

9. Diffraction Grating

Background

Fraunhofer diffraction
Fresnel diffraction
Angular dispersion
Resolving power
Spectral lines

Aim of the experiment

To determine the wavelengths of the prominent lines of mercury by a plane transmission diffraction grating, hence to find (a) the chromatic resolving power of the plane transmission diffraction grating and (b) the dispersive power of the grating.

Apparatus required

Spectrometer
Plane transmission diffraction grating
Mercury-lamp
Spirit level

Theory

If a parallel beam of monochromatic light is incident normally on the face of a plane transmission diffraction grating, bright diffraction maxima are observed on the other side of the grating. These diffraction maxima satisfy the grating condition :

$$(a + b) \times \sin \theta_n = n\lambda, \quad \dots(1)$$

where $(a+b)$ = the grating element ($=2.54/N$, N being the number of rulings per inch of the grating),

θ_n = the angle of diffraction of the n^{th} maximum

n = the order of spectrum which can take values $0, \pm 1, \pm 2, \pm 3, \dots$

λ = the wavelength of the incident light

Clearly, the diffraction is symmetrical about $\theta_0 = 0$. If the incident beam contains different colours of light, there will be different θ_n corresponding to different λ in the same order n . By measuring θ_n and knowing N , λ can be calculated.

Chromatic resolving power of a grating is defined as its power of distinguishing two nearby spectral lines and is defined as

$$\text{Chromatic } R.P = \frac{\lambda}{\Delta\lambda} \quad \dots(2)$$

Where $\Delta\lambda$ is the separation of two wavelengths which the grating can resolve; the smaller the value of $\Delta\lambda$, the larger the resolving power. Employing Rayleigh's criterion for the limit of resolution, one can show in the case of a grating

$$R.P = \frac{\lambda}{\Delta\lambda} = nN. \quad \dots \quad (3)$$

The angular dispersion or dispersive power of a grating is defined as the rate of change of angle of diffraction with the change of wavelength in a particular order of the spectrum. Differentiating eqn. (1) with respect to λ , we get

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}. \quad \dots \quad (4)$$

Eqn.(4) shows that for a given small wavelength difference $\Delta\lambda$ the angular separation $\Delta\theta$ is directly proportional to the order n . When θ is small (less than 6°), $\cos\theta$ is constant and hence $\Delta\theta$ is proportional to $\Delta\lambda$. Such a spectrum is called a normal spectrum.

Procedure

(a) Adjustment of the Collimator and the Telescope :

1. Level the prism table, telescope and collimator with spirit level such that telescope axis and collimator, axis intersect the principal vertical axis of the spectrometer. A prism may be used for this purpose.
2. Focus the eye-piece of the telescope on the cross-wire by drawing it in or out of the telescope tube until the cross-wire is seen clearly.
3. **Use Schuster's method for focusing telescope and collimator for parallel rays [see Page no. 117, topic (v)]**

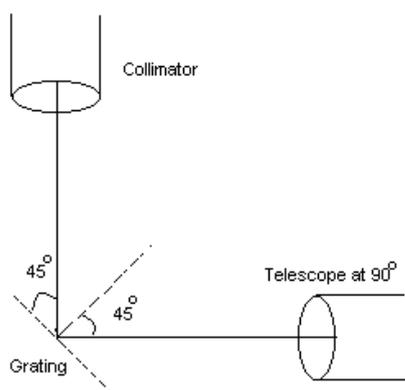


Fig. 1 Normal position of the diffraction grating

(b) Adjustment of the Grating :

- The grating is to be adjusted on the prism table such that light from the collimator falls normally on it. For achieving this :
4. First the collimator and the telescope are brought in one line and the image of the slit is focused on the vertical cross-wire. The corresponding reading on both the verniers is noted.
 5. The telescope is rotated through 90° .

6. Mount the grating on the prism table and rotate the prism table so that the reflected image is seen on the vertical cross-wire in the telescope. Take the vernier readings.
7. Turn the prism table from this position through 45° or 135° , so that 'writing' on the grating is away from the collimator. In this position, the grating is normal to the incident beam (see Fig.1).
8. The slit is rotated in its place till the spectral lines are very sharp and bright. This brings the slit parallel to the lines of grating.

