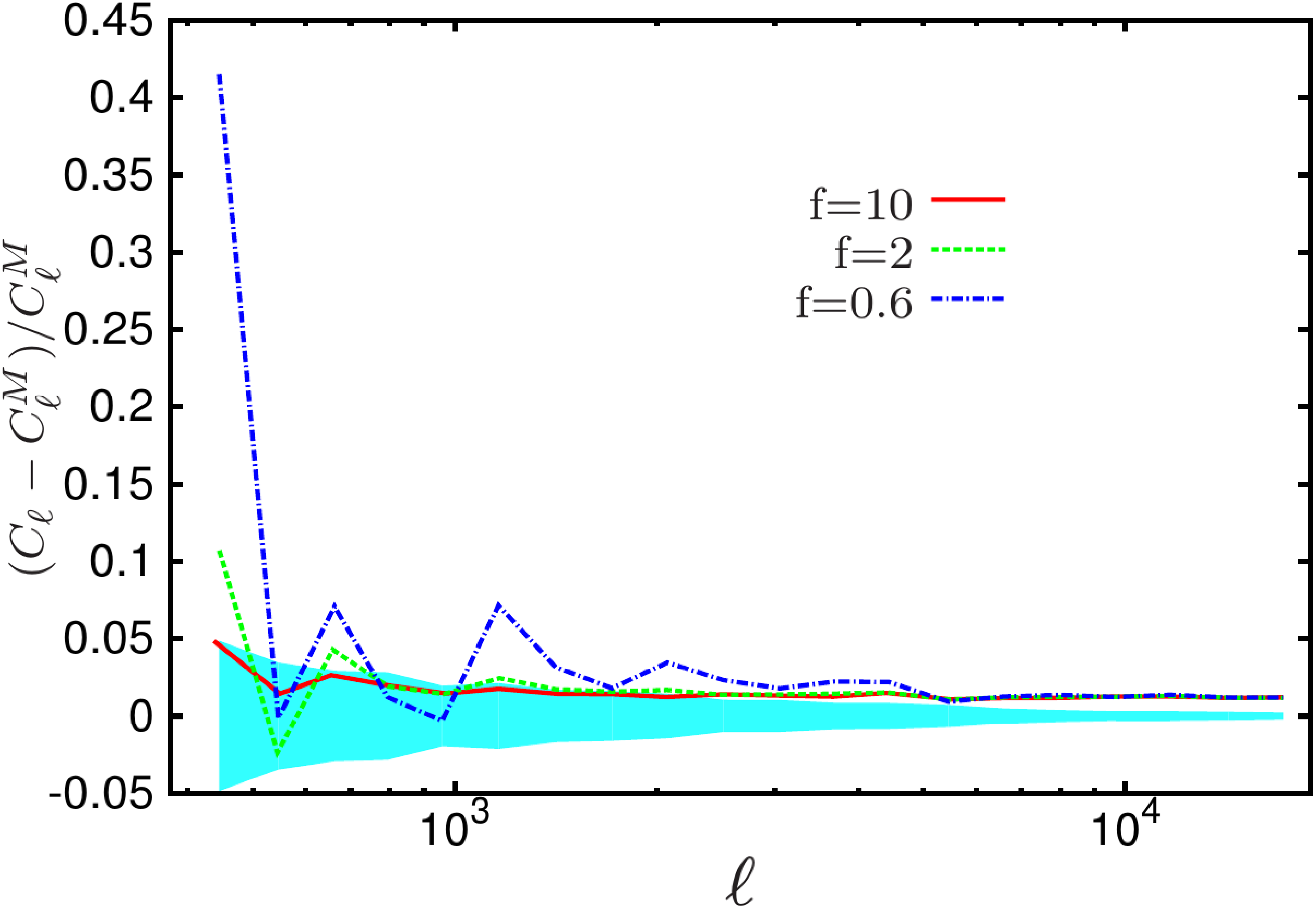

Tapered Gridded Estimator
C_ℓ^M
Random
10^9
10^8
10^7
10^3
10^4
ℓ

An Improved TGE

$$\left\langle \left(\mid \mathcal{V}_{cg} \mid^2 - \sum_i \mid \tilde{w}(\mathbf{U}_g - \mathbf{U}_i) \mid^2 \mid \mathcal{V}_i \mid^2 \right) \right\rangle = M_g C_{2\pi U_g}$$

$$M_g = \left\langle \left(\mid \mathcal{V}_{cg} \mid^2 - \sum_i \mid \tilde{w}(\mathbf{U}_g - \mathbf{U}_i) \mid^2 \langle \mid \mathcal{V}_i \mid^2 \right) \right\rangle_{\mathrm{UPAS}}$$

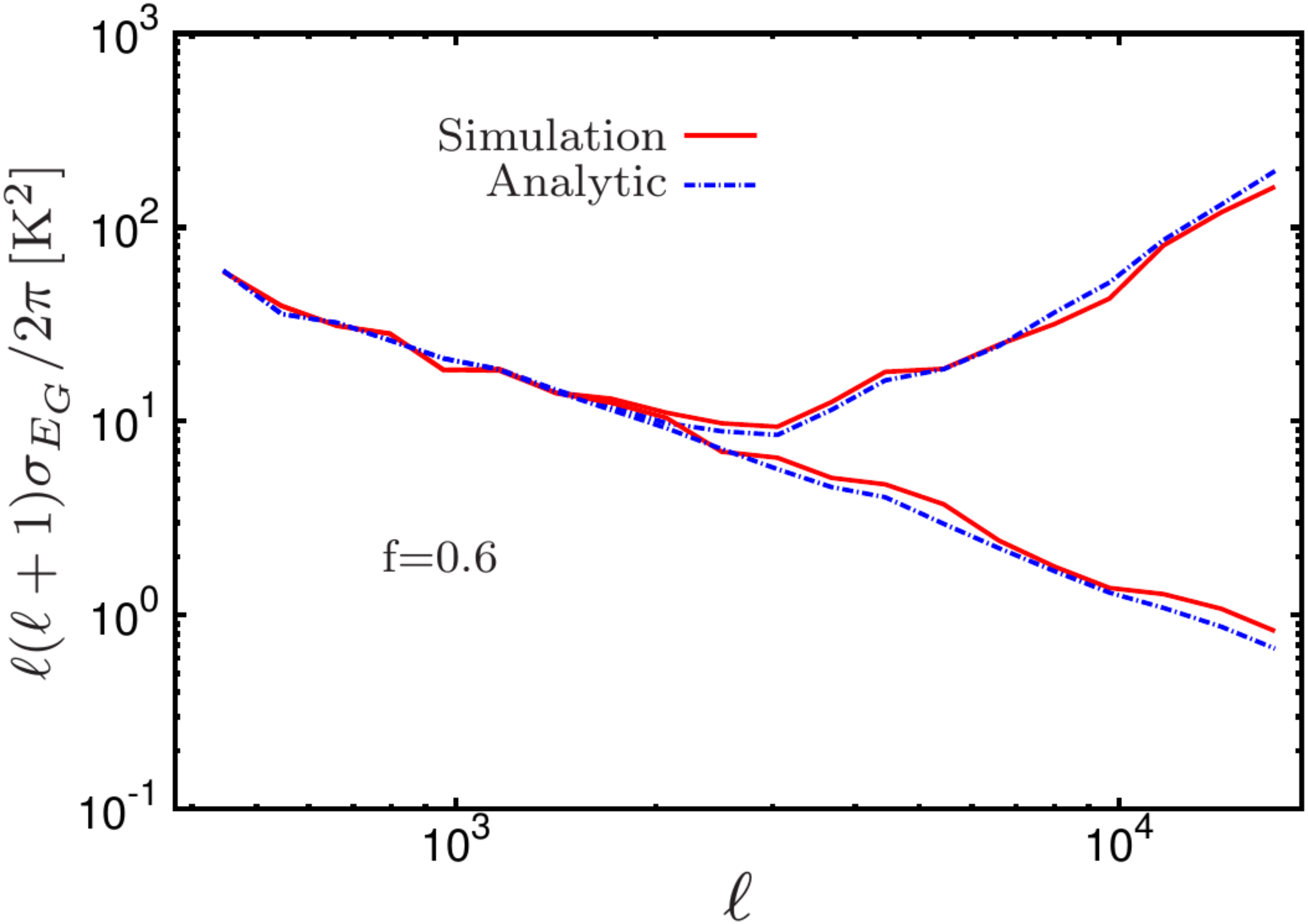

$\ell(\ell+1)C_\ell/2\pi\,[\mathrm{K}^2]$
10^3
10^2
10^1
10^3
10^4
ℓ
Model
f=10
f=2
f=0.6



0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
$(C_\ell - C_\ell^M)/C_\ell^M$
10^3
10^4
ℓ
f=10
f=2
f=0.6

Variance

$$\sigma^2_{E_G}(a) = \frac{\sum_{gg'} w_g w_{g'} M_g^{-1} M_{g'}^{-1} \mid \langle \mathcal{V}_{cg} \mathcal{V}^*_{cg'} \rangle \mid^2}{[\sum_g w_g]^2}$$



3Dimensions

$$P(k_\perp, k_\parallel)$$

Visibility Correlation

$$v_i(\tau_m) = (\Delta\nu_c) \sum_a e^{2\pi i \tau_m \nu_a} \mathcal{V}_i(\nu_a)$$ for fixed τ_m

$$\langle | \, v_i \, |^2 \rangle = V_0 \left[\frac{\mathrm{B}}{r^2 r'} P(k_\perp, k_\parallel) \right] + \sum_a \langle | \, \mathcal{N}_i(\nu_a) \, |^2 \rangle$$

$$\langle | \, \mathcal{V}_i \, |^2 \rangle = V_0 C_{\,2\pi U} + \langle | \, \mathcal{N}_i \, |^2 \rangle$$