(c) ***Measuring the Diffraction Angles :***

The spectrum is shown in Fig. 2.

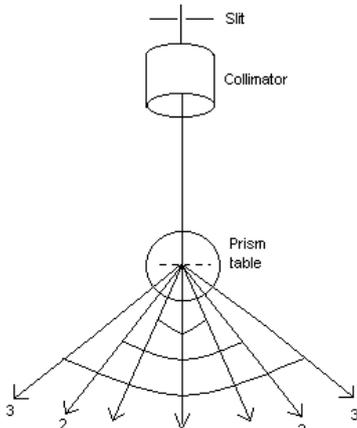


Fig. 2 The spectrum by diffraction grating

9. Rotate the telescope to the left side of the direct image and adjust it on different spectral lines (starting with first order blue line and finishing with second order yellow line) turn by turn. It should be taken care that the movement of telescope is in one direction.

10. Note the vernier readings V_1 and V_2 .

11. Now rotate the telescope to the right side of the direct image and repeat steps 9 and 10. The difference of corresponding vernier readings will give twice the angle of diffraction.

12. Find the angles of diffraction for prominent lines in the first and the second order spectra.

Observations

Vernier constant of the spectrometer (Least Count) :

Number of lines per inch of the grating (N) :

$$\text{Grating element } (a+b) = \frac{2.54}{N} \text{ cm}$$

Table 1

To set the unruled surface of the grating for normal incidence

Direct reading of the telescope without grating			Telescope is rotated through 90° and set at angle	Reading of the prism table when the angle of incidence is 45°			Prism table is rotated through 45° or 135° and set at angle
Main Scale (M)	Vernier (V)	Total (T=M+V)		Main Scale (M)	Vernier (V)	Total (T=M+V)	

Table 2
Determination of the angles of diffraction for the lines of different colour and order

Order No. (<i>n</i>)	Colour of the line	Vernier No.	Readings for the diffracted images with the telescope at the					Difference between the left and right readings of vernier (2θ)	Mean (2θ) $(\frac{a+b}{2})$	Angle of Diffraction (θ)	
			Left			Right					
			Main scale (M)	Vernier (V)	Total ($T=M+V$)	Main scale (M)	Vernier (V)				Total ($T=M+V$)
1	Blue	1st			(= a_1)			(= a_2)	($a_1 \sim a_2$) (=a)		
		2 nd			(= b_1)			(= b_2)	($b_1 \sim b_2$) (=b)		
2	Blue	1st			(= a_1)			(= a_2)	($a_1 \sim a_2$) (=a)		
		2 nd			(= b_1)			(= b_2)	($b_1 \sim b_2$) (=b)		

Diffraction Grating

Order No. (<i>n</i>)	Colour of the line	Vernier No.	Readings for the diffracted images with the telescope at the				Difference between the left and right readings of vernier (2θ)	Mean (2θ) $(\frac{a+b}{2})$	Angle of Diffraction (θ)
			Left		Right				
			Main scale (M)	Vernier (V)	Total ($T=M+V$)	Main scale (M)			
1	Green	1st			(= a_1)			(= a_2)	($a_1 \sim a_2$) (=a)
		2 nd			(= b_1)			(= b_2)	($b_1 \sim b_2$) (=b)
2	Green	1st			(= a_1)			(= a_2)	($a_1 \sim a_2$) (=a)
		2 nd			(= b_1)			(= b_2)	($b_1 \sim b_2$) (=b)

Diffraction Grating

Order No. (<i>n</i>)	Colour of the line	Vernier No.	Readings for the diffracted images with the telescope at the					Difference between the left and right readings of vernier (2θ)	Mean (2θ) $(\frac{a+b}{2})$	Angle of Diffraction (θ)	
			Left			Right					
			Main scale (M)	Vernier (V)	Total (T=M+V)	Main scale (M)	Vernier (V)				Total (T=M+V)
1	Yellow	1st			(= a_1)			(= a_2)	($a_1 - a_2$) (=a)		
		2nd			(= b_1)			(= b_2)	($b_1 - b_2$) (=b)		
2	Yellow	1st			(= a_1)			(= a_2)	($a_1 - a_2$) (=a)		
		2nd			(= b_1)			(= b_2)	($b_1 - b_2$) (=b)		