Main Point

$v_i\ (\tau_m)$ for fixed τ_m

is equivalent to

$V_i\ (\upsilon_a)$ for fixed υ_a

Tapered Gridded Estimator

$$\mathcal{V}_{cg} = \sum_i \tilde{w}(\mathbf{U}_g - \mathbf{U}_i)\,\mathcal{V}_i$$

$$\hat{E}_g = M_g^{-1} \left(\mid \mathcal{V}_{cg} \mid^2 - \sum_i \mid \tilde{w}(\mathbf{U}_g - \mathbf{U}_i) \mid^2 \mid \mathcal{V}_i \mid^2 \right)$$

$$v_{cg} = \sum_i \tilde{w}(\mathbf{U}_g - \mathbf{U}_i)\,v_i$$

For a fixed τ_m

$$\hat{P}_g = \left(\frac{M_g \mathrm{B}}{r^2 r'} \right)^{-1} \left(\mid v_{cg} \mid^2 - \sum_i \mid \tilde{w}(\mathbf{U}_g - \mathbf{U}_i) \mid^2 \langle \mid v_i \mid^2 \right)$$

TG Estimator

$$\langle \hat{P}_g(\tau_m) \rangle = P(k_{\perp g}, k_{\parallel m})$$

$$\tau_m \text{ and } U_g \longrightarrow k_{\parallel m} \text{ and } k_{\perp g}$$

$$k_{\parallel} = \frac{2\pi\tau_m}{r'} \qquad \mathbf{k}_{\perp} = \frac{2\pi\mathbf{U}}{r}$$

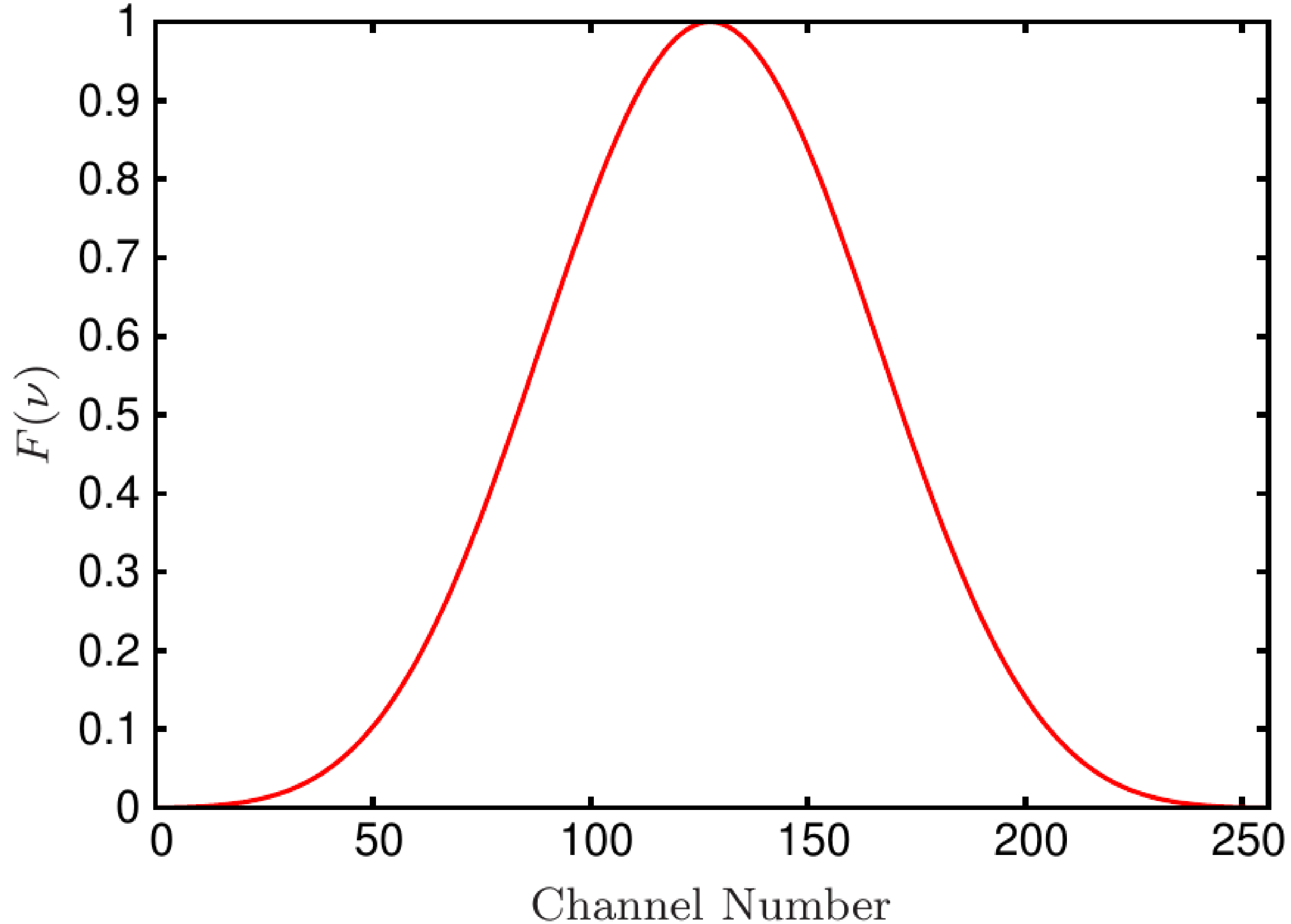
TG Estimator

$$\langle \hat{P}_g(\tau_m) \rangle = P(k_{\perp g}, k_{\parallel m})$$

$$\tau_m \text{ and } U_g \longrightarrow k_{\parallel m} \text{ and } k_{\perp g}$$

$$k_{\parallel} = \frac{2\pi\tau_m}{r'} \qquad \mathbf{k}_{\perp} = \frac{2\pi\mathbf{U}}{r}$$

$$v_i^f(\tau_m) = (\Delta\nu_c) \sum e^{2\pi i \tau_m \nu_a} F(\nu_a)\mathcal{V}_i(\nu_a)$$



$$v_i^f(\tau_m) = \frac{1}{\mathrm{B_{bw}}} \sum_n \tilde{f}(\tau_m - \tau_n) v_i(\tau_n)$$

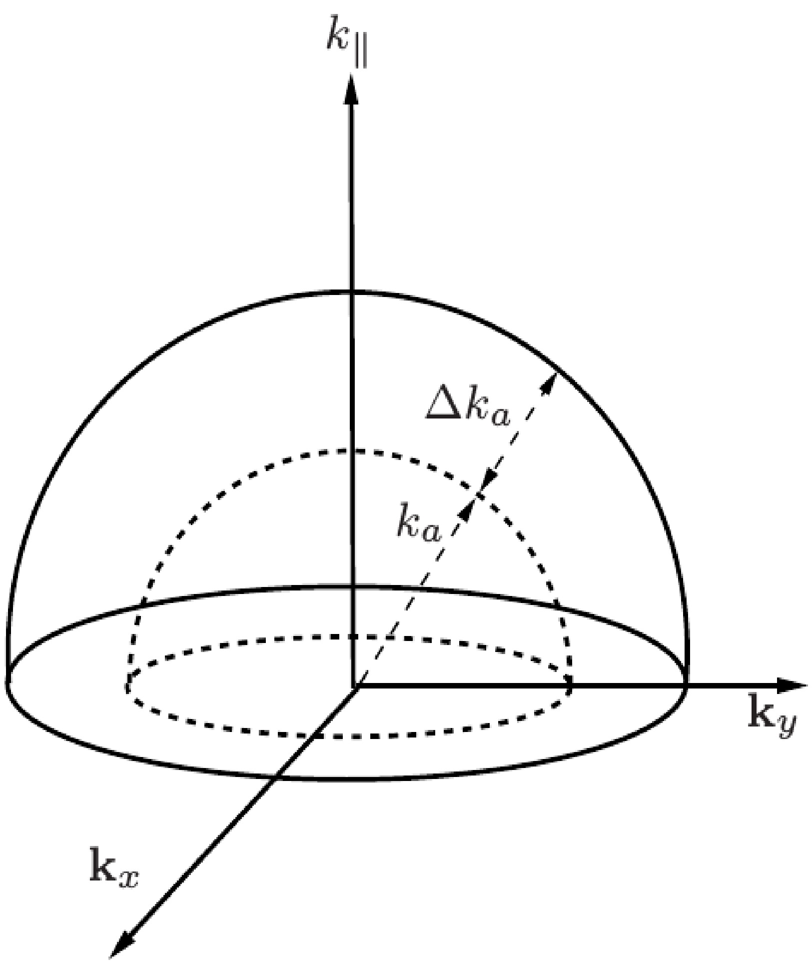
$$\langle| v_i^f(\tau_m) |^2\rangle = \frac{1}{\mathrm{B_{bw}}^2} \sum_a | \tilde{f}(\tau_m - \tau_a) |^2 \langle| v_i(\tau_a) |^2\rangle .$$