Calculation and Results

Table 3
Determination of wavelength of unknown lines

No. of lines per cm of the grating surface (N) (given)	Colour of the line	Order no. (n)	Angle of diffraction (θ) (From Table 2)	Wavelength of the spectral line (λ) (\AA)	Mean λ (\AA)
	Blue	1			
		2			
	Green	1			
		2			
	Yellow	1			
		2			

Table 4

Determination of Resolving power and dispersive power of the grating

Colour of the line	Order No. (n)	Angle of diffraction θ (from Table 2)	No. of grating lines illuminated by the collimator (N)	Resolving power of the grating = nN	Angular dispersion of the grating = $\frac{nN}{2.54 \cos \theta}$

Error calculation

The wavelength of unknown spectral line is determined from the relation:

$$\lambda = \frac{\sin\theta}{nN}$$

Therefore, the maximum proportional error in the determination of λ is

$$\frac{\delta\lambda}{\lambda} = \frac{\cos\theta}{\sin\theta} \delta\theta$$
$$\frac{\delta\lambda}{\lambda} = \frac{\delta\theta}{\tan\theta} \quad \dots(A)$$

2θ measured from the difference between two readings corresponding to two positions of the telescope. Hence $\delta\theta$ is equal to the value of one vernier constant (in radian). Substituting the measured values of θ and the value of $\delta\theta$ in eqn. (A) and multiplying by 100, the maximum percentage error in λ can be calculated.

Discussion

- (i) When mounting the grating on the prism table, if the ruled surface of the grating is towards the collimator, two images are viewed in the telescope placed with its axis normal to that of the collimator. The two images are formed by reflection at the front and back surfaces of the grating. In this case, work is to be done with the front surface image. To distinguish between the front and back surface images, an electrical lamp is to be placed behind the sodium flame. Both the monochromatic sodium light and the white light are incident on the grating. The image formed by reflection of the white light from the back surface of the grating will be coloured. This image is ignored and the adjustments for making the plane of the grating vertical are to be done with the other image.
- (ii) When the ruled surface of the grating is in the side of the collimator, the prism table is to be rotated through 45° in the proper direction to make the unruled surface of grating normal to the rays from the collimator. Also, it should be placed on the prism table so as to get the maximum area of the surface exposed to the incident light.
- (iii) The slit should be made very narrow to increase the brightness of the higher order diffracted images.
- (iv) The source position should be so adjusted as to make the diffracted images on both sides of the central one equally bright.
- (v) If necessary, the slit illumination can be increased by forming an image of the source on the slit by inserting a convex lens of short focal length between the slit and the source.
- (vi) While rotating the telescope, it should be moved always in the same direction so as to avoid any back-lash error.

Questions

1. In this experiment, how does diffraction occur?
2. What is a plane transmission diffraction grating?
3. What is a reflection grating?
4. How are commercial gratings made?
5. What type of grating do you use for your experiment?
6. Define grating element and corresponding points.
7. What is the effect of increasing the number of lines per cm on the grating?
8. What do you understand by the angular dispersive power of the grating?
9. How does the angular dispersive power of the grating vary with (i) the order number n of the spectrum, (ii) the grating element or the number of lines per cm in the grating, and (iii) the wavelength λ ?
10. Distinguish between a grating spectrum and a prismatic spectrum.
11. What will happen if the slit is illuminated with white light?
12. What will happen if the rulings of the grating are not parallel and the distance between two consecutive rulings is not constant?
13. What is the SI unit of wavelength?
14. What happens if the ruled surface of the grating faces the collimator?

15. What do you mean by the resolving power of a grating?
16. How can you experimentally verify that the incident rays are normal to the grating surface?
17. What are the uses of a diffraction grating?

References

1. Fundamental of Optics by F. Jenkins and H. White 535 JEN/F
2. Optics by A.Ghatak 535 GHA/O
3. Optics by E. Hecht 535 HEC/O