$$\langle| v_i^f(\tau_m) |^2\rangle = A_f(0) \langle| v_i(\tau_m) |^2\rangle$$

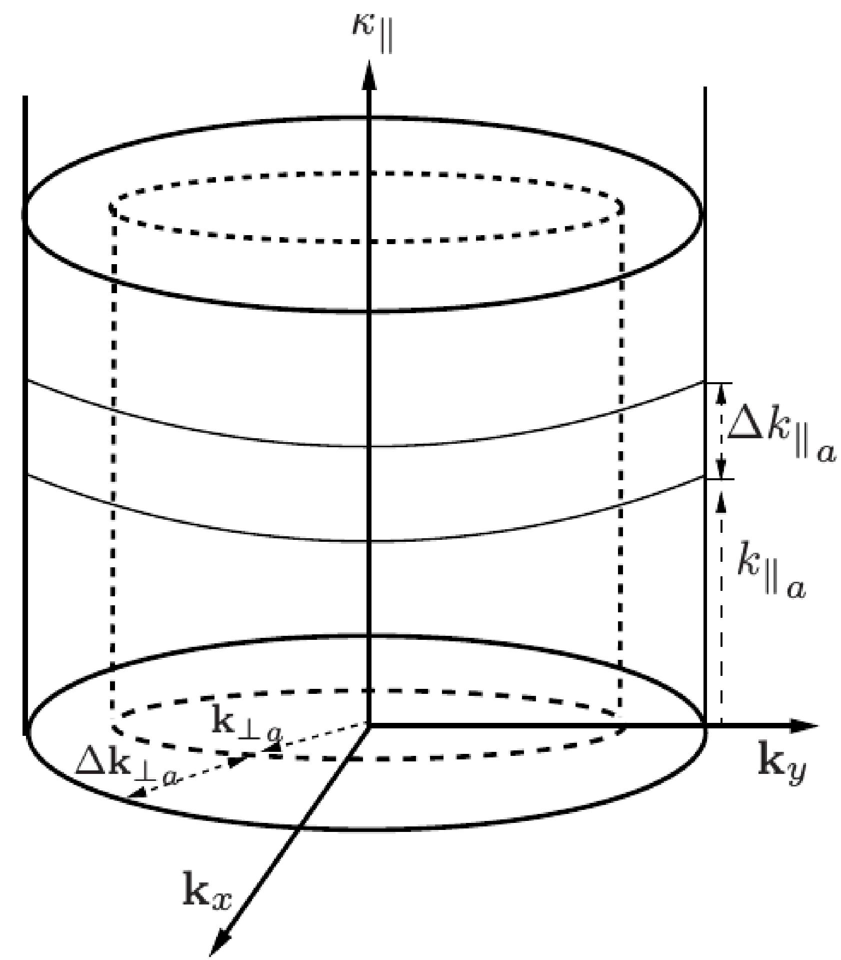
$$A_f(0) = \frac{1}{\mathrm{B_{bw}^2}} \sum_n | \tilde{f}(\tau_n) |^2$$

$$\hat{P}_g = \left(\frac{M_g \mathrm{B}}{r^2 r'}\right)^{-1} \left(\mid v_{cg} \mid^2 - \sum_i \mid \tilde{w}(\mathbf{U}_g - \mathbf{U}_i) \mid^2 \langle\mid v_i \mid^2\right)$$

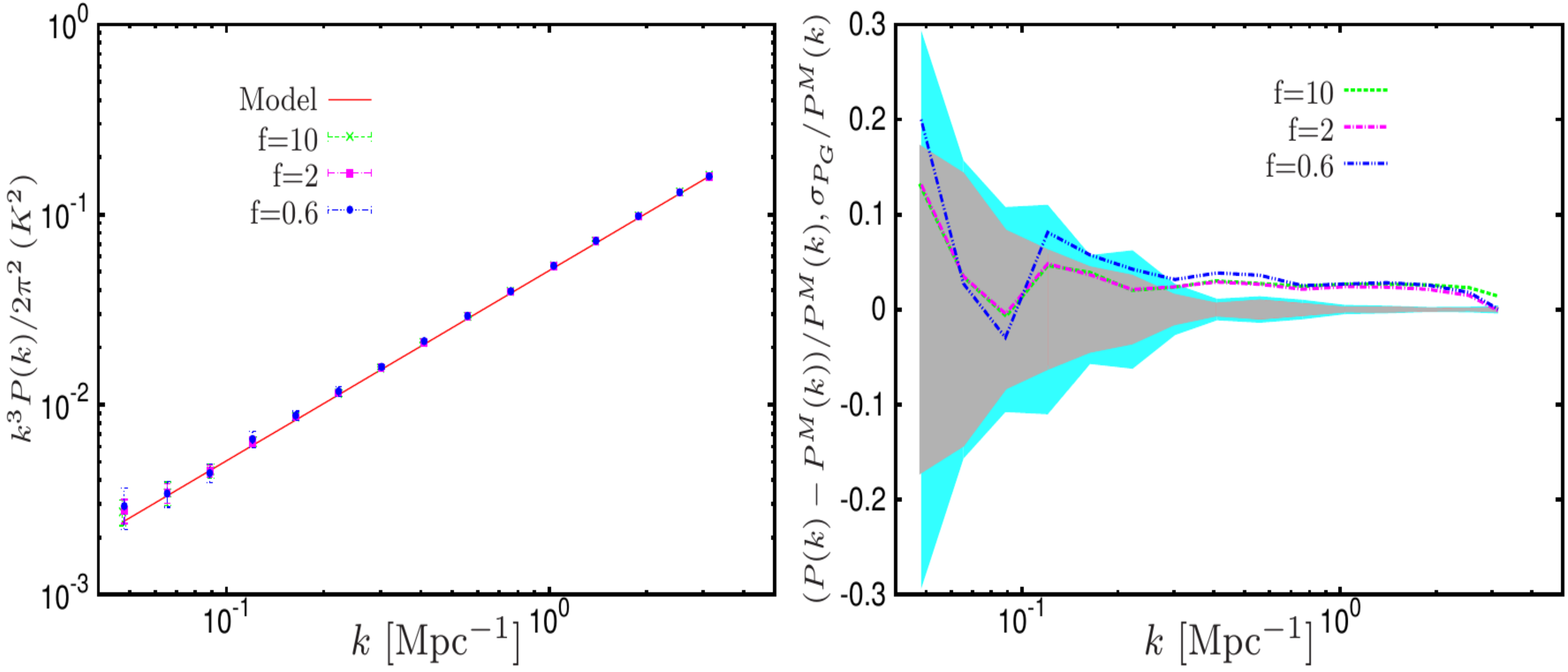
$$\hat{P}_g(\tau_m) = \left(\frac{M_g \mathrm{B}_{\mathrm{bw}}\, A_f(0)}{r^2 r'}\right)^{-1} \left(\mid v_{cg}^f(\tau_m) \mid^2 - \sum_i \mid \tilde{w}(\mathbf{U}_g - \mathbf{U}_i) \mid^2 \mid v_i^f(\tau_m) \mid^2\right)$$


$k_{\|}$
Δk_a
k_a
$\mathbf{k}_y$
$\mathbf{k}_x$

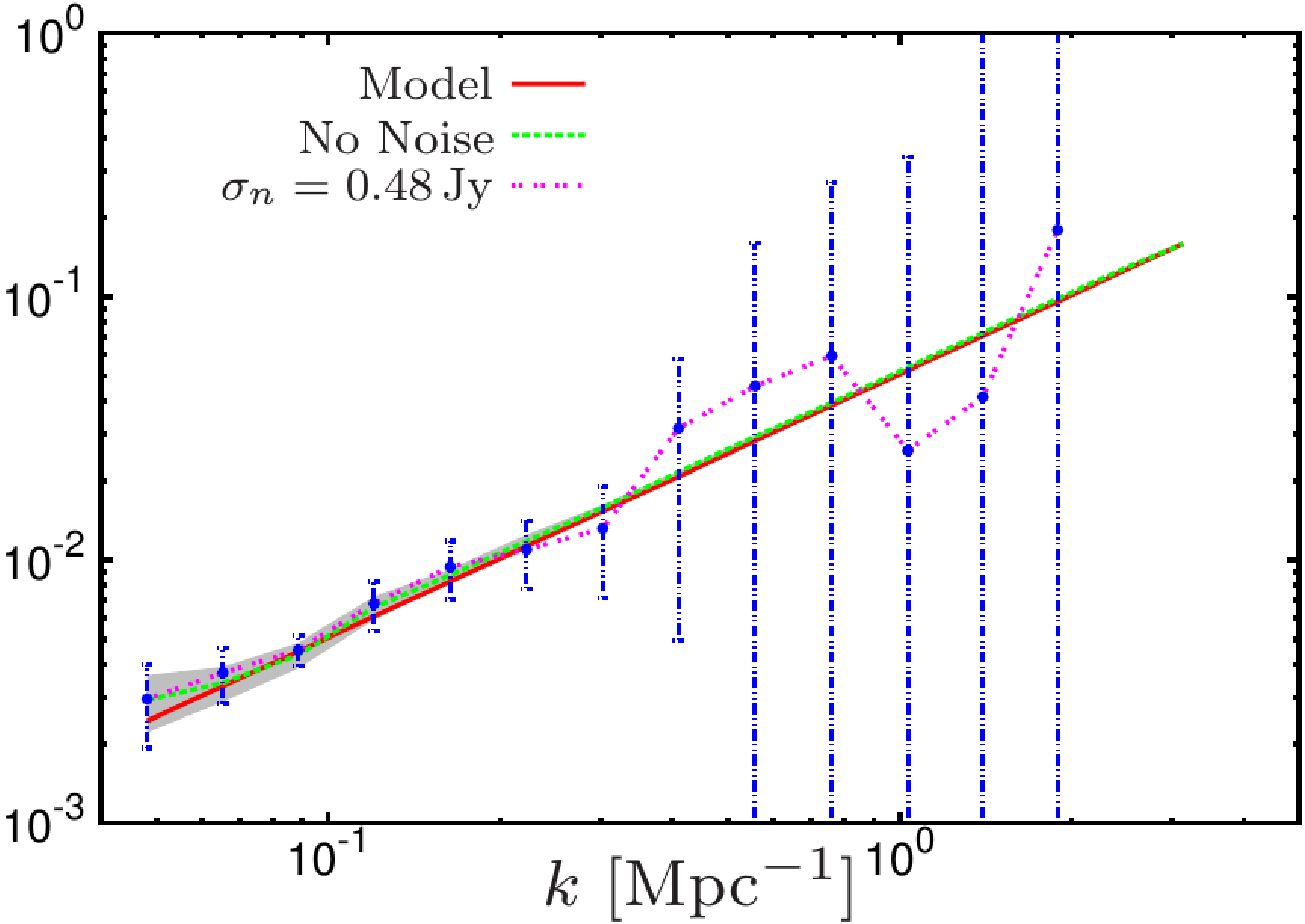
(a)


$\kappa_{\|}$
$\Delta k_{\|a}$
$k_{\|a}$
$\mathbf{k}_{\perp a}$
$\Delta \mathbf{k}_{\perp a}$
$\mathbf{k}_y$
$\mathbf{k}_x$

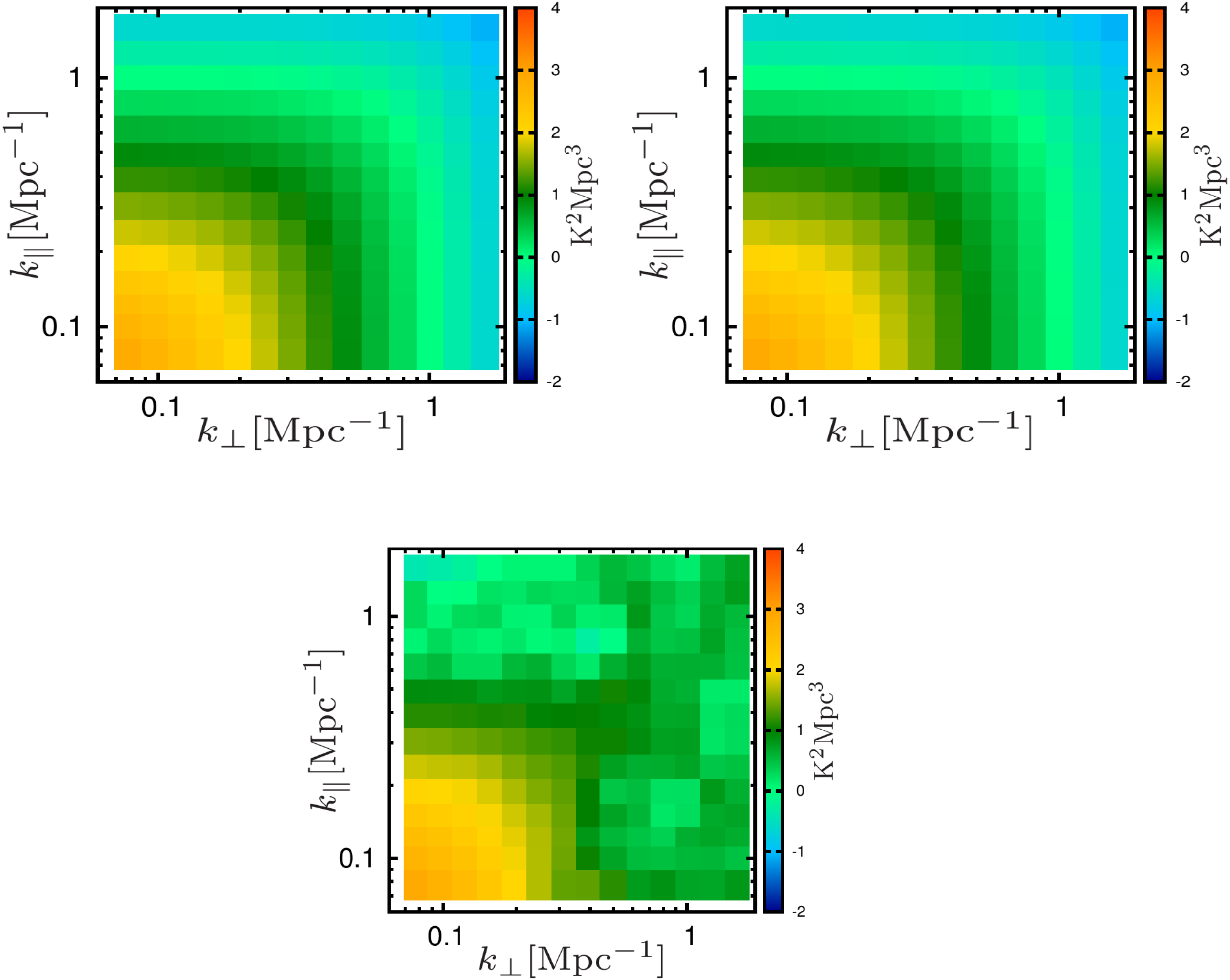
(b)



Model
f=10
f=2
f=0.6
$k^3 P(k)/2\pi^2$ (K^2)
k [Mpc^{-1}]
10^0
10^{-1}
10^{-2}
10^{-3}
f=10
f=2
f=0.6
$(P(k) - P^M(k))/P^M(k), \sigma_{P_G}/P^M(k)$
k [Mpc^{-1}]
0.3
0.2
0.1
0
-0.1
-0.2
-0.3



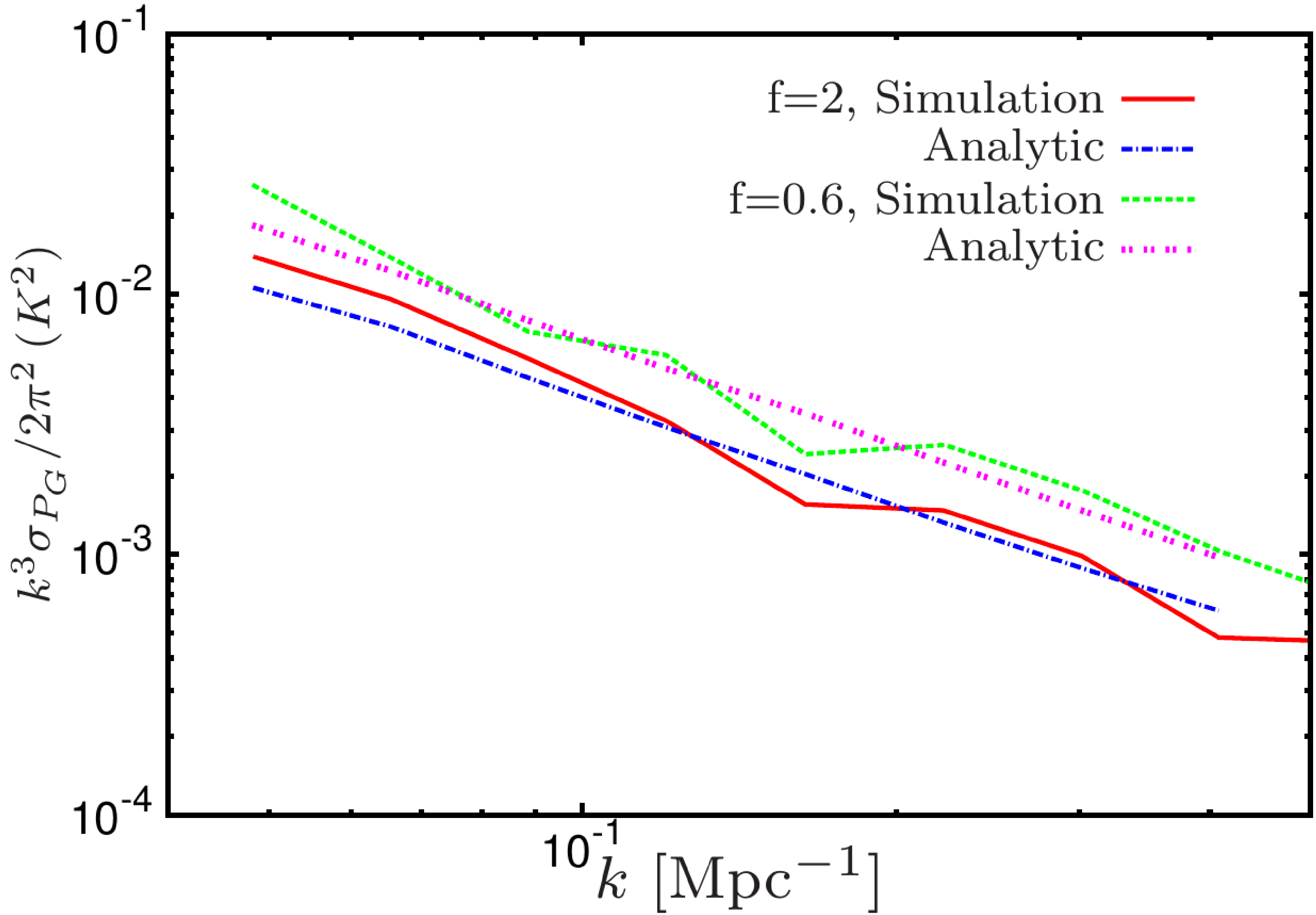
Model
No Noise
$\sigma_n = 0.48\,\mathrm{Jy}$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-1}
10^0
$k\ [\mathrm{Mpc}^{-1}]$



$k_{||}\,[\mathrm{Mpc}^{-1}]$
$k_{\perp}\,[\mathrm{Mpc}^{-1}]$
$\mathrm{K}^2\mathrm{Mpc}^3$
$k_{||}\,[\mathrm{Mpc}^{-1}]$
$k_{\perp}\,[\mathrm{Mpc}^{-1}]$
$\mathrm{K}^2\mathrm{Mpc}^3$
$k_{||}\,[\mathrm{Mpc}^{-1}]$
$k_{\perp}\,[\mathrm{Mpc}^{-1}]$
$\mathrm{K}^2\mathrm{Mpc}^3$

Thank You

$$\sigma_{P_G}^2 = \left(\frac{\mathrm{B_{bw}}\, A_f(0)}{r^2 r'}\right)^{-2} \frac{\sum_{gm,g'm'} \; w_{gm} w_{g'm'} M_g^{-1} M_{g'}^{-1} \mid \langle v_{cg}^f(\tau_m) v_{cg'}^{f*}(\tau_{m'}) \rangle \mid^2}{[\sum_{gm} w_{gm}]^2}$$



$$\sigma^2_{P_G} = \left(\frac{\mathrm{B_{bw}}\, A_f(0)}{r^2 r'}\right)^{-2} \frac{\sum_{gm,g'm'} \; w_{gm} w_{g'm'} M_g^{-1} M_{g'}^{-1} \mid \langle v^f_{cg}(\tau_m) v^{f*}_{cg'}(\tau_{m'}) \rangle \mid^2}{[\sum_{gm} w_{gm}]^2}.$